Identification and Characterization of Loads using Voltage, Current and Power Characteristics


Abstract—This paper presents a graphical way to identify and characterize loads using the α-β-0 frame and the generalized power tensor theory in three phase three-wire or four-wire systems. The α-β-0 frame is based on the Clarke's Transform and it takes three phase voltage or current signals, and leads it to a convenient space vector, spreading zero-sequence component of the alpha and beta components that describe the three phase signals. Tensor theory is based on the dyadic or tensor product between voltage and current instantaneous vectors; this definition allows us to represent the phenomena of power quality produced by the load operation, through the deformation of a cube and the trajectory of one of its characteristic vectors. These new ways for identification and characterization could help researchers to construct better models of three phase loads. Results are reached by implementing simulations in MATLAB® and experimental implementation.

Index terms — Power quality, Vector trajectory, Power tensor theory, Patterns, Voltage-current characteristic, Loads identification, Loads characterization.

I. INTRODUCTION

Identification and characterization of loads has been a useful tool for several applications like, dimensioning of facilities, loads modeling, power conservation and energy efficiency [1], diagnosis of machines [2] [3], etc. Modeling is the most important part in the simulation analysis; also a good identification of loads is a useful input for network operation and planning [4]. Some methods of identification and characterization of loads use data analysis [5], or voltage-current characteristic [6] [7]; but most of such methods only apply for single phase loads; those that involve three phase systems don’t involve voltage and current at the same time [8]. This paper proposes a way for identification and characterization of loads, using 3-dimentional trajectories for three phase voltages, three phase currents and three phase power, instead of single phase voltages or currents diagrams, using the α-β-0 frame and the generalized power tensor theory.

II. BACKGROUND OF OTHER METHODS

A. Voltage – Current Characteristic

Traditionally, for single phase loads, the voltage-current characteristic or voltage vs. current diagram, has been the most popular method for identification and characterization of loads [1] [6] [7]; even on three phase systems, it is still use, graphing the V-I characteristic for each phase. In Figure 1 it can be seen three typical single phase V-I characteristics corresponding to (a) resistive load, (b) an inductive load, and (c) a inductive non-linear load. Note that the pure resistive load draws a 45º line.

This is not a suitable approach for identify three phase loads, because is not possible to see the whole three phase effect caused by the operation of loads. Also, is hard to determine which harmonic is imposing the load to the network.

B. Alpha – Beta Characteristic or Concordia Patterns

Due to the need to identify and characterize three phase loads on a single diagram, for a better understanding and analysis, some authors have proposed the α-β frame [8], this approach uses the Clarke's Transform for α-β components or Concordia Transform, and their paths to draw the three phase voltage or current vector characteristic.

The Concordia Transform is based on the equation (1.1), derived from the Clarke's Transform, it takes the
measurements of a three phase system and leads it to a two
axes coordinate frame of reference (α-β frame), allowing to
obtain the two-dimensional patterns useful for loads
identification and characterization.

\[
i_{\alpha} = \frac{\sqrt{3}}{2} i_a - \frac{1}{\sqrt{6}} i_b - \frac{1}{\sqrt{6}} i_c
\]
\[
i_{\beta} = \frac{\sqrt{2}}{2} i_b - \frac{1}{\sqrt{2}} i_c
\]  

(1.1)

Figure 2 (a) shows current patterns of a three phase load
without unbalances, neither harmonics nor reactive power;
corresponding to the signals stated in equation (1.2). This
pattern could be taken as reference or ideal condition. Figure 2
(b) shows a load that causes three phase current with 5th
harmonic.

\[
i_a = I_m \sin(\omega t)
\]
\[
i_b = I_m \sin(\omega t - 2\pi / 3)
\]
\[
i_c = I_m \sin(\omega t + 2\pi / 3)
\]  

(1.2)

A more recent approach, based on the tensor theory has been
published in [9], it uses the dyadic product of the voltage and
current instantaneous vectors, to get the Power Tensor, it
allows us to know all the phenomena of power quality just
analyzing the different characteristic values of the formed
matrix, or measuring the deformations of a cube. The equation
(1.3) defines the power tensor.

\[\varphi_{ij} = v_i \otimes i_j\]  

(1.3)

Then, for a three phase system, the power tensor can be
written as is stated in equation (1.4).

\[
\varphi_{ij} = \begin{bmatrix} v_a & i_a \\ v_b & i_b \\ v_c & i_c \end{bmatrix} \begin{bmatrix} i_a & i_a \\ i_b & i_b \\ i_c & i_c \end{bmatrix} = \begin{bmatrix} v_a i_a & v_a i_b & v_a i_c \\ v_b i_a & v_b i_b & v_b i_c \\ v_c i_a & v_c i_b & v_c i_c \end{bmatrix}
\]  

(1.4)

Rewriting equation (1.4) leads to equation (1.5).

\[
\varphi_{ij} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix} = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}
\]  

(1.5)

Where \(d_1, d_2, d_3\) are the column vectors that form the power
tensor. These vectors depicted the forces that shift each cube
face, having three director vectors given by the reference axes,
\(\hat{x}, \hat{y}, \hat{z}\) named \(e_1, e_2, e_3\) as is shown in Figure 3.

![Figure 3: Geometrical representation of the Power Tensor in R3](image)

A cube without deformation corresponds to a system without
power transference. The cube becomes deformed even with
ideal operation conditions, i.e. the matrix \(\varphi_{ij}\) is formed by
instantaneous vectors of voltage and current with positive
sequence, fundamental component and perfectly on phase.
Also, the instantaneous power tensor will produce additional
cube deformation according to the system characteristics, as is
described in [9]. In this approach, the cube is spinning at \(2\omega\)
with network fundamental frequency.

With the aim to get a spatial vector derived of the power
tensor, is desirable to reduce the order of the tensor. This
spatial vector allows us to describe the characteristic trajectory
in \(\mathbb{R}^3\) of the power tensor. Reduction of the tensor order is
described in [9], and leads to the expression stated in equation
(1.6), where the tensor characteristic equation is obtained.

\[
\lambda^3 (\lambda - a_{(1)}) = 0
\]  

(1.6)

Where:

\[
a_{(1)} = \varphi_{11} + \varphi_{22} + \varphi_{33} = v_a i_a + v_b i_b + v_c i_c
\]  

(1.7)

And is an eigenvalue of the power tensor. Solution of the
cubic equation (1.6) is:

\[
\lambda_{(1)} = a_{(1)}; \lambda_{(2)} = \lambda_{(3)} = 0
\]  

(1.8)
Figure 4, shows the trajectory corresponding to the projections generated, making the internal product between each instantaneous power tensor and their eigenvectors, as is stated in equation (1.9).

\[ y_i = \mathbf{S} y_x = \lambda_i x_i = a_i x_i \tag{1.9} \]

These projections (the end point of the first order tensor at each time) are named characteristic trajectories.

Figure 5, shows the characterization of a three phase load with 5th harmonic in current, here the number of bumps is equal to \( n+1 \), where \( n \) is the harmonic order.

III. ALPHABET–CERO CHARACTERISTIC TRAJECTORY

One of the proposed approaches is using the Clarke Transform or \( \alpha-\beta-0 \) frame; it allows us to get a 3-dimentional representation of a three phase voltage or current. The \( \alpha-\beta-0 \) Transform is applied, following the equations (1.10) and (1.11) for voltage and current respectively, where the voltage or current can be represented with the trajectory or pattern of the resultant \( \alpha-\beta-0 \) vector.

\[
\begin{bmatrix}
    v_0 \\
    v_\alpha \\
    v_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}}
\begin{bmatrix}
    \sqrt{2} \\
    1 \\
    0
\end{bmatrix}
\begin{bmatrix}
    v_\alpha \\
    v_\beta \\
    v_c
\end{bmatrix} \tag{1.10}
\]

\[
\begin{bmatrix}
    i_0 \\
    i_\alpha \\
    i_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}}
\begin{bmatrix}
    \sqrt{2} \\
    1 \\
    0
\end{bmatrix}
\begin{bmatrix}
    i_\alpha \\
    i_\beta \\
    i_c
\end{bmatrix} \tag{1.11}
\]

Figure 6 shows the resultant pattern or trajectory generated by the operation of an ideal three phase load.

Trajectories are drawing over a plane that is normal to the director cero axis, and parallel to \( \alpha-\beta \) directors axes, it allows us to see if the pattern crosses the plane; this situation is possible when a load draws an unbalanced current or voltage even having harmonics.

An example of an unbalanced load with 5th harmonic in current can be seen in Figure 7, note how the pattern crosses the plane.
Due the inclusion of zero axis, it can be possible to characterize loads that draw 3\textsuperscript{rd} harmonic; Figure 8, shows a comparison between the first order tensor trajectory and (α-β-0) vector trajectory. Note how the trajectory (in black color) cut the plane upward and downward.

IV. MODIFIED CHARACTERISTIC TRAJECTORY OF THE POWER TENSOR

The method depicted in chapter II. shows cube deformation with ideal operation conditions (sinusoidal balanced voltages and currents and perfectly on phase); also, draws a perfect cube having non-power transference. Thus, is necessary to define a non-deformed cube using vector algebra to get one that represents an ideal power transference conditions.

In Figure 9 is depicted the reference cube and the deformed cube by action of the tensors conditions for an ideal operation case, where the voltage and currents signals are generated either mathematically or from a phase locked loop. The reference cube is formed by director vectors \( e_1, e_2, e_3 \); these are the main edges or interior edges of the cube. The main edges of the deformed cube can be depicted as \( X_p, Y_p, Z_p \) which are the sum of vectors \( d_a, d_b, d_c \) and \( e_1, e_2, e_3 \) respectively.

Thereby, constructing the cube with unitary vectors comes to equation (1.12).

\[
\begin{align*}
    e_1 &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\
    e_2 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\
    e_3 &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{align*}
\]

Then, vectors \( d_a, d_b, d_c \) must be scaled, proportionally to the unitary cube dimensions, dividing them by the mean power as is depicted in equation (1.13)

\[
\begin{align*}
    X_p &= e_1 + d_a / P_{\text{mean}} \\
    Y_p &= e_2 + d_b / P_{\text{mean}} \\
    Z_p &= e_3 + d_c / P_{\text{mean}}
\end{align*}
\]

A. Modifier Vectors

Then, the modifier vectors \( d_1, d_2, d_3 \) can be gathered as is depicted in equation (1.14).

\[
\begin{align*}
    d_1 &= X_p - e_1 \\
    d_2 &= Y_p - e_2 \\
    d_3 &= Z_p - e_3
\end{align*}
\]

Vectors \( d_1, d_2, d_3 \) are used to modify or adjust the real power tensor, such that if the real power tensor corresponds to a load drawing balanced sinusoidal voltages and currents and perfectly on phase, this cube will be the same cube formed by director vectors \( e_1, e_2, e_3 \).

The complete procedure for a real three phase system can be seen as follows (\( V_{ar} \) is the real voltage at phase \( a \), and so on):

\[
\begin{bmatrix}
    \Phi_{ij} \\
    \Phi_{ij}
\end{bmatrix}
= \begin{bmatrix}
    v_{ar} \\
    v_{br}
\end{bmatrix} \otimes \begin{bmatrix}
    i_{ar} \\
    i_{br}
\end{bmatrix}
= \begin{bmatrix}
    \Phi_{11} & \Phi_{12} & \Phi_{13} \\
    \Phi_{21} & \Phi_{22} & \Phi_{23} \\
    \Phi_{31} & \Phi_{32} & \Phi_{33}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    d_{ar} \\
    d_{br}
\end{bmatrix}
= \begin{bmatrix}
    \Phi_{11} & \Phi_{12} & \Phi_{13} \\
    \Phi_{21} & \Phi_{22} & \Phi_{23} \\
    \Phi_{31} & \Phi_{32} & \Phi_{33}
\end{bmatrix}
\]
Applying the definition in (1.13), for the real system and adjusting it with the modifier vectors; the director vectors $X$, $Y$, $Z$, can be gathered as is depicted in equation (1.16).

$$X = \left( e_1 + d_{\text{mean}} \right) \, d_1$$
$$Y = \left( e_2 + d_{\text{mean}} \right) \, d_2$$
$$Z = \left( e_3 + d_{\text{mean}} \right) \, d_3$$

From the definition of the equation (1.16), and having a load that draws sinusoidal balanced voltages and currents signals at positive sequence and perfectly on phase, the resultant cube will be drawn as a perfect cube. Figure 10 shows the comparison between the original power tensor approach and the modified tensor approach.

B. Characteristic Vector Trajectory

In order to draw the 3-dimentional characteristic trajectory, and therefor to identify the effects of loads operation; is necessary to get a first order tensor or vector that can be affected by all the phenomena in the power characteristics imposed by the load. In this sense, the chosen vector is the opposite diagonal of the cube shown in Figure 12, and is described by the equation (1.17).

$$V = X + Y - Z$$

This vector only projects a point without following a trajectory having an ideal load operation; due to the non-deformed cube definition in IV. The ideal condition can be seen in Figure 13.

The trajectory drawn by the vector $V$ can be gathered with either single or a combination of multiple power issues caused by different loads.

On the other hand, if the load has a non-linear behavior, the resultant cube will be drawn with deformations as shown in Figure 11.

In Figure 14-16, several three phase loads have been characterized. Note in Figure 14, that this method can show zero sequence components, it means 3rd harmonic or unbalances, even higher order harmonics at the same time, like $\alpha-\beta-0$ vector trajectory does, considering that $\alpha-\beta-0$ can’t characterize three phase loads with 3rd harmonic.
Figure 14: Trajectory of a balanced three phase load with 3\textsuperscript{rd} harmonic in current.

Figure 15: Trajectory of a balanced three phase load with 5\textsuperscript{th} harmonic in current, all the bumps are the same size.

Figure 16: Trajectory of a balanced three phase load with more 5\textsuperscript{th} harmonic than 7\textsuperscript{th} in current.

Figure 17: Four typical loads with a combination of different power issues

In Figure 17, it can be seen the trajectory generated by four three phase loads (a) $pf = 0.86$ lagging, (b) $pf = 0.86$ leading, (c) $pf = 0.86$ lagging and unbalanced, (d) $df = 0.86$ (displacement factor) lagging with 5\textsuperscript{th} harmonic in current.

Also, is possible to characterize loads with a complex combination of power issues, this is done by looking at the whole 3-D view of the generated trajectory and comparing these trajectories with the known ones. Also is possible take note about the drawn 3-D patterns for make additional comparisons.
V. EXPERIMENTAL RESULTS

Two real industrial loads with several power issues have been characterized, to test the “modified trajectory of the instantaneous power tensor”. Measurements have been done with the AEMC Power Pad-3945, and processed with MATLAB®, using the proposed method.

Figure 18 shows the characterization of a central communication, measurements have been taken from the central’s PCC. This is a three phase load with several harmonics ($5^{th}$ predominantly), and with a lagging displacement factor. Note that the predominant harmonic order establish the number of bumps, which must be closed paths; in the right-down corner of the Figure 18, a zoom to the X-Y view is performed and it can be seen that the orientation of the whole trajectory is to the right, just like is shown in Figure 17 (b) for an inductive three phase load; the soft trace is due that the trajectory is crossing the plane, which means presence of a $3^{rd}$ harmonic.

![Figure 18: Characterization of a central communication with several harmonics and lagging displacement factor.](image)

Figure 19 shows the characterization of a three phase adjustable speed drive connected to a hoist. This load draws $5^{th}$ predominant harmonic, and also evidence of $3^{rd}$ harmonic due to the traces crossing the plane. In the right-down corner of the Figure 18, a zoom to the X-Y view was performed and it can be seen that $11^{th}$ harmonic and $0^{th}$ harmonic are just taken away the symmetry to the whole trajectory, but, five bumps, corresponding to the predominant harmonic order, can be count. Also, the reactive power of this load is overcompensated because there is evidence of leading displacement factor (the whole trajectory is slightly left oriented).

![Figure 19: Characterization of a Hoist adjustable speed driver.](image)

VI. CONCLUSIONS

- The characteristic trajectory $\alpha-\beta-0$ method adds additional capabilities to Concordia patterns method, since the zero axis handles the zero-sequence components and allows to see the operation of an unbalanced and non-linear load at same time.
- The modified characteristic trajectory of the power tensor can depict many phenomena of loads operation; odd harmonics are depicted more precisely with $n$ bumps as harmonic order.
- The modified characteristic trajectory of the power tensor is the only one that can depict $3^{rd}$ harmonic with all the physical meaning (3 bumps and cero sequence evidence), and can give information about harmonic content when a load imposes more than one harmonic, and state the dominant harmonic order. Also, can characterize reactive non-linear and unbalanced loads.
- Loads with more complex combination of power issues can be characterized with the modified characteristic trajectory of the instantaneous power tensor, even for a system with different loads having different characteristics on each phase.
VII. BIBLIOGRAPHY


