Effects of Lightning Channel Tortuosity on Generated Electric Fields

Juan Pulgarín, Camilo Younes, Diego del Río
Department of Electrical, Electronic and Computer Engineering
Universidad Nacional de Colombia
Manizales, Colombia
Email: jdpulgarinr@unal.edu.co

***Abstract*—In this paper it is presented a simulation analysis of electric fields generated by lightning channels with tortuosity. The tortuosity of the channel is reproduced by means of a numerical algorithm that takes into account statistical characterizations made by different authors observing real lightning channels, so the macroscale tortuosity of the simulated channel is similar to the one recorded in nature. Later it is used an analytic model which allows the calculation of the electromagnetic fields generated by lightning channels with tortuous geometries, once the geometry is known. The electromagnetic fields are calculated assuming a Heaviside step function as the current traveling along the channel. The objective of this work is to develop a methodology for the calculation of electromagnetic fields, considering a more realistic geometry for the lightning channel than that of a straight and vertical one, and to analyze some characteristics of the generated fields in terms of wave peak value and wave-form for different points around the lightning channel. This methodology is pretended to be used in a future work in order to determine the effect of the lightning channel tortuosity on the location accuracy of lightning locating systems.**

***Keywords*—lightning electromagnetic fields; lightning channel geometry; lightning locating systems.**

I. Introduction

Lightning electromagnetic fields modeling is an important task when they are considered the induced effects that can cause damage to electrical apparatus, electrical distribution networks and also for the design and evaluation performance of lightning locating systems. The majority of the models of today do not consider the geometry of the lightning channel when they try to reproduce real lightning electromagnetic fields, instead of that, they treat the lightning channel as straight and vertical to a flat ground.

However, some authors have analyzed the effect of the lightning channel geometry in the modeling of different stages. Some of the principal works have considered the characterization of tortuosity by means of laboratory experiments with short sparks [1], the lightning propagation modeling by means of simulation [2], the effect of the tortuousity on the induced voltages [3], the effect of the lightning channel tortuosity on the radiated electromagnetic fields [4]–[9] and also the effect of both, the tortuosity and branching on the electromagnetic fields [10]. The authors of the present work have been involved with a research aimed to model the lightning stepped leader stage, including the geometry of the channel, and as a result it has been developed a model, which is described in [11], [12]. This model enables among other things, the simulation of random channels with a macroscale tortuosity similar to the one reported for natural lightning observations [13]–[15].

When the lightning channel is represented as straight and vertical to a flat and perfectly conducting ground, the electromagnetic fields generated that way have the same shape all around the channel for the same distance from its base. In this paper the authors simulate a lightning channel with tortuosity, making use of the model for the generation of such channels described in [11], [12], and of the technique described in [4] for the calculation of electromagnetic fields produced by tortuous lightning channels. The objective of this work is to show the influence of the channel geometry on two parameters of the electromagnetic wave propagation that could cause some error in the time-of-arrival technique for lightning localization, namely: the shape of the electric field wave around the channel for points at the same distance from its base, and changes in the magnitude of the electric field wave when the azimuth of the observation point is changed but the distance from the channel base remains the same.

II. Channel Geometry Simulation

The model described in [11], [12] is a model for the stepped leader stage, including the tortuosity and branching of the channel. The model is able to generate random channels with tortuosity and with or without branches. For the generation of these random geometries, the model makes use of the data reported from real lightning observations.

The generation of random channel geometries is based on the statistical characterizations of natural lightning reported by Hill [13], [14] and by Idone and Orville [15]. Therefore, a simple procedure to reproduce channels with a macroscale tortuosity similar to the one recorded in nature was developed as follows. The z-axis was set as the main direction of lightning propagation toward the ground in agreement with Hill's observations, and the leader starts from the origin point in the cloud. Using spherical coordinates (Fig. 1), ϕ was randomly generated from a Gaussian distribution whose mean was 180°and the standard deviation was fitted in order to obtain a mean absolute value around 17°, in accordance with Hill's and Idone's findings. The coordinate θ was uniformly generated between 0°and 360°, which allows the channel to

VII Simposio Internacional sobre Calidad de la Energía Eléctrica 2013 SICEL
Noviembre 27,28 y 29
Medellín Colombia

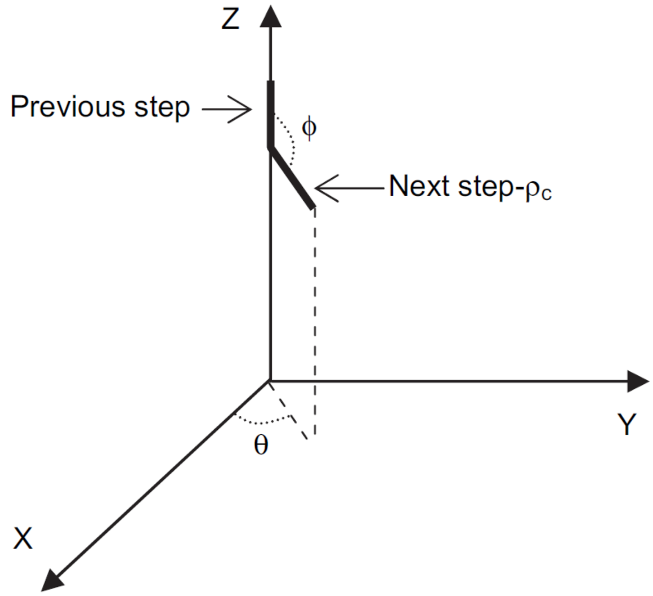


Figure 1. Geometry for the Generation of Random Tortuous Channels

extend in three dimensions. Finally, the step length is defined by the coordinate ρ_C; thus, this was generated on the basis of a uniform distribution in which the segment length lies between 50 and 100 m at the beginning, and it is gradually reduced to be between 10 and 30 m at ground level. This procedure was consecutively repeated up to the completion of the path to the ground.

The random channel obtained with this method is presented in Fig. 2, and is the geometry used for the simulation.

III. Electromagnetic Fields Calculation Methodology

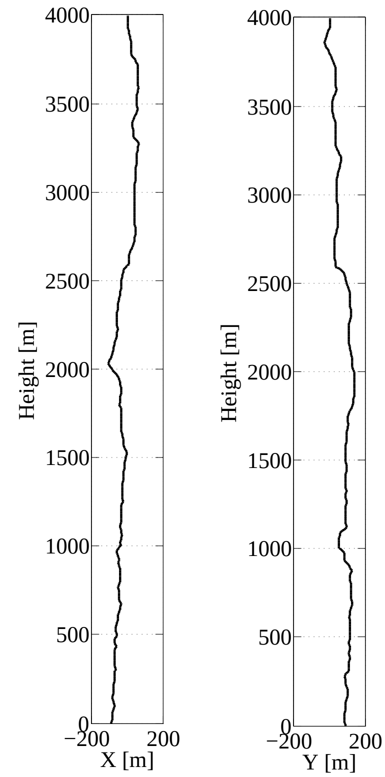
The methodology utilized in order to calculate the electromagnetic fields produced by lightning channels with tortuosity is the one described in [4]. That methodology is briefly described here, adding the modifications made for the present application.

A. Return Stroke Current Representation

This methodology assumes a Heaviside step function for the current traveling along the lightning channel, this choice is made in order to obtain analytic expressions for the fields. The mathematical expression for the current pulse is:

$$i(\lambda, t) = I_0 \left[u\left(t - \frac{\lambda}{v} \right) \right] \times \left[u(\lambda) - u(\lambda - h) \right] \qquad (1)$$

where u is the Heaviside step function, v is the speed of the current wave, I_0 is the amplitude of the current pulse, h is the length of the segment and λ is a new coordinate, which runs along the tortuous channel, changing its direction when it passes from one segment to another. More specifically λ is the path from the ground to a point in the channel, along the tortuous geometry and it is measured in meters.



(a) xz-view (b) yz-view

Figure 2. Random Tortuous Channel

Thus, the current is represented by a Heaviside step function which propagates along the channel without distortion and with a constant speed that causes a time delay for the currents at different heights.

This representation for the current is a reduced approximation to the real case, but it allows to obtain analytic expressions for the fields, this choice is also supported by the fact that what is under analysis here is the effect of the geometry on the generated electromagnetic fields, this work does not seek to develop a new model for either the electromagnetic fields or the return stroke current.

B. Coordinates Transformation

Once the return stroke current expression has been defined, it is necessary to define an adequate methodology to deal with the tortuosity of the lightning channel. The approach defined in [4] is the same used and summarized here.

In first place, the analytic expressions for the electromagnetic fields generated by the current function described above are obtained and presented in [4]. These equations were obtained for a straight and vertical channel, but they can be used to evaluate the electromagnetic fields of a tortuous channel by superimposing the contribution of every segment of the channel and by means of a proper coordinates transformation as described below.

The general methodology consists of making a translation of the coordinate system (x, y, z) to the point at which a segment

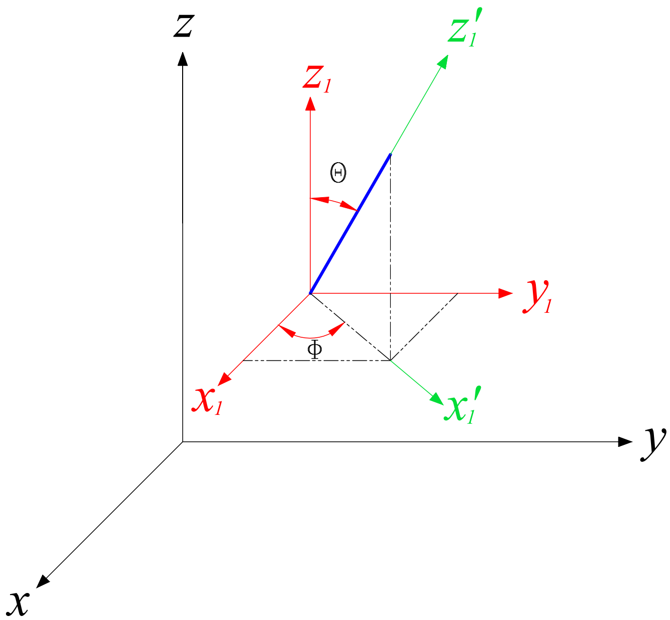


Figure 3. Geometry for the Rotation of the Coordinate System

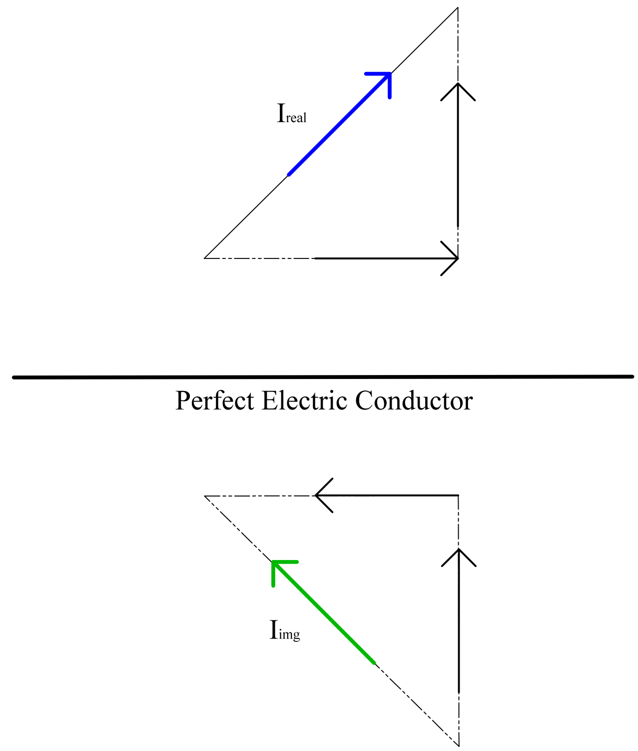


Figure 4. Method of Images for an Inclined Segment of Current

of the tortuous channel starts, resulting the coordinate system (x_1, y_1, z_1) as it can be seen in the Fig. 3, then the coordinate system is rotated by an angle θ on the plane formed by the constant angle ϕ, resulting the coordinate system (x_1', y_1', z_1').

Thus, the last coordinate system (the one consisting of the axes (x_1', y_1', z_1')), has its z_1' axis along the channel segment.

Finally, the coordinates of the observation point are calculated in the new coordinate system and then the electromagnetic fields for the segment can be evaluated by means of the equations developed in [4] for a straight channel.

This process is repeated for every segment forming the tortuous channel and the contribution of each is added to obtain the fields produced by the complete channel.

The electromagnetic fields calculated for each segment must be transformed properly to the fields in the original coordinate system before the addition of the fields produced by each segment. In order to achieve this task, they are used the relations between the units vectors in the coordinate systems (x_1, y_1, z_1) and (x_1', y_1', z_1') which are presented in (2).

$$A = \begin{bmatrix} \cos\theta\cos\phi^2 + \sin\phi^2 \\ \cos\theta\cos\phi\sin\phi - \cos\phi\sin\phi \\ -\sin\theta\cos\phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\theta\cos\phi\sin\phi - \cos\phi\sin\phi \\ \cos\phi\sin\phi^2 + \cos\phi^2 \\ -\sin\theta\sin\phi \end{bmatrix}$$

$$C = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \hat{i}_1 \\ \hat{j}_2 \\ \hat{k}_3 \end{bmatrix} = \begin{bmatrix} A & B & C \end{bmatrix} \begin{bmatrix} \hat{i}_1' \\ \hat{j}_2' \\ \hat{k}_3' \end{bmatrix} \quad (2)$$

By means of (2) it can be made the transformation between the two coordinate systems, using either the matrix formed by the three vectors A, B and C, or the inverse of this matrix.

C. Effect of Ground

In this work, it is assumed a perfectly conducing and flat ground, in order to be able to use the method of images with the proper considerations that have to be made for a segment with arbitrary location and slope.

In Fig. 4 the method of images for a segment of current with arbitrary location and slope over a perfectly conducting plane is depicted. In this figure it can be seen the direction of the image current for a particular segment, which has been deducted after decomposing the original current in its vertical and horizontal components.

IV. Results

By means of the methodology described, they were simulated the electromagnetic fields generated by a Heaviside current with a magnitude different than 1, traveling along the tortuous channel of Fig. 2. The objective of this paper is to analyze the effect of the tortuosity of the lightning channel on the electromagnetic fields and the possible effect that the tortuosity could have on the location accuracy of a lightning locating network operating by the time-of-arrival technique, so the figures here show only the electric field. It was simulated the total component of the electric field, although the method described in this paper is able to calculate any of the components of the electric or magnetic field.

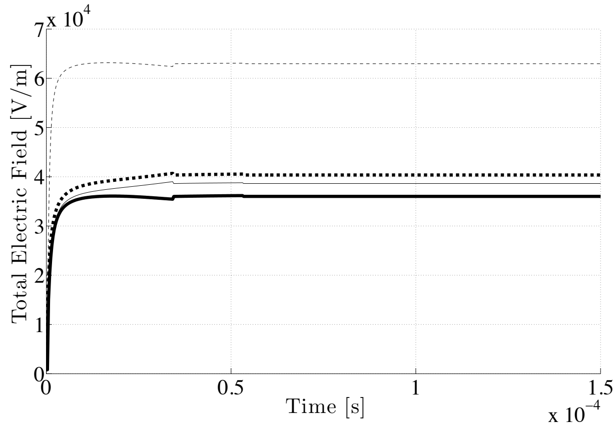


Figure 5. Total electric field at ground, for $r = 100$ m from the base of the channel. Solid thick line, $\phi = 0$; dashed slim line, $\phi = \pi/2$; solid slim line, $\phi = \pi$; dashed thick line, $\phi = 3\pi/2$

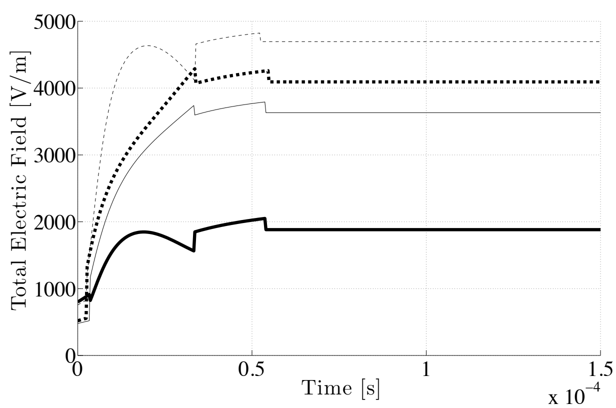


Figure 6. Total electric field at ground, for $r = 1000$ m from the base of the channel. Solid thick line, $\phi = 0$; dashed slim line, $\phi = \pi/2$; solid slim line, $\phi = \pi$; dashed thick line, $\phi = 3\pi/2$

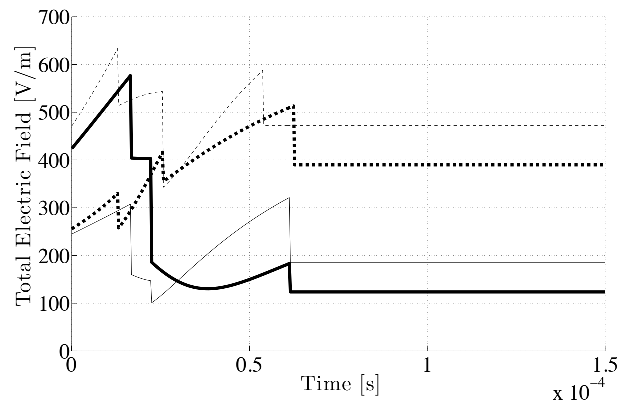


Figure 7. Total electric field at ground, for $r = 5000$ m from the base of the channel. Solid thick line, $\phi = 0$; dashed slim line, $\phi = \pi/2$; solid slim line, $\phi = \pi$; dashed thick line, $\phi = 3\pi/2$

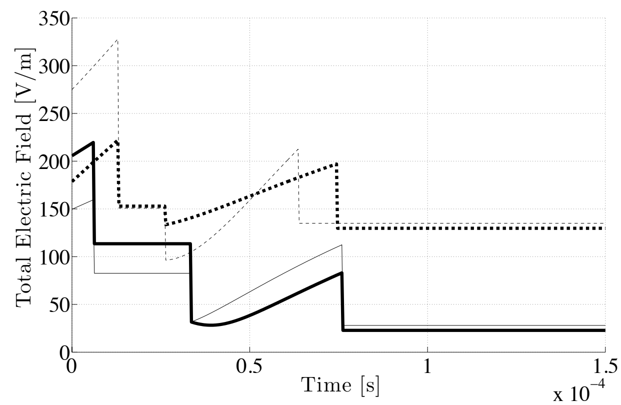


Figure 8. Total electric field at ground, for $r = 10000$ m from the base of the channel. Solid thick line, $\phi = 0$; dashed slim line, $\phi = \pi/2$; solid slim line, $\phi = \pi$; dashed thick line, $\phi = 3\pi/2$

The values of the parameters utilized during the simulation were: $I_0 = 30000$ [A], $v = c/3$ [m/s].

The observation points selected, using a cylindrical coordinate system are presented in 3.

$$\begin{aligned} r &= \{100, 1000, 5000, 10000\} \\ \phi &= \{0, \pi/2, \pi, 3\pi/2\} \\ z &= \{0\} \end{aligned} \tag{3}$$

As it can be seen in the results shown from Fig. 5 to Fig. 8, the total electric field generated by a tortuous lightning channel is not uniform for points at the same distance from the base of the lightning channel and with a different azimuth angle around it.

In general, they can be observed two important effects, first, the shape of the electric field wave changes for points at the same distance but different azimuth angle, presenting a more notorious effect for the points that are more distant from the base of the channel. In second place, the magnitudes of the electric field wave present also a change, when one moves around the channel for points at the same distance from its base. In all the cases they can be seen percentage differences of more than 170%.

Table I summarizes the different changes in the maximum magnitude of the electric field obtained in this simulation.

TABLE I
RELATIONS OF MAXIMUM MAGNITUDES FOR DIFFERENT ANGLES AROUND THE CHANNEL

r [m]	$\phi = 0$	$\phi = \pi/2$	$\phi = \pi$	$\phi = 3\pi/2$	%
100	3.60×10^4	6.31×10^4	3.90×10^4	4.04×10^4	175%
1000	2047	4818	3789	4291	235%
5000	576	629	320	510	196%
10000	158	327	158	221	200%

It is worth noting here that the simulated electric fields have a nonzero initial value, which could lead to think that the electric field wave travels instantaneously from the source to the observation point. This result is due to the assumptions made in the model, which assumed a nonzero value for the initial charge distribution at the two ends of the lightning channel, assuming that this nonzero value of electric charge is neutralized during the return stroke process.

V. Conclusions

In this work it was defined a methodology for the generation of lightning channels with a macroscale tortuosity similar to the one recorded in nature, this methodology allows the simulation of lightning channels with toruosity and with or without branches. By means of this methodology it was simulated a lightning channel with tortuosity, this geometry was utilized as an input parameter for the calculation of the electromagnetic fields generated when a Heaviside step function travels along the tortuous channel.

The results obtained this way for the total electric field at different points showed two important results that could cause an error when it is used the time-of-arrival technique for lightning localization. The two results are due to the non-uniformity of the electromagnetic fields generated by tortuous channels when the azimuth of the observation point is changed. One of the changes that was observed is the change in the shape of the electric field wave and the other is an important change in the maximum magnitude of the electric field wave.

The time-of-arrival technique is one of the two important techniques used by modern lightning locating systems, which report among other things the peak value of the lightning return stroke current, so it is important to note that the tortuosity of the lightning could introduce an important effect on the mentioned calculations, given the results obtained in this work, but further analysis is needed in order to quantify these effects, which were described here only in a qualitative way.

It is important to note that the current used in this work was a Heaviside step function, and some important results were obtained showing the important influence of the lightning channel geometry on the generated electromagnetic fields, so it is important to continue developing this methodology in order to obtain results for more realistic representations of the return stroke current.

Acknowledgment

The authors would like to thank Universidad Nacional de Colombia and COLCIENCIAS for their support in the development of this work.

References

[1] D. Amarasinghe, U. Sonnadara, M. Berg, and V. Cooray, "Channel tortuosity of long laboratory sparks," *Journal of Electrostatics*, vol. 65, no. 8, pp. 521–526, July 2007.

[2] A. Gulyás and N. Szedenik, "3d simulation of the lightning path using a mixed physical-probabilistic model - the open source lightning model," *Journal of Electrostatics*, vol. 67, no. 2-3, pp. 518–523, May 2009.

[3] A. Andreotti, U. D. Martinis, C. Petrarca, V. A. Rakov, and L. Verolino, "Lightning electromagnetic fields and induced voltages: Influence of channel tortuosity," in *2011 XXXth URSI General Assembly and Scientific Symposium*, 2011.

[4] G. Lupò, C. Petrarca, V. Tucci, and M. Vitelli, "Em fields generated by lightning channels with arbitrary location and slope," *IEEE Transactions on Electromagnetic Compatibility*, vol. 42, no. 1, pp. 39–53, February 2000.

[5] Z. kuo Zhao and Q. lin Zhang, "Influence of channel tortuosity on the lightning return stroke electromagnetic field in the time domain," *Atmospheric Research*, vol. 91, no. 2-4, pp. 404–409, February 2009.

[6] D. K. L. Chia and A. C. Liew, "Effect of tortuosity of lightning stroke path on lightning electromagnetic fields," in *2008 Asia-Pacific Sympsoium on Electromagnetic Compatibility & 19th International Zurich Symposium on Electromagnetic Compatibility, 19-22 May 2008, Singapore*, 2008.

[7] F. Guo and B. Zhou, "Influence of branch geometry on radiation field characteristics from lightning return strokes in the time domain," in *2009. CEEM 2009. 5th Asia-Pacific Conference on Environmental Electromagnetics*, 2009.

[8] M. SAITO, M. ISHII, and N. ITAMOTO, "Influence of geometrical shape of return stroke channel on associated electromagnetic fields," in *2011 7th Asia-Pacific International Conference on Lightning, November 1-4, 2011, Chengdu, China*, 2011.

[9] M. Saito, M. Ishii, and N. Itamoto, "Influence of geometry of - gc strokes on associated electromagnetic waveforms," in *2011 International Symposium on Lightning Protection (XI SIPDA), Fortaleza, Brazil, October 3-7, 2011*, 2011.

[10] G. Lupò, C. Petrarca, V. Tucci, and M. Vitelli, "Em fields associated with lightning channels: On the effect of tortuosity and branching," *IEEE Transactions on Electromagnetic Compatibility*, vol. 42, no. 4, pp. 394–404, November 2000.

[11] M. Vargas and H. Torres, "On the development of a lightning leader model for tortuous or branched channels - part i: Model description," *Journal of Electrostatics*, vol. 66, pp. 482–488, 2008.

[12] ——, "On the development of a lightning leader model for tortuous or branched channels - part ii: Model results," *Journal of Electrostatics*, vol. 66, pp. 489–495, 2008.

[13] R. Hill, "Tortuosity of lightning," *Atmospheric Research*, vol. 23, no. 3, pp. 217–233, October 1988.

[14] R. D. Hill, "Analysis of irregular paths of lightning channels," *Journal of Geophysical Research*, vol. 73, no. 6, pp. 1897–1906, January 1968.

[15] V. P. Idone and R. E. Orville, "Channel tortuosity variation in florida triggered lightning," *Geophysical Research Letters*, vol. 15, no. 7, pp. 645–648, July 1988.