VII Simposio Internacional sobre Calidad de la Energía Eléctrica SICEL 2013

A Portfolio Theory Approach for the Power Supply Contracts to Paraguayan Electric System by Itaipu

Paulo E. B. Arce, Anastacio S. Arce E., and Adriano A. F. M. Carneiro

***Abstract*— Itaipu is a binational enterprise undertaking Paraguay and Brazil. The electrical power produced by Itaipu hydroelectric power plant is sold to Administración Nacional de Electricidad (ANDE) from Paraguay, and Eletrobras from Brazil, through an annual contract. The main source of power supply for Paraguay's Electrical Power System is the Itaipu Hydroelectric Power Plant. Historically, Itaipu produces more power than the expected and sells this surplus power for lower prices than the contract prices. However, surplus power depends on the natural inflows, occurring more often on dry periods of the year and it is possible that surplus power won't occur or would be less than expected. In this context, ANDE has two possible options of contracts with Itaipu: an *ex-ante* risk-free option through an annual contract and a short-term option, an *ex-post* risky option at lower prices. A third option for the supply to meet the demand is through the cession of Power Contracted by Eletrobras. This option can be very expensive due to the high cost when used for a period greater than 4 hours on the same day. For this problem, it is proposed a mean-variance model for ANDE's portfolio optimization, obtaining an efficient frontier to describe which portfolio options achieve the lower costs for each level of risk. The model indicates the optimal decision, based on monthly power contracts that would lead to best economic achievements.**

***Index Terms*—power contracts, portfolio theory, mean-variance, risk management, portfolio optimization.**

I. Introduction

The Itaipu Hydroelectric Power Plant is a binational enterprise undertaking Paraguay and Brazil. It has an installed capacity of 14,000 MW, distributed in 20 generating units, each rated at 700 MW. Of the 20 generating units, 10 units produce in 50 Hz, frequency of Paraguayan Electric Power System (PEPS), the other half, 10 generating units, produce in 60 Hz, frequency of Brazilian Electric Power System (BEPS). The Itaipu 60 Hz connection with BEPS is made through a 500kV and a 765kV transmission system, transmitting Itaipu's 60 Hz production to the state of Sao Paulo, Brazil's biggest electric energy consumption center. Itaipu's 50 Hz production is transmitted to the Margen Derecha Substation, where a part of this energy is delivered to Paraguay through a 220 kV transmission system. The other part is conducted to the Foz de Iguaçu Substation, where the alternating current is rectified and transmitted in direct current to the state of Sao Paulo. Itaipu's energy production in 2011 was 92.500 GWh, of which 8.306 GWh were supplied to Paraguay and the rest, 83.487 GWh, were supplied to Brazil [1].

Itaipu's energy supply to PEPS represents 74 % of Paraguay's electric energy consumption while in the BEPS, it represents 17 %. PEPS counts with other sources of generation besides Itaipu, like Acaray and Yacyreta hydroelectric power plants, with 200 and 3.200 MW installed capacity respectively. Paraguay owns Acaray. Yacyreta is a binational enterprise between Paraguay and Argentina [2]. Despite having other sources of generation, Paraguay still has Itaipu as his main energy supplier. The Itaipu and Yacyreta interconnection is impossible today due to limitations of the transmission systems, avoiding the parallel operation of the electrical power systems of Brazil and Argentina through the PEPS.

Itaipu sells its generated power to the PEPS through the Administración Nacional de Electricidad (ANDE), and to the BEPS through Eletrobrás. In PEPS context, ANDE acts as a Distribution Company (DISTCO) and a Generation Company (GENCO), since it is Paraguay's single electricity company. Due to these characteristics, Paraguay's electrical energy demand is met mainly by Itaipu's power supply which is formalized by ANDE-Itaipu power contracts. These power contracts are discretized in monthly basis, in order to meet monthly power demand, in the period of one year.

As a DISTCO, ANDE objectives to achieve the lower costs possible in power contracting. ANDE makes its contract decisions based on experience and empirical methods but there is no formal mathematical methodology to the referred procedure. In this context a Portfolio approach is appropriate to manage this problem. Diverse works such as [3], [4], [5] and others, achieve important results treating power contracts problems by the Portfolio Theory.

This work proposes a Portfolio Optimization model to minimize ANDE's power contracting costs, based on Markowitz's mean-variance model.

P. E. B. Arce is with University of Sao Paulo (USP), Electric Engineering Department, São Carlos, SP 13566-590 Brazil (e-mail: paulo.arce@usp.br).

A.S. Arce E. is with Eastern National University, Polytechnic Faculty, Ciudad del Este, Paraguay (arce@fpune.edu.py).

A. A. F. M. Carneiro is with University of Sao Paulo (USP), Electric Engineering Department, São Carlos, SP 13566-590 Brazil (e-mail: adriano@sc.usp.br).

A. *Itaipu Hydroelectric Power Plant*

Itaipu has an installed capacity of 14,000 MW, distributed in 20 generating units, each rated at 700 MW. The first generating units started operating in 1984 and power generation has increased since then, as can be observed in figure 1:

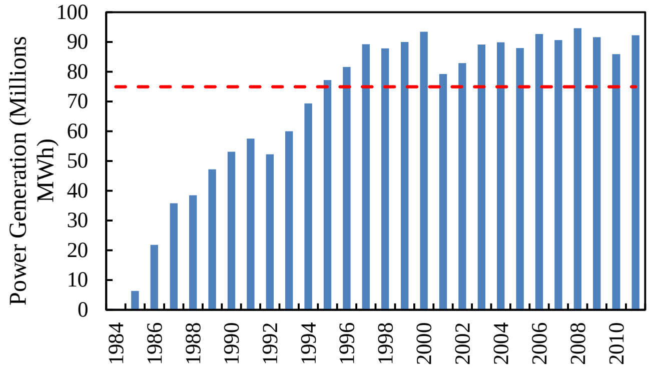

Fig. 1. Itaipu's Annual Power Generation. The red line indicates Itaipu's guaranteed energy (75,000 MWh per year).

From 1995, Itaipu's power generation has always been superior to its guaranteed energy (75,000 MWh per year). Although Itaipu generates more than its guaranteed energy over the year, in a monthly basis the generation varies seasonally leading sometimes to generation levels below the rated 700 MW per generating unit.

In the dry period of the year going from May to September, the power availability increases, since gross head increases. In the wet period of the year, gross head decreases, so power availability decreases too. Figure 2 shows power availability per generating unit, for two different years (1984 and 2010), also the average power availability per generating unit for the 1984-2011 historical period.

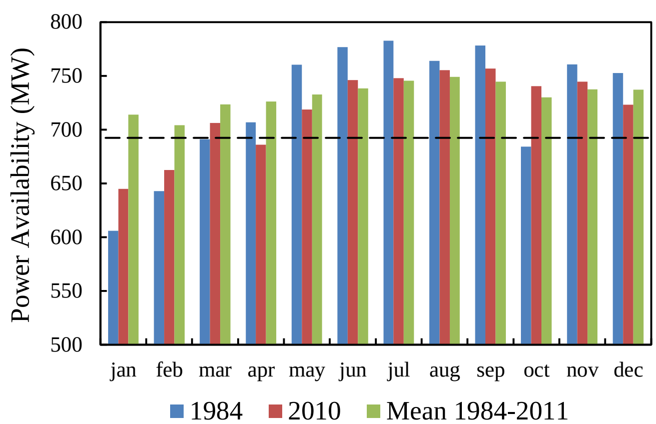

Fig. 2. Itaipu's Power Availability per generating unit, for the years of 1984, 2010 and 1984-2011 monthly average.

Figure 2 indicates that power availability is seasonal. The highlighted line indicates the rated power per generating unit (700 MW). In the dry period (May-Sep) availability is bigger. In the wet period (Oct-Apr) availability is lower. Historically, the monthly power available is superior to the rated per generating unit (700 MW), as shown in the 1984-2011 average. Although historically the power availability is above nominal, it can occur that natural inflows are too high, so the gross head decreases and power availability would be below the rated 700 MW per generating unit, as can be seen in several months in the years of 1984 and 2010. In the year of 1984, this situation occurs in the months of January, February, March and October. In 2010, power availability is below rated in the months of January, February and April. On the other months, for both 1984 and 2010, power available is above rated. The available power above rated is called Surplus Power. Thus, for instance, in July 2010 power available is 747 MW, meaning that Surplus Power is 47 MW per generating unit.

From figure 2 analysis it is concluded that power availability in Itaipu is superior to the rated per generating unit (700 MW), along the year, especially on the dry period of the year. Although in some months it is possible that the generation levels are lower than the rated per generating unit. This characteristic is very important to ANDE's portfolio management, since it influences directly on the energy price as will be seen in the next sections.

B. *Administración Nacional de Electricidad (ANDE)*

ANDE (Administración Nacional de Electricidad) is a Paraguayan Electricity company created in 1964 that operates in the generation, transmission, distribution and commercialization of electricity. It is Paraguay's only DISTCO, and part-owner of Itaipu Binacional. ANDE is responsible for PEPS power supply. The PEPS has an increasing annual demand of about 10% per year and about 78% of the power supply comes from Itaipu, as shown in Table I:

TABLE I
PARAGUAY'S POWER SUPPLY SOURCES

	Itaipu	Yacyreta	Acaray	Other Sources
Power Supply (MW)	7.000	1.750	200	110

Paraguay's power demand is very seasonal since it is mostly a residential load, which makes it very sensitive to climate conditions. Residential load corresponds to about 42% of the total demand:

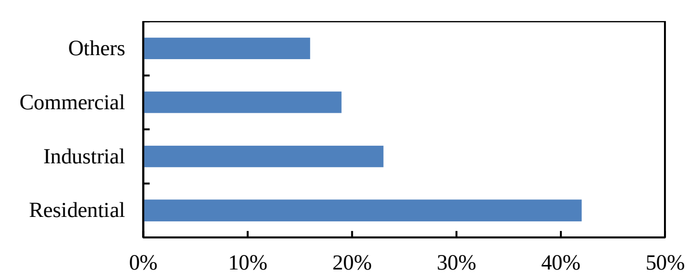

Fig. 3. Paraguay's load classification.

Paraguay's power supply by Itaipu is made through an annual contract, celebrated between ANDE and Itaipu. Thus, ANDE must decide every year the monthly amount of power contracted with Itaipu for the next year. The following section explains formally how this decision is made.

II. Commercialization Rules Between ANDE and Itaipu

A. Itaipu's Available Power for Commercialization

Itaipu has an installed capacity of 14000 MW of which only a part is available for commercialization with ANDE and Eletrobrás. Of the total installed capacity, a part is reserved for maintenance (*PM*), a part is reserved for own consumption of the power plant (*DCP*) and a part is reserved for frequency regulation (*RPO*). So, the Available Power for Commercialization (*PDI*) is defined by (1):

$$PDI = PI - PM - DCP - RPO \quad (1)$$

In 2011, *PM*, *DCP* and *RPO* were 1400 MW, 38 MW and 427 MW respectively. Thus:

$$PDI = 14000 - 1400 - 38 - 427 = 12135\ MW \quad (2)$$

Thereby, for 2011, 12135 MW were available for commercialization with ANDE and Eletrobrás.

B. Contracted Power

Every year ANDE and Eletrobrás must declare the monthly amount of power to be contracted from Itaipu, based on the *PDI*. ANDE declares first the monthly amounts contracted (PC) and Eletrobrás contracts the rest of the *PDI*, so that the whole *PDI* is always contracted by ANDE and Eletrobrás. To illustrate this dynamic, let's suppose that in January, 2011, ANDE declared a total 700 MW for that month. Then Eletrobrás must contract the rest:

$$PC_{Eletrobrás} = PDI - PC \quad (3)$$

Thus, for the given example, Eletrobrás would contract:

$$PC_{Eletrobrás} = 12135 - 700 = 11435\ MW \quad (4)$$

Both ANDE and Eletrobrás have the right to use the energy that can be produced by Itaipu. The amount of energy associated to Power Contracted is the guaranteed energy and it is approximately 70 % of the Power Contracted value, in MWmed. Hence, considering a month with 720 hours, and 700 MW contracted, the energy associated to the power contracted is:

$$700\ MW\ x\ 0.7\ x\ 720\ h = 352800\ MWh \quad (5)$$

In order to represent the energy units in monthly base, it will be considered in$MWmed$, and obtained by (6):

$$352800\ MWh = \frac{352800\ MWh}{720\ h} = 490\ MWmed \quad (6)$$

The price paid for the contracted power is fixed, and depends on several factors, that can be precisely appreciated in [6] and [7]. There is no variation on the contracted power price which is defined by (7):

$$CUSE = 22{,}6\ \frac{US\$}{kWmes} \quad (7)$$

The fact that the energy cost doesn't vary along the year, makes the Contracted Power (PC) a risk-free option. Also, since the Contracted Power is defined the year before its delivery, it can be considered an *ex-ante* modality of contract.

C. Surplus Power

As shown in figure 2, in several months, Itaipu's Power Availability (*PA*) exceeds rated power per generating unit. The power that exceeds rated power per generating unit is called Surplus Power. The rated power per generating unit (*RPG*) corresponds to 700 MW, so the monthly Available Surplus Power (*PED*) can be calculated by (8):

$$PED = PA - RPG \quad (8)$$

The surplus power can be commercialized with ANDE and Eletrobrás when the Contracted Power is not enough to meet the system's demand.

In the case that ANDE's Contracted Power is not enough to meet the PEPS demand, the subcontracted portion is provided by the Surplus Power, when it exists.

The price paid for the Surplus Power is relative to royalties for water usage, leading to a very low cost. The price of the Surplus Power in 2011 is defined in (9), and for ease of nomenclature will be called β:

$$Monthly\ Surplus\ Power\ Cost = \beta = 3{,}92\frac{US\$}{MWmes} \quad (9)$$

However, as shown in figure 2, the Surplus Power can be lower than expected or in a more drastic situation available power can be lower than the rated power per generating unit. In this case, there is no Available Surplus Power (PED) and the subcontracted demand, could be provided by Power Transfer, made by Eletrobrás to ANDE, where a part of Eletrobrás' Contracted Power ($PC_{Eletrobrás}$) is ceded to ANDE. This modality of contract has a penalty involved, which consists in charging the whole day when the Transferred Power (*PL*) is practiced for more than 4 hours, in that same day. This situation can lead to very high costs on the portfolio.

To illustrate this dynamic, let's suppose that in January, 2011, ANDE contracted 700 MW ($PC = 700\ MW$) and the system's demand was 1000 MW. Let's also suppose that Available Surplus Power (PED) was 100 MW ($PED = 100\ MW$). The subcontracted demand corresponds to the difference between the demand and the Contracted Power:

$$1000\ MW - 700\ MW = 300\ MW \quad (10)$$

Of the 300 MW, a part will be met by Surplus Power, namely 100 MW. The rest, 200 MW, will be transferred by Eletrobrás' Contracted Power ($PL = 200\ MW$).

From the previous analysis, it can be concluded that the Surplus Power is an *ex-post* risky option, since the energy is valued after its delivery and the price of the energy can be a very low one when Surplus Power is available. On the other hand, when it is not available, it is necessary to use part of Eletrobrás Power Contracted which is subject to the referred penalty. In this case the price paid for the energy can be very high. Therefore, the variation on Power Availability causes a variation on the energy price paid by ANDE.

Thus, ANDE's decision making can be interpreted as a Portfolio Optimization, which intends to obtain the minimum cost possible in power contracts when meeting the system's demand. For this purpose, Markowitz's Modern Portfolio Theory is adapted for the conditions of ANDE's problem. The next section gives a brief explanation of Markowitz's Portfolio Theory.

III. Portfolio Theory

In 1952, economist Harry Markowitz enunciated the portfolio theory in the paper entitled *Portfolio Selection* [8]. The objective of the analysis made was to contribute in decision making in financial markets exposed to contract price risks. Presently, portfolio analysis is largely used when analyzing energy contracts ([3], [4] and [5]).

It is supposed that an investor must maximize its expected return, or minimize its expected cost. Also it is supposed that the investor must be risk-averse when taking risks is not necessary.

Markowitz defines the expected return, or the expected cost as the historic average of observed return, or observed cost. The expected return or expected cost (E_i) for a security *i* is defined by (11):

$$E_i = X_i \mu_i \tag{11}$$

Where X_i is the relative amount invested in security *i*, and μ_i is the historic average of returns for security *i*. The expected portfolio return as a whole (E_{Rp}) is the sum of all *N* expected returns for the *N* securities:

$$E_{Rp} = X_1\mu_1 + X_2\mu_2 + \cdots + X_N\mu_N = \sum_{i=1}^{N} X_i\mu_i \tag{12}$$

Markowitz states that the risk can be interpreted as the variation over the expected return. This uncertainty about future returns or future costs can be expressed as the Variance of the historic series. To represent the Variance of all securities available, and to represent the covariance between two different securities, Markowitz defines the covariance matrix (13):

$$\boldsymbol{COV} = \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ \sigma_{i1} & & \sigma_{ij} \end{bmatrix} \tag{13}$$

Where:

$$\sigma_{ij} = \begin{cases} \sigma^2, if\ i = j \\ cov_{ij}, if\ i \neq j \end{cases} \tag{14}$$

So in the diagonal of the ***COV*** matrix, we have variance of the securities, and on the other elements we have the covariance between two different elements. In [6] it is stated that the portfolio's Variance (*V*) as a whole is defined by (15):

$$V = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{ij} X_i X_j \tag{15}$$

Where X_i is the relative amount invested in security *i*, and X_j is the relative amount invested in security *j*.

Since expected return is considered the average of historic returns, and risk is measured by the Variance of expected return, Markowitz's model is often referred as the *Mean-Variance* model.

Markowitz states that the portfolio that maximizes return or minimizes cost not necessarily is the one that has the minimum risk. In this sense Markowitz's mean-variance model can be interpreted as a non-linear optimization problem, minimizing risk subject to a minimum level of return, or maximum level of cost. Alternatively, this can be modeled to maximize return or minimize cost, subject to a maximum level of risk. The last statement is the one used in solving ANDE's portfolio optimization.

IV. Model Proposed

ANDE's decision making problem consists in define monthly Contracted Power (PC_t), trying to meet the system's demand (d_t) for a period of one year with the minimum cost possible. In other words ANDE must minimize the Portfolio Expected cost and manage its exposure to the Surplus Power availability risk and complementation with Eletrobrás' Contracted Power ($PC_{Eletrobrás}$) transfer. In order to minimize the cost, ANDE must let a part of its monthly demand subcontracted, betting there will be enough Surplus Power to meet the Subcontracted Demand (*PL*). The uncertainty about Surplus Power availability reflects on an uncertainty on the ANDE's portfolio cost, which will be low in case Surplus Power is enough to meet the Subcontracted Demand (*PL*), and will be very high in case that Surplus Power is not enough. In the last case, it will be needed Power Transfer from Eletrobrás to ANDE. This mechanism is subject to heavy price penalty as referred is section II.

A. Objective Function

The objective is to minimize ANDE's portfolio Expected Cost. The annual cost will be the sum of monthly costs, which will depend on the amount of Contracted Power (PC_t) times its price ($CUSE$) and the Subcontracted Demands' (*PL*) cost, which include the amount of Surplus Power times its price (β) and the eventual Power Transferred from Eletrobrás price.

The variation on Power Availability causes a variation on the energy price paid by ANDE. For a bigger Power Surplus availability, there is a lower possibility to use Power Transferred from Eletrobrás to meet its demand; consequently a lower ANDE's portfolio cost is expected. On the other hand, for lower Power Surplus availability, there is a bigger chance to use Power Transferred from Eletrobrás to meet its demand; consequently a bigger ANDE's portfolio cost is expected. To represent the (*PL*) cost, it is proposed a cost function that takes into account the amount of Available Power in Itaipu and the penalty involved with the Power Transferred from Eletrobras ($\mu(PED)_t$). Let us consider the historic monthly Available Power in Itaipu, which can be represented by the matrix in (16):

$$\begin{bmatrix} PED_{jan,1983} & \cdots & PED_{dec,1983} \\ \vdots & \ddots & \vdots \\ PED_{jan,2011} & \cdots & PED_{dec,2011} \end{bmatrix} \quad (16)$$

The maximum value observed in the matrix will be associated to the lowest value possible for subcontracted demands' cost $\mu(PED)_t$. Associated with the minimum value in the matrix will be considered the penalty for Power Transferred being applied for the 30 days of the month, and for the worst case, being used only for 4 hours. The function cost of PL represents the dynamic by which the power cost increases as the Available power decreases. The function cost is modeled by a linear function as illustrated in figure 4:

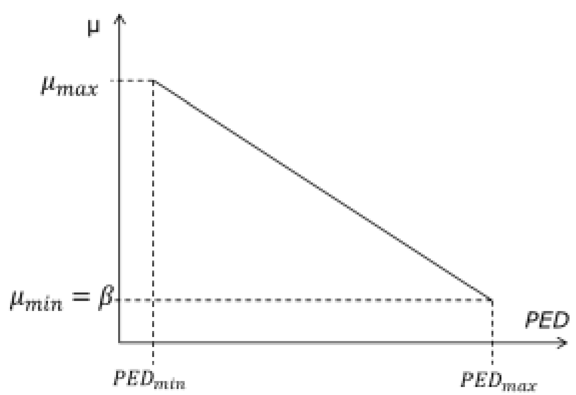

Fig. 4. Cost Function for Transferred Power from Eletrobrás to ANDE.

By applying the cost function to (16), it is obtained a cost matrix based on the historicall PED. This leads to (17):

$$\begin{bmatrix} \mu(PED)_{jan,1983} & \cdots & \mu(PED)_{dec,1983} \\ \vdots & \ddots & \vdots \\ \mu(PED)_{jan,2011} & \cdots & \mu(PED)_{dec,2011} \end{bmatrix} \quad (17)$$

The expected monthly cost of subcontracted demand will be the monthly average from (17). Thus:

$$\mu(PED)_t = \sum_{n=1}^{29} \mu(PED)_{t,n} \quad (18)$$

Where t corresponds to the months, and n corresponds to the years of the PED historic, going from 1983 to 2011 (29 years). Equation (18) corresponds to an adaptation of Markowitz's (11). The objective function is defined in equation (19).

B. *Constraints*

Firstly, the risk constraint is indicated in equations (20) and (21). This means that the portfolio risk will be measured by the Variance of the whole portfolio. PL_i and PL_j are the monthly Subcontracted Demands for months *i* and *j* and σ_{ij} is the element of the ***COV*** matrix. Equation (20) represents an adaptation of Markowitz's equation (15) for the conditions of ANDE's portfolio.

Equation (21) corresponds to the maximum level of risk to which ANDE is willing to be exposed. This means that V_{max} is a parameter, defined by the investor (in this work, ANDE) which will determine the maximum Variance for the portfolio. ANDE can define V_{max} in a large range, and the value defined will depend on its aversion to risk.

The second constraint (22) refers to the obligation to meet demand. Thus, for every month *t*, the sum of Contracted Power (PC_t), Surplus Power and Transferred Power, represented by (PL) must equal that month's demand (d_t).

The last constraint refers to a minimum level of Contracted Power (PC_{min}), as indicated in equation (23). This obligation is imposed by Eletrobrás to ANDE, and refers to the previous year minimum demand. In 2011, PC_{min} corresponded to 600 MW (467 MW*med*), which was 2010 minimum demand.

The proposed model intends to assist ANDE in deciding *how much of its demand should be contracted ex-ante (PC) and how much should be subcontracted, betting on high Surplus Power availability in order to minimize its power contracts portfolio.* The proposed model is defined by a non-linear optimization, expressed by equations (19) to (23):

$$\boldsymbol{Min} \sum_{t=1}^{12} (CUSE * PC_t + \mu(PED)_t * PL_t) \quad (19)$$

$s.t.$:

$$V = \sum_{i=1}^{12} \sum_{j=1}^{12} (PL_i \, PL_j \sigma_{ij}) \quad (20)$$

$$V \leq V_{max} \quad (21)$$

$$PC_t + PL_t = d_t \quad (22)$$

$$PC_t \geq PC_{min} \quad (23)$$

$$t = 1, \ldots, 12$$

The optimization model is solved using LINGO 7, a linear and non-linear solver. LINGO has an efficient set of tools and functions which allows it to be used with *MS Excel*, in order to plot the functions and handle data. Further information on non-linear programming can be found in [10].

V. Results

A. *Efficient Frontier*

The optimization process can be performed for various levels of risk, defined by V_{max} in equation (21). For each V_{max} defined there is a corresponding optimal cost. In other words, for each level of risk there is an optimal portfolio which minimizes expected cost for that level of risk.

The Efficient Frontier defines the minimum cost portfolios for each level of risk. This means that ANDE must decide how to contract power from Itaipu based on the portfolios contained on the Efficient Frontier which correspond to the optimal portfolios. Figure 5 corresponds to the Efficient Frontier of the proposed model. It was obtained performing the optimization process with several V_{max} values.

As indicated in figure 5, as risk (Variance) increases, expected cost decreases. This means that exposure to risk is recompensed by the expectation of better (lower) portfolio costs.

Portfolios that lie on the Efficient Frontier are called Efficient Portfolios, since they provide the minimum cost for the level of risk. They are also called optimal portfolios. Portfolios that cluster to the right of the Efficient Frontier are sub-optimal portfolios or Inefficient Portfolios, since there are portfolios that provide lower costs for the level of risk or alternatively, lower risk for the expected cost. This is the case of ANDE 2011 portfolio, highlighted on figure 5:

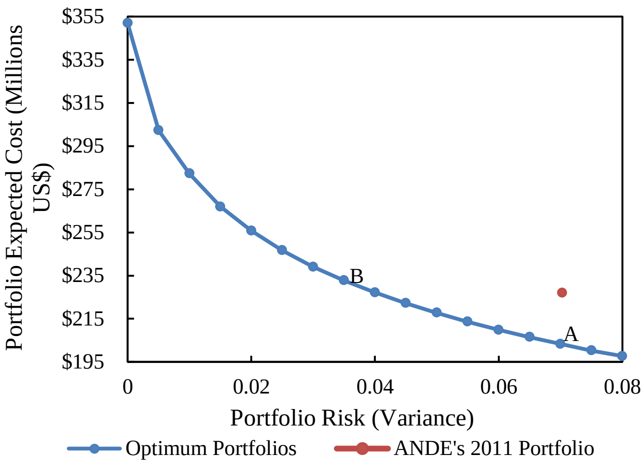

Fig. 5. Efficient Frontier. The highlighted portfolio (red) corresponds to ANDE's 2011 portfolio.

Figure 5 shows that ANDE's 2011 practiced portfolio corresponds to an inefficient portfolio, since it is not on the Efficient Frontier. It is possible to state that the decision made by ANDE in 2011 led to the inefficiency of investments. For the same variance (0.07 or 7%), there is a portfolio on the Efficient Frontier with a lower cost (*A*). Also, there is a portfolio with the same cost but lower risk (*B*).

The former analysis confirms the idea that ANDE must make its contractual decision based on the Efficient Frontier Portfolios, obtained by the optimization model proposed.

Although the Efficient Frontier indicates all optimal portfolios possible for the conditions of the problem, it gives information only about risk level (variance level) and cost. It doesn't provide information about the monthly Contracted Power which is a major concern in terms of ANDE's decision making. In this sense the subsequent analyses are necessary for this purpose.

B. Optimal Contracted Power

The most important result in terms of decision making is the optimal result for Contracted Power (PC_t). The optimization process can be performed for several risk levels by setting different V_{max} levels in equation (21). As indicated in figure 5, to each level of risk (V_{max}) corresponds an optimal minimum portfolio cost. This is consequence of a difference between the amounts of Contracted Power (PC_t), obtained by each level of risk. Since Surplus Power and Transferred Power are *ex-post* forms of contract and depend on Available Surplus Power (*PED*), ANDE has little direct control over them. The main form of portfolio management by ANDE is an accurate and efficient definition of Contracted Power levels for each month of the planning period (the next year). Figure 6 illustrates this decision for three levels of risk:

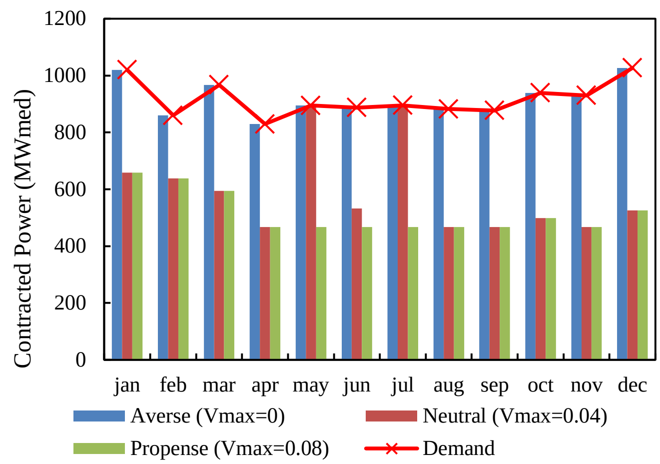

Fig. 6. Contracted Power for three levels of risk.

As the risk (V_{max}) increases, the Contracted Power decreases. This happens because as risk is increased, a higher fraction of the demand is let subcontracted, betting on a good Available Power scenario, which will lower the portfolio cost by *ex-post* contracts by Surplus Power, since it has a much lower cost in comparison with Contracted Power. In the first case (risk-averse), risk is defined as zero ($V_{max} = 0$). This leads to 100% of the demand to be contracted *ex-ante* (by Contracted Power), meaning that in this case ANDE would have total risk protection. In one hand ANDE has in this case a secure positioning but on the other hand the portfolio expected cost is very high (about 352 US$ millions) as indicated in figure 5. For all months, the Contracted Power is equal to the Demand curve as shown in figure 6.

Increasing the risk ($V_{max} = 0.04$ and $V_{max} = 0.08$) the Contracted power decreases. The first case shows a neutral positioning and the last, a risk-propense positioning. In both cases there is a minimum Contracted Power (467 MWmed) which corresponds to constraint (23). The difference between these two results can be observed from May to July, since these months are the most expensive and show the highest variance in the ***COV*** matrix. In this sense they have a major contribution to the portfolio Variance in equation (20). As the risk is increased, the portfolio expected cost decreases, since a larger fraction of the monthly Demand is subcontracted, betting on good Available Power scenarios, which will allow *ex-post* contracts by Surplus Power.

The analysis of Optimal Contracted Power is very important and in terms of costs it is related to the expected cost equation (19), which considers an average cost of all historic data (mean-variance model). However, it is important to study the impact of practicing the Optimal Contracted

Power decision on the historic scenarios individually, since Surplus Power can occur below expected, leading to Transferred Power *ex-post* contract, which will have the opposite effect intended over the portfolio cost. The further analysis illustrates this point.

C. Permanence Curves

For each level of risk corresponds an expected cost, which is obtained by the average of all historical data available. However, for each year of the historic, there is a different cost due to different levels of Available Surplus Power (*PED*). The permanence curves define the portfolio cost for each year of the available historic. For this analysis, optimization was performed for three risk levels, corresponding to risk-averse positioning, neutral positioning and risk-propense positioning. Also, ANDE's 2011 decision was analyzed for the historical data to prove the proposed model's efficiency:

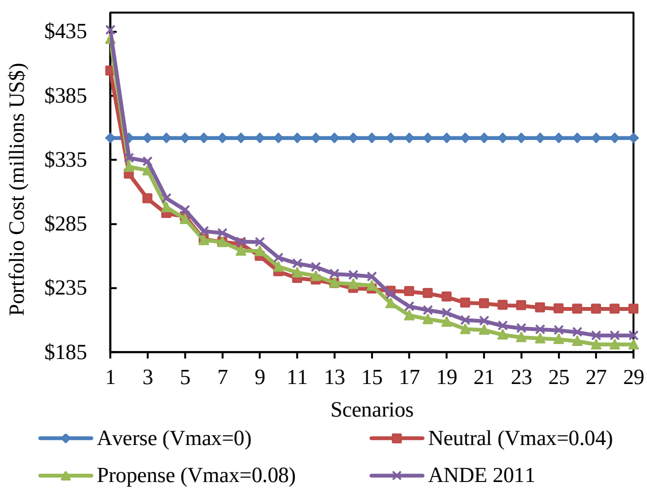

Fig. 7. Portfolio Cost Permanence Curves.

Instead of displaying the years (1983, 1984… 2011), the horizontal axis is displayed from 1 to 29. The reason for this choice is that the permanence curves are easier to interpret when sorted from the highest cost to the lowest (descendent order of costs), and the chronological order of the years (scenarios) is not necessarily the one that leads to the descendent order of costs. For instance, 2011 is not necessarily the year that leads to the lowest cost. Namely, 2011 corresponds to scenario 7.

When the risk level chosen is zero ($V_{max} = 0$), all scenarios have the same cost. This is consequence of having no subcontracted demand. In other words, 100% of the demand is met by Contracted Power (*PC*), and no Surplus Power or Transferred Power is needed. As a result there is no variation on the cost (Variance is zero). This is clear as the Averse curve is flat. In this case ANDE obtains a total risk protection. On the other hand the cost is very high. Independently on the Available Power (*PED*), the portfolio cost is high (about US$ 352 millions). This is a good characteristic when the scenario is not favorable (monthly *PED* levels are low), namely scenario 1, for the historic data available. For the other 28 scenarios, a more risk-propense positioning leads to better (lower) portfolio costs.

When the risk level is increased ($V_{max} = 0.04$), the permanence curve (Neutral) shows a large range of costs depending on the scenarios. Increasing risk level to its maximum ($V_{max} = 0.08$) it is possible to obtain the most risk propense curve (Propense). It is important to notice that the more the risk is increased, the more it is possible to obtain low costs, when scenarios are favorable (high levels of *PED*). On the other hand the more risk is increased, the more it is possible to obtain high costs when scenarios are not favorable. This statement can be verified comparing Neutral and Propense curves in figure 7. For scenarios 16 to 29, the propense positioning leads to better costs than the neutral positioning. This means that for the referred scenarios, risk exposition was offset. On the other hand, for scenarios 1 to 14, except scenario 8, the neutral positioning obtained lower costs. In these cases the protection from risk represented by the neutral positioning compared to the propense was offset.

Finally it is made a comparison with ANDE's 2011 portfolio. It is clear that the optimized decision obtained by the proposed model leads to better results. For all 29 scenarios, the optimal decision in the Propense case leads to lower costs. Even the neutral positioning leads to better results from scenarios 1 to 15.

D. Comparison between the Model's Result and the 2011 practiced portfolio

Finally, a comparison between ANDE's 2011 practiced portfolio and the model's result for the same level of risk will be made, to show the model's efficiency against a real portfolio practiced by ANDE. Figures 8 and 9 show the comparison:

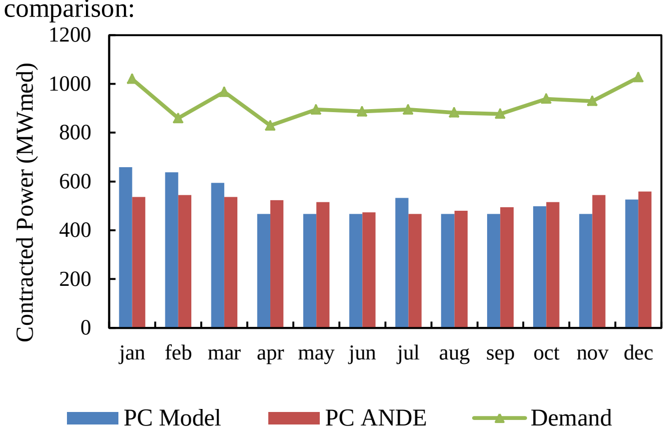

Fig. 8. Comparison between the Model's result and ANDE's 2011 portfolio, for the same level of risk in terms of Contracted Power (*PC*).

Figure 8 compares the Contracted Power by the model (*PC Model*) and the Contracted Power by ANDE in the year of 2011 (*PC ANDE 2011*). The demand series used is the 2011 ANDE's real demand.

To make an accurate comparison, both Model result and ANDE's practiced portfolio must have the same level of risk, namely the same Variance. To obtain, ANDE's 2011 portfolio Variance, the subcontracted demand (*PL*) is used in equation (20), obtaining a total Variance of 7%. To obtain the

Model's result for a Variance of 7%, the optimization is made using $V_{max} = 0.07$ in equation (21).

As can be observed in figure 8, both results show subcontracted demand for all months. This subcontracted demand can be billed *ex-post* as Surplus Power, if available or as Transferred Power from Eletrobrás when Surplus Power is not available or is not enough. Figure 9 illustrates this result:

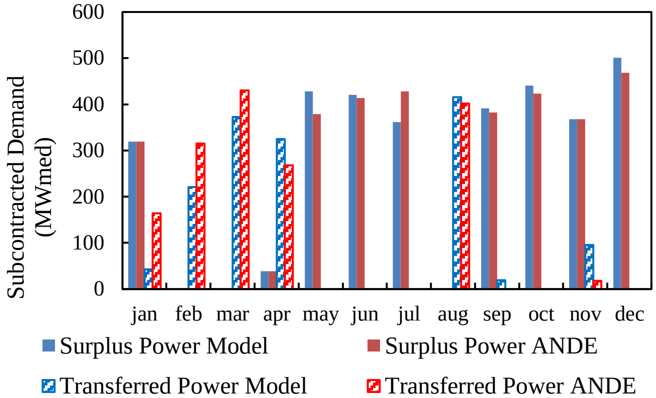

Fig. 9. Comparison between the Model's result and ANDE's 2011 portfolio, for the same level of risk, in terms of Surplus Power and Transferred Power.

Figure 9 compares the Surplus Power and the Transferred Power for the Model's result and ANDE's practiced portfolio in 2011. The analysis of figure 9 shows that the optimized result leads to higher levels of Surplus Power to meet subcontracted demand. This happens to all months of the year except February, March and August, when Surplus Power availability was zero. On the rest of the year, the optimized decision makes best use of the available Surplus Power than the ANDE's practiced decision. Consequently, ANDE's decision leads to a higher Portfolio Expected Cost, since a higher part of its demand is met either by Contracted Power or Transferred Power, which are much expensive than Surplus Power.

In terms of Portfolio Expected Cost, table II compares the costs of the portfolios from figures 8 and 9:

TABLE II
COMPARISON OF COSTS (ANDE 2011 X OPTIMIZED PORTFOLIO)

	***Portfolio* Total Cost (US$)**
ANDE (2011)	342,716,965.23
Model (Optimized Decision)	328,106,319.87
Difference	14,610,645.36

Table II makes clear the efficiency of the proposed model. As can be depicted in the table, the difference between the actual practiced portfolio by ANDE in 2011 and the result of using the proposed model leads to a saving of more than 14 million US$.

VI. CONCLUSIONS

This paper presented a model for annual power contracts decision between ANDE and Itaipu, based on non-linear optimization and risk management. A Markowitz's mean-variance theory adaptation is made for this purpose, minimizing the total cost of power contracts, subjected to a maximum risk level. The risk of the portfolio comes from the variance observed on Surplus Power contracts. Results show that the model's contracting decision leads to lower portfolio costs that can reach a saving of the order of 14.6 million dollars, for the same risk level adopted, as can be seen on the Efficient Frontier. The model also provides the possibility of an accurate risk management, allowing ANDE to decide its positioning towards Power Availability risk.

REFERENCES

[1] Itaipu Binational, Generation, transmission, hydrology and meteorological data, http//www.itaipu.gov.py/. Accessed September (2012).
[2] National Electricity Administration, ANDE (*Administracion Nacional de Electricidad*) Generation, transmission and distribution data, http//www.ande.gov.py/. Accessed August (2012).
[3] Awerbuch, S., Berger, M. Energy Diversity and Security in the EU: Applying portfolio theory to EU electricity planning and policy-making, Paris: International Energy Agency, February; 2003, Report No. EET/2003/03. IEA, February 2003.
[4] Awerbuch, S., Jansen, J. Building Capacity for Portfolio-Based Energy Planning in Developing Countries. REEEP - *Renewable Energy & Energy Efficiency Partnership*, August, 2004.
[5] Arce, P. et. al. Contratação de Energia no Ambiente de Contratação Livre. Congresso Latino-americano de Geração e Transmissão de Energia Elétrica, Mar del Plata, ARG. Anais do CLAGTEE 2011, 2011.
[6] Itaipu Binacional, Technical Report – RIAT. August, 2012.
[7] Itaipu Binational treaty between Brasil and Paraguay (1973).
[8] Markowitz, H. Portfolio Selection; efficient diversification of investiment. New York, John Wiley, 1959.
[9] Arce, A.; "Optimal Dispatch of Generating Units of the Itaipu Hydroelectric Plant", Power Engineering Review, IEEE, 2001.
[10] D. Luemberger; "Introduction to Linear and Non-Linear Programming", Second Edition, Addison-Wesley, Reading Mass., 1984.

Paulo Eduardo Bassi Arce was born in Foz do Iguaçú, Brazil, in 1987. He received the B. S. degree in electric engineering in Sao Paulo University (USP) in 2011. He is currently pursuing de M. S. degree in electric engineering in the same university. Since 2008 he is a researcher in the LSEE/USP (Electric Power Systems Laboratory). His research interests include electric power systems operation and planning, electric energy markets, power commercialization, risk management.

Anastacio Sebastián Arce Encina was born in Puerto Casado, Paraguay, in1961. He received his B.Sc. degree in Electrical Engineering from UNICAMP, Brazil, in 1986. Since 1987 he has been on the staff of the Operational Systems Departament of the Itaipu Binacional Company. He received his M.Sc.and Ph.D degrees in Electric Engineering from UNICAMP, Brazil, in 1999 and 2005, respectively. He joined the staff of UNE (Eastern National University), Paraguay in 1997. His area of interest concerns planning and operation of electrical power systems, electrical energy market.

Adriano Alber de França Mendes Carneiro was born in Belém, Brazil. He received his B. S. degree in electric engineering in the Pontificia Unversidade Católica de Sao Paulo (PUC-SP) in 1970. He received his M. S. degree in the Itajubá State University (UNIFEI) and Ph.D. degree in electric engineering in the Campinas State University in 1980 and 1991 respectively. His research interests include power systems planning and power systems operation.