Evaluating Power Quality through the Sample Entropy Algorithm

S. Valencia Ramírez, J. H. Estrada, E. A. Cano – Plata

Abstract—This paper presents a brief summary of previous work, where a new method to measure the power quality through electromagnetic fields radiated by distribution systems, was proposed. It has been shown that is possible to measure the wave form irregularity of electrical signals radiated by electrical system by means of the sample entropy algorithm. This work shows the behavior of entropy when it is used to evaluate the phenomena that deteriorate the power quality. Distortions, transformers aging and overloading are detected by means of the entropy concept.

Index Terms—Entropy, electromagnetic fields, power quality, distortions, irregularity

I. INTRODUCTION

In electrical engineering, the study of interruptions and wave disturbances in the electrical supply creates the concept of power quality. This approach has gained increasing interest in power distribution companies. Phenomena such as harmonics, swells, sags, flicker, etc. are analyzed through voltage and current signals and currently the power systems are permanently monitored by data acquisition systems. The different electrical disturbances affect elements of the distribution system, specially the electric transformer [1].

The electrical system monitoring is fundamental to guarantee a reliable and quality supply. Currently there exist different kind of equipment, which are able to obtain power quality indexes by invasive or non-invasive methods, which means, some of them require to be connected to the system and some of them does not [2][3].

The voltage and current measurements are done through direct interventions with invasive equipment on electrical installations for detecting incipient faults in the networks elements (transformers, generators, filters, loads). This kind of measurements could be highly expensive if it is needed to evaluate equipment that is found far away from load centers of electrical system. On the other hand, the non-invasive methods can be better candidates for evaluating an electrical system in a fast and practical way, the acoustic emission and the infrared method are two of them.

These last are a good solution for the detection of possible power quality issues through partial monitoring of electrical and mechanics signals produced by the system. The signals are read by sensors located nearby to the elements that are part of the network, making them safe and practical to use [4][5].

Estrada et al [6] have been developing a new method that allows to obtain a power quality index. This index is taken from the electromagnetic waves radiated by lines and transformers, as a non-invasive method. Estrada et al demonstrated that the entropy of magnetic and electrical signals, rises when the respective wave suffers deformations owing to distortions produced by harmonics. The previous works have focused in taking measurements from power and distribution systems. The present work goes further in the study of the entropy behavior on several signals simulated under disturbances of periodical type.

II. ENTROPY

Entropy is a concept that has been applied in different disciplines, basically it measures the tendency of a system toward the chaos, (destruction, disorder, irregularity, insolvent systems) [7].

A. Sample Entropy – SampEn

Richman and Moorman [8] took the studies made by Pincus [9] and proposed the Sample Entropy as an algorithm for evaluating the complexity of chaotic systems, which is based on a fast calculation of the variability from non-lineal time series. The SampEn algorithm measures time series samples probability of being located at a certain distance.

The Sample Entropy of a signal is obtained in the following way [10]:

1. From a time series \( X = \{x_1, \ldots, x_N\} \) of length \( N \) the following vectors are obtained
Let $X = \{x_1, \ldots, x_N\}$ then it is constructed a new time series $y^j(\tau)$ whose values are averages of the consecutive elements of $X$ without any overlaps, where $\tau$ is the scale factor. Each element generated from the time series is calculated through the equation:

$$y^j = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_i, \quad 1 \leq j \leq \frac{N}{\tau} \quad (5)$$

Where $\tau = 1$ corresponds to $y(1)$ and this is the original time series. Each series’ length is the $X$ length divided by the $\tau$ factor. This operation is appreciated in Figure 1.

![Figure 1](image)

**Fig. 1 Upper scales calculation from a time series.**

Finally, each new time series represents a new scale function of the $\tau$ factor, which will be processed by SampEn. This procedure is called Multi-scales entropy.

It is important not to forget that all the calculations depend on the parameters $m$ and $r$, that are chosen as well as the data sample frequency. Theses parameter should not be changed for the evaluation of a study case because the results entropy will be biased.

A important feature of the entropy is that their values are not affected by the amplitude of the signal, the SampEn only evaluates the wave form that the signal analyzed has along its series [10]. This means that a signal of 220V and another signal of 440V, which have the same level of harmonic distortion, will have the same entropy value.

### III. Entropy Measurements

#### A. SampEn

It is possible to evaluate the irregularity that an electrical wave has, through the SampEn algorithm, the greater the irregularity, the higher the entropy. In order to appreciate what was said before, four signals, which are described in Table I, were constructed using Matlab, with a frequency of 60Hz and a sample rate of 400 Hz, then the signals are processed by the Sample Entropy algorithm.

\[ U_m(i) = \{x_i, x_{i+1}, \ldots, x_{i+m-1}\}, \quad 1 \leq i \leq N - m \]

2. $n^m_i(r)$ is defined as the number of vectors $U_m(j)$ that are close to the vector $U_m(i)$. The distance that separates the vectors is calculated as the maximum absolute value of the difference between vector’s components.

\[ d[x(i), x(j)] = \max_{k=1, \ldots, m}[|U(i) - U(j)|] \quad (1) \]

3. Then $C^m_r(i)$ is calculated as the regularity of any vector $U_m(j)$ to be close the vector $U_m(i)$ within a $r$ tolerance.

\[ C^m_r(i) = \frac{n^m_i}{N - m + 1} \quad (2) \]

4. It is considered the natural logarithm of each $C^m_r(i)$ and then it is averaged for $i$.

\[ \Phi^m_r = \frac{1}{N - m} \sum_{i=1}^{N-m} \ln C^m_r(i) \quad (3) \]

5. It is increased the dimension to $m+1$ and the steps from 1 to 4 are repeated.

6. So, the SampEn is calculated as:

\[ \text{SampEn}(m, r) = \lim_{N \to \infty} - \ln \frac{\Phi^{m+1}_r}{\Phi^m_r} \quad (4) \]

The SampEn estimated value depends on $m$ and $r$. In this work $m = 2$ and $r$ is estimated as $0.2SD_x$, where $SD_x$ is the standard deviation of the original data sequence, $X$. This values are suggested by Pincus [9] and are validated by Richman[8] and Madalena [10].

#### B. Multi-scale Entropy – MSE

Multi-Scale entropy MSE is a tool that can evaluate the time series in upper scales. The MSE proposed by Madalena Costa et al [10] has been widely used in studies of physiological signals in which it allows to recognize between healthy and unhealthy organisms. One of the most important features of the MSE is that it can differentiate different kind of signals, as for instance, the pink noise that is present in a lot of phenomena such as earthquakes, stock market behavior, distribution of mountains, etc. [11][12][13][14]. Madalena proposed to consider the structure and organization of the time series in upper scales.
Table II shows the results of the entropy value obtained for each of the signals. Signal 1 is an ideal sinusoidal wave; its SampEn value is smaller than other signals. Signal 2 has a higher irregularity, this is due to the 3rd harmonic added to the wave, which causes an increase in the entropy value. The other signals even are more irregular than the first one, which causes higher entropy values.

<table>
<thead>
<tr>
<th>Time series</th>
<th>Harmonics</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal 1</td>
<td>0</td>
<td>sin(wt)</td>
</tr>
<tr>
<td>Signal 2</td>
<td>$3^{rd}$</td>
<td>$0.5 \sin(wt) + 0.5 \sin(3wt)$</td>
</tr>
<tr>
<td>Signal 3</td>
<td>$3^{rd} + 5^{th} + 7^{th}$</td>
<td>$0.5 \sin(wt) + 0.5 \sin(3wt) + 0.5 \sin(5wt) + 0.5 \sin(7wt)$</td>
</tr>
<tr>
<td>Signal 4</td>
<td>Gaussian White noise</td>
<td>$\sin(wt) + N$</td>
</tr>
</tbody>
</table>

Figure 2 shows the constructed signals:

![Fig. 2 Graphic representation of the signals to be treated by entropy](image)

Table III shows the number of harmonics in each signal. The entropy results are shown in Figure 3.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal 1</td>
<td>0</td>
</tr>
<tr>
<td>signal 2</td>
<td>$2^{nd}$</td>
</tr>
<tr>
<td>signal 3</td>
<td>$2^{nd} + 3^{rd}$</td>
</tr>
<tr>
<td>signal 4</td>
<td>$2^{nd} + 3^{rd} + 4^{th}$</td>
</tr>
<tr>
<td>signal 5</td>
<td>$2^{nd} + 3^{rd} + 4^{th} + 5^{th}$</td>
</tr>
<tr>
<td>signal 6</td>
<td>$2^{nd} + 3^{rd} + 4^{th} + 5^{th} + 6^{th}$</td>
</tr>
<tr>
<td>signal 7</td>
<td>$2^{nd} + 3^{rd} + 4^{th} + 5^{th} + 6^{th} + 7^{th}$</td>
</tr>
</tbody>
</table>

B. MSE

In order to observe the entropy behavior when it comes to the evaluation of power quality phenomena, this work just tried with steady state disturbances: Harmonics, flicker and notching. This is the case because the entropy evaluates the irregularity that the signal has along the time series. All the signals were computed and evaluated through the MSE algorithm (This algorithm calculates 20 upper scales).

1) Harmonics

In order to evaluate harmonics disturbances through the entropy, seven signals were generated. Figure 3 shows the signals constructed, signal 1 corresponds to an ideal sinusoidal wave, and the rest of them were constructed by adding a new harmonic component, but conserving the disturbances that were present in the previous signal. See table III. Figure 3 shows how the signal is gets distorted as harmonic components are increased.

![Fig. 3 Graphic representation of the signals with harmonic distortion to be treated by entropy](image)

Fig. 3 Graphic representation of the signals with harmonic distortion to be treated by entropy

These seven signals were processed by means of the SampEn algorithm, with the 20 upper scales from the time series of each signal; the MSE curves are shown in Figure 4. The results indicate that the more complex the signal, meaning, the more distorted it is a by harmonic components, the higher the entropy value.

![Fig. 4 Multi-scale entropy curves, obtained for the signals described in Table III](image)

The MSE results can vary from a signal to another, this is due to the scaling change where the samples that make the signal are averaged and the signal suffers a new discretization of its temporal series. This discretization can produce different behaviors to the original MSE curve. To obtain a result that integrates all MSE values, the 20 scales of each signal are averaged, see Figure 5.
Fig. 5 Entropy increases directly to the injection of harmonic components to the fundamental 60 Hz wave.

Observing the MSE average curves in Figure 5 it can be seen a reaction of the entropy when the signal is more and more complex, due to the increase of harmonics in the signal.

2) Flicker

The voltage fluctuations are another periodic phenomenon that alters the wave form in electrical systems. This disturbance has two variables that produce a higher complexity wave-form, which are its amplitude and its frequency variation. These variations produce periodical changes in the voltage magnitude. In the same way of that the detection of harmonics irregularities is done, the MSE can be used to detect Flicker irregularities.

Two scenarios are analyzed:

1. Flicker with variations in voltage magnitude and fixed frequency of 10Hz.
2. Flicker with variations in frequency and fixed voltage drops up to 50% of the original signal amplitude.

For each scenario the signals were constructed with a frequency of 60 Hz, sample rate of 24.000Hz, and a total number of cycles to be evaluated equal to 30.

Scenario 1: For this simulation, 6 waves were constructed with different variation voltages caused by flicker and a fixed frequency of 10 HZ for each case; the first signal corresponds to a sinusoidal wave without flicker. A brief representation of these signals is shown in Figure 6, and the results are presented in Table IV.

![Signal without Flicker](Image)

![Signal with Flicker and drop voltage of 25%](Image)

![Signal with Flicker and drop voltage of 50%](Image)

Fig. 6 Constructed signals with variations due to Flicker

<table>
<thead>
<tr>
<th>Signal</th>
<th>Amplitude deviation [%]</th>
<th>MSE average [Nats]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.1813</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.1816</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.1837</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.1913</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>0.2515</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>0.3365</td>
</tr>
</tbody>
</table>

A similar result to the one obtained for the case of harmonic distortion is given, as soon as the wave is affected by the variation in the voltage amplitude, an increase in the signal entropy value starts to develop, the greater the signal irregularity, the higher the entropy value. As it is shown in Figure 7, the signal with 50% of deviation from the peak value, has a more chaotic behavior when compared with other signals. The more distant is the signal from the ideal case, the higher the entropy value.

![MSE](Image)

Fig. 7 Average MSE for each signal VS. drop level caused by flicker

Scenario 2: In this scenario the MSE behavior is observed when the flicker frequency is varied, sustaining a fixed drop voltage of 5% for each simulation case. The chosen frequency values and resultant entropy are shown in Table V.

![Signal with different frequency by flicker and MSE results](Image)

<table>
<thead>
<tr>
<th>Signal</th>
<th>Frequency [Hz]</th>
<th>MSE average [Nats]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.1837</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.2249</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.3371</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.3352</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.3365</td>
</tr>
</tbody>
</table>

The signal with a 0 Hz frequency represents the wave with the lowest entropy; this can be appreciated in the Figure 8. The next signal in order of irregularity is the wave with a flicker of 0.5Hz; this is due to the variation of the cycles along the time series. An interesting fact is that for frequencies higher than 2 Hz, the MSE values are similar between each other.
3) Notching
Another periodical phenomenon is the voltage notching, which similar to harmonic and flicker phenomena, affects the power quality with periodical distortions in the wave-form. In order to simulate the notching phenomenon it was decided to construct eight signals with different percentage in the notches depth, which are defined in Table VI. Every case has a notch occurrence with a frequency of 120 Hz.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Notch Depth (%)</th>
<th>MSE average [Nats]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.1813</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.1815</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.1819</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>0.1833</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>0.1853</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>0.1882</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>0.1916</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Likewise as the above cases, it is confirmed the fact that whatever the irregularity present in the wave-form, it will be detected by the entropy value and the higher the levels of distortion, the higher entropy values. As it is shown in Figure 9, the signal with the highest entropy value is the wave with a notch depth of 70%.

So far, it has been observed that the entropy is a parameter that varies in a direct relationship with signal irregularity. Previous works [6], [15], developed by the authors of this paper have measured the electromagnetic fields from industrial and dwelling circuits. The industrial circuit entropy was higher than the entropy value of dwelling circuit. This is caused due to the irregularities in the wave-form due to the high quantity of non-lineal loads connected to the industrial network, producing larger disturbances when compared to the dwelling network.

Applying the same techniques, Estrada et al. explored the transformer behavior when aging effects affect it. Figure 10 proposes that electrical stress causes the entropy of magnetic radiated waves to decrease [15].

Up to now, these works present an opportunity for development of a new method to measure the electric system conditions from the point of view of harmonic distortion and aging of transformers through the analysis of the radiated electromagnetic signals. It is possible to estimate power quality issues that the electrical current signal can present and to determinate it by means of magnetic field measurements, in the same way it is possible to estimate the voltage signal distortion through the electric field measurements. The fields are treated separately because the distance that exists between the observer and the emission point is less than a wave length [16].

IV. CONCLUSIONS
The entropy is a strong candidate to be a new indicator of the power quality from the viewpoint of signal irregularity. Distortions like harmonics, flicker and notching contribute to deflect the signal from its ideal sinusoidal shape. When the wave loses its ideal form, the regularity of the time series is affected, which leads to injurious wave shapes. These irregularities can be detected setting an ideal value of entropy. If the irregularities appear in the signal, the entropy will do the same.

The great advantage of the Sample Entropy method is that It
does not require knowing the signal amplitude, it is only needed to know the signal waveform. This is a motivation to continue developing this methodology in order to achieve a mean of obtaining electrical signals through noninvasive tools, which would permit to evaluate the irregularity of the signal through of radiated signals of the circuit.

In order to find initiatives for the development of new ways to measure the power quality and to detect incipient faults, researchers have been done, which have consisted in field measurements through non-invasive methods. Previous works carried out by the authors of this paper have focused in studies about radiated electromagnetic signals. It has been determined that entropy is a tool that measures the waveform irregularity promoted by phenomena that deteriorate it.

REFERENCE


