Dynamic load modeling for small disturbances using measurement-based parameter estimation method

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Abstract—Load modeling is an important task in power system stability analysis and control. Taking this into account, in this paper an improved particle swarm optimization (PSO) is applied to obtain dynamic load models based on the measurement approach. To achieve this, a measurement-based parameter estimation method is used for identification of the exponential recovery load model. Measurements are obtained performing dynamic simulation of an IEEE 14-bus test system under several disturbances, and the response of the obtained models is validated using different data sets. An adequate load modeling improves the comprehension of load behavior and the capability of reproducing transient events on power systems.

Index Terms—Measurement-based load modeling, particle swarm optimization, parameter estimation, exponential recovery load model.

I. INTRODUCTION

A n adequate planning and correct operation of power systems is strongly related with the adequate understanding of each one of elements connected to the network. The model quality of each component in the power system affects considerably the simulation veracity. In contrast to transmission lines, synchronous generators, transformers or compensation devices, which are characterized through deterministic and widely accepted models, the load model continues under study, due to its stochastic and time-varying characteristics [1].

Voltage stability is one of the studies that are highly affected by load modeling. Although static load models, such as ZIP or exponential, are widely used for power systems stability analysis, those models are unable to reproduce the dynamic response of load in the case of transient events and often show deviation from field measurements. This indicates the inefficiency of static load models, due to some characteristics as under-voltage motor protection, thermostatic control, under-load tap changers or slow voltage recovery, which require of dynamic models to have an adequate representation [2].

Load modeling can be achieved via component-based or measurement-based approaches [3][4]. Component-based approaches develop a load model gathering information of load class and composition surveys from load substations. Later, each part of the load composition is replaced using a static representation based on parameters obtained from experimentation. On the other hand, measurement-based approaches develop a load model using voltage, active and reactive power measurements acquired from the load substation. Those measures are used to tune the parameters of a model structure, so the mismatch between the real power system and model response is minimal [4][5].

Measurement-based strategy is an interesting approach for load modeling, since gathering the information of load composition may be troublesome for large-scale power systems. Also, due to the availability of measurement units such as Digital Fault Recorders (DFR) or Phasor Measurement Units (PMU) for power system monitoring, it is easy to obtain measures of the transient behavior. According to this, in this paper the measurement-based method was used for dynamic load modeling.

Theoretical foundation of the measurement-based approach is system identification. Several authors proposed different strategies for load parameter estimation, where search algorithms have been used [6-10]. Examples of these include: Search algorithms based on statistical techniques such as Least Square (LS) based parameter estimation [6] and Weighted Least Square (WLS) based parameter estimation [7]. One of the difficulties of these methods consists on the convergence into a local minimum and its high sensitivity to the initial value of the parameters (e.g. proper weights). In other category are included the search algorithms that use metaheuristic techniques such as Simulated Annealing [8], Genetic Algorithms [9], Neural Networks [10], among others.

Particle swarm optimization (PSO) algorithm is an evolutionary technique for optimization that has been used in several applications related to power systems [11-14]. An application for measurement-based load modeling is shown in [15], where a parameter estimation process is solved using a standard particle swarm optimization algorithm for a
composite load with a high penetration of distributed generation.

Considering the previously exposed, this paper is focused on presenting an improved PSO algorithm which is used to develop a methodology for measurement-based dynamic load modeling for voltage stability studies, based on an exponential recovery load model structure.

This paper is organized as follows: first the load model is described in Section II. The measurement-based approach is presented in Section III. Next, the optimization technique for the parameter estimation process is presented in Section IV. Section V contains the proposed methodology for the development of load models based on measurements. The simulation results are presented in Section VI, and finally conclusions are given in Section VII.

II. DYNAMIC LOAD MODELING

Several dynamic load model structures have been proposed and most of them only were applicable to a particular system or for a specific study case [3].

Two of the most widely used dynamic load models are the Exponential Recovery Load (ERL) model [5] and the Composite Load (CL) model [16], which are specially designed for specific applications. For example, the ERL model is commonly used to approximate loads that recover slowly over a time period, which varies from several seconds to tens of minutes. Meanwhile, the CL model is employed in cases where Induction Motors (IM) are a dominant component. According to [3], due to the IM are responsible for consumption of approximately 60% to 70% of the total energy supplied by a power system, the CL model will quite often be applicable, but this is not a mandatory condition. Other characteristics must be evaluated in the load model selection, such as low computational effort, availability of measurements and comparison to determine if the model structure selected is better than others, in case of a specific set of measurements.

A. Composite load model

This structure is proposed in [16] and has been the object of many studies, where load is represented as a combination of a static ZIP for the representation of load static behavior, and a third-order induction motor model for the representation of load dynamic behavior. Dynamic part of the load model is represented using third-order induction motor equations, as is shown in (3) and (4).

\[
\begin{align*}
\frac{dE'_d}{dt} & = -\frac{1}{T_e}(E'_d + (X - X')I_d) - (\omega - 1)E'_q \\
\frac{dE'_q}{dt} & = -\frac{1}{T_e}(E'_q + (X - X')I_d) + (\omega - 1)E'_d \\
\frac{d\omega}{dt} & = -\frac{1}{2H}[T_0 - (E'_d I_d + E'_q I_q)]
\end{align*}
\]

where \( E'_d \) is the d-axis internal EMF, \( E'_q \) is the q-axis internal EMF, \( \omega \) is the mechanical speed, \( I_d \) and \( I_q \) are the d- and q-axis stator currents, respectively, \( R_s, X_s, R_m, X_m \) are the stator resistance, stator reactance, rotor resistance and rotor reactance, respectively and \( H \) is the inertia constant.

The static part of the load model is represented using a static ZIP load model, where load is modeled as a combination of constant impedance, constant current and constant power loads, as is presented in (6).

\[
\begin{align*}
P_{ZIP} & = P_{baseZIP} \left[ a_0 + a_1 \left( \frac{V}{V_0} \right) + a_2 \left( \frac{V}{V_0} \right)^2 \right] \\
Q_{ZIP} & = Q_{baseZIP} \left[ b_0 + b_1 \left( \frac{V}{V_0} \right) + b_2 \left( \frac{V}{V_0} \right)^2 \right]
\end{align*}
\]

Where \( P_{baseZIP}, Q_{baseZIP} \) are steady state ZIP load active and reactive power, \( a_0, a_1, a_2 \) are the constant power, constant current and constant impedance coefficients for active power, respectively, and \( b_0, b_1, b_2 \) are the constant power, constant current and constant impedance coefficients for reactive power, respectively. A detailed description of this model is presented in [16].

Using this load model structure, there are eleven parameters to be estimated: \( [R_s, X_s, X_m, R_m, X_m, H, T_0, a_0, a_1, a_2, b_0] \). The first seven parameters correspond to induction motor parameters, and the last four parameters correspond to static load parameters.

B. Exponential recovery load model

An exponential recovery load model, proposed in [17], is based on active and reactive power exponential response after a step disturbance at the bus voltage. Non-linear first-order equations are deployed for the representation of load response, which are shown in (7) and (8).

\[
\begin{align*}
T_p \frac{dx_p}{dt} & = -x_p + P_0 \left( \frac{V}{V_0} \right)^{Np} - P_0 \left( \frac{V}{V_0} \right)^{Np} \\
P_d & = x_p + P_0 \left( \frac{V}{V_0} \right)^{Np} \\
T_q \frac{dx_q}{dt} & = -x_q + Q_0 \left( \frac{V}{V_0} \right)^{Nq} - Q_0 \left( \frac{V}{V_0} \right)^{Nq} \\
Q_d & = x_q + Q_0 \left( \frac{V}{V_0} \right)^{Nq}
\end{align*}
\]
where $V_0, P_0, Q_0$ are the nominal bus voltage, active power and reactive power at the load bus respectively, $x_p, x_q$ are state variables related to active and reactive power dynamics, $T_p, T_q$ are time constants of the exponential recovery response, $N_{ps}, N_{qs}$ are exponents related to the steady state load response, $N_{pt}, N_{qt}$ are exponents related to the transient load response.

Based on (7) and (8), there are six parameters in total for the ERL model that must be estimated: $T_p, N_{ps}, N_{pt}, T_q, N_{qs}, N_{qt}$. In the next section, the measurement-based load approach to derive these load parameters is presented.

III. MEASUREMENT-BASED LOAD MODELING APPROACH

The measurement-based load modeling approach is developed in a two-stage procedure. Once measurements are obtained, a model structure is selected to represent load behavior. Later, a parameter estimation method is applied, to determine a set of parameters that minimizes the difference between measured data from the power system, and simulated data from the load model [2][18], as shown in the figure 1.

![Fig. 1. Block diagram of parameter estimation process for load modeling.](image)

After an event occurs in the power system, a dataset of voltage $V$, active power $P_{km}$ and reactive power $Q_{km}$ is acquired from load response. Voltage measurements can be used to calculate load model response for active and reactive power, denoted as $P_{ls}$ and $Q_{ls}$, respectively. Both measured and calculated values are used to evaluate the objective function of the optimization problem, and using this value, the optimization algorithm updates the load parameters $\theta$, to minimize the objective function [19]. In this paper, the PSO algorithm was used for this purpose, and the next section presents its basic formulation.

IV. PARTICLE SWARM OPTIMIZATION

PSO algorithm is a meta-heuristic optimization technique based on the behavior of birds and fish. Individuals of the population (particles) fly around in a multidimensional search space, combining local search methods and global search methods as the particles position is adjusted according to its experience and the experience of its neighboring particles, tending to the best position encountered by itself or its neighbors [20][21].

The initial population for the optimization algorithm is generated randomly or heuristically. Each particle is represented by a position vector, a speed vector and fitness value. Initial speed are generated randomly, and each speed vector is updated in every iteration, using the information of the relative position of the particles respect to the best position found by itself and the best position found by the swarm, as represented in (9).

$$v^{(t)}_k = \omega v^{(t-1)}_k + b_1 r_1 (P^{(t)}_k - x^{(t)}_k) + b_2 r_2 (P^{(2)}_k - x^{(t)}_k)$$

$$x^{(t+1)}_k = x^{(t)}_k + v^{(t)}_k$$

Where $v^{(t)}_k$ is the speed of particle $k$ at current iteration, $v^{(t-1)}_k$ is the speed of particle $k$ at previous iteration, $P^{(1)}_k$ is the best position found by the particle $k$ until the current iteration, $P^{(2)}_k$ is the best position found by the swarm until the current iteration and $x^{(t)}_k$ is the position of particle $k$ at current iteration. Then, particle speed is calculated as a function of the speed of the previous iteration, the relative position of the particle with respect to the best position found by itself, and the relative position of the particle with respect to the best position found by the swarm, where each of these components is weighted with the inertia weighting factor $\omega$, the cognitive coefficient $b_1$ and the social coefficient $b_2$. Variables $r_1$ and $r_2$ are uniformly distributed random numbers.

V. PROPOSED METHODOLOGY

The methodology used in this paper is described below.

A. Selected load model

As previously mentioned, to obtain a load model using measurements require, as initial step, the definition of the model structure. When the CL model is used to represent the dynamic characteristics of load under system disturbances, it is necessary to estimate eleven parameters of the model, according to the model described in Section II-A. Instead, when the ERL is used, only six parameters must be estimated. This represents a considerable difference in computational time. For this reason, an ERL model is used in this paper for the representation of the dynamic load response.

Later, a parameter estimation method is applied as a second stage. For this case, an improved PSO algorithm was used.

B. Improved PSO algorithm

It is observed that during PSO evolution, the particle speeds start with a high value to improve the exploration of the search space. Later, as the evolution process continues, the speeds of the particles decrease as these moves towards the optimal solutions. However, the magnitudes of the speed vectors may decrease in such a way that particles remain static in the search space [22]. This implies that a lot of computational time is required but the solutions may not improve after some iterations. For this reason, a speed restart mechanism is applied to generate a new set of speed vectors...
when the particle speeds reach low values.

If the maximum of all the speed vectors components is lower than a tolerance, the particles are moved around their current average position, using the vectors of relative position of each particle respect to the average position as the new set of speeds. By means of this mechanism, particle are moved based on the calculated speed vectors, so better solutions may be found because of the exploration of new areas in the search space. Fig. 2 illustrates the speed restart procedure.

![Image](image-url)

**Fig. 2. Illustration of the speed restart procedure**

The PSO algorithm implemented in this paper, featuring the speed restart mechanism explained above, is presented below:

**Initial adjusts:** define the number of particles $N$, inertia weighting factor $\omega$, cognitive coefficient $b_1$ and social coefficient $b_2$.

**Step 1:** Generate a set of initial $N$ particles and speeds.

**Step 2:** Evaluate the fitness function of each particle at their current position and initialize the information of the best particle position and the best swarm position.

**Step 3:** Move the particles using the speed vectors generated in Step 1.

**Step 4:** For the new position, evaluate fitness function. If a particle finds a better solution than the value stored in its own memory, the best value and position of the particle are updated. If the swarm finds a better fitness function than the best value stored in the swarm memory, the best value and position of the swarm are updated.

**Step 5:** Check the speed restart criterion. If true, go to step 6; otherwise, go to step 7.

**Step 6:** Determine the average position of the particles, calculate the vectors of relative position of the particles with respect to the average position and move the particles using the calculated vectors. Go to step 8.

**Step 7:** Calculate a new set of speeds using (9) and move the particles.

**Step 8:** Check the stopping criterion. If the stopping criterion is satisfied, stop. Otherwise, go to Step 4.

Considering the PSO algorithm, the initial population is generated using uniformly distributed random numbers in specified intervals, as is recommended in [23] and also adjusted experimentally. The parameters are shown in Table I.

### Table I: Intervals for initial population

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>0.001</td>
<td>0.1</td>
</tr>
<tr>
<td>$N_{P\mu}$</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$N_{Q\mu}$</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$T_g$</td>
<td>0.001</td>
<td>0.1</td>
</tr>
<tr>
<td>$N_{P\epsilon}$</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$N_{Q\epsilon}$</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

C. **Objective function and stopping criterion**

The objective for the optimization process is aimed to minimize the sum of the square difference of the measured data and the simulated data. Objective function is shown in (11).

$$
\min \frac{1}{N} \sum_{i=1}^{N} \left[ (P_{i,m} - P_{i,s})^2 + (Q_{i,m} - Q_{i,s})^2 \right]
$$

where $N$ is the numbers of samples [19]. A penalty factor is added to the fitness function if any component of the particle vector is lower than zero. The stopping criterion for the optimization process is based on monitoring the evolution of the best solution of the swarm; if this solution does not improve after a specified number of iterations, the algorithm stops. Moreover, the algorithm is stopped if a maximum number of iterations are reached.

D. **Initialization of PSO algorithm**

The parameters of the PSO algorithm were determined by exhaustive testing, where the parameters that presented the best performance are summarized in Table II. Inertia weighting factor $\omega$ was changed from 0.5 to 1.0, cognitive coefficient $b_1$ and social coefficient $b_2$ were changed from 1.0 to 2.0 [13].

### Table II: Parameters of the PSO algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia weighting factor $\omega$</td>
<td>0.7</td>
</tr>
<tr>
<td>Cognitive coefficient $b_1$</td>
<td>1.5</td>
</tr>
<tr>
<td>Social coefficient $b_2$</td>
<td>1.5</td>
</tr>
<tr>
<td>Maximum number of iterations</td>
<td>1000</td>
</tr>
<tr>
<td>Number of iterations to stop the algorithm is solution is not improved</td>
<td>50</td>
</tr>
</tbody>
</table>

For the purpose of this paper, parameters of different load models are identified using the PSO algorithm and measurements of voltage, active and reactive power after a disturbance. Once the models are estimated, a validation test is carried out, in order to analyze the model response using datasets that were not included on the estimation process.

VI. **RESULTS AND DISCUSSION**

Tests were carried out in a modified IEEE 14 bus test system shown in Fig. 3, where induction motors were included at load buses 5, 10, 11 and 12 to represent a dynamic load. Those composite loads are represented using exponential recovery load models.
A. Case 1: Model estimation procedure

Initially, the parameters of the exponential recovery load models for the dynamic loads are estimated using measurements obtained after a disturbance (three-phase fault at line 10-11). Due to the space limitation, only loads models for buses 10 and 12 are presented in this paper. Voltage responses at the analyzed buses are depicted in Fig. 4.

The load model responses using the estimated parameters of Table III and the voltage measurement of Fig. 4 are shown in Fig. 5 and 6 for loads at buses 10 and 12, respectively.

The estimated parameters for each load and the estimation errors are summarized in Table III.

<table>
<thead>
<tr>
<th>Parameters after the optimization process</th>
<th>Bus 10</th>
<th>Bus 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>0.00611595</td>
<td>0.00867302</td>
</tr>
<tr>
<td>$N_{ns}$</td>
<td>1.81624098</td>
<td>1.55338333</td>
</tr>
<tr>
<td>$N_{ps}$</td>
<td>2.1273348</td>
<td>2.13001792</td>
</tr>
<tr>
<td>$T_q$</td>
<td>0.00592844</td>
<td>0.01905618</td>
</tr>
<tr>
<td>$N_{qs}$</td>
<td>1.9227372</td>
<td>1.23120496</td>
</tr>
<tr>
<td>$N_{qt}$</td>
<td>2.0475567</td>
<td>2.0094539</td>
</tr>
<tr>
<td>Error</td>
<td>1.0367E-06</td>
<td>1.5403E-05</td>
</tr>
</tbody>
</table>

As seen in Figs. 5 and 6, the model response accurately fits to the measurements obtained from the power system under the evaluated condition, for both steady state and transient
response, which is also observed in the estimation error value of Table III.

B. Case 2: Model validation after an event located near to the analyzed buses.

To validate the obtained load model, a new disturbance is applied to the power system, and load response is compared with the model response using the new measurements and the estimated parameters of Table III. An outage of line 9-10 in t=0.1s is simulated and the respective voltage measurements at buses 10 and 12 are depicted in Fig. 7.

![Voltage measurement at buses 10 and 12 after a line outage.](image)

The estimation results of active and reactive power at load buses 10 and 12 are shown in Fig. 8 and 9.

![Comparison of estimated and measured active and reactive power at bus 12 after a line outage near to the study buses.](image)

It is clear from Figs. 8 and 9 that the load model response is adequate for the representation of load behavior, even in case of a small disturbance such as a line outage. The estimation errors are summarized in Table IV.

<table>
<thead>
<tr>
<th>Loads</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 10</td>
<td>4,3310E-07</td>
</tr>
<tr>
<td>Bus 12</td>
<td>6,8213E-06</td>
</tr>
</tbody>
</table>

C. Case 3: Model validation after an event located far from the analyzed buses.

An additional test is carried out to validate the model response after a disturbance that occurs far from the studied buses, using the parameters shown in Table III and a new set of measurements of a three-phase fault at line 2-5 with duration of 100ms. Voltage measurements at buses 10 and 12 are depicted in Fig. 10.

![Voltage measurement at buses 10 and 12 after a line outage.](image)

Using these measurements, the estimation results of active and reactive power at load buses 10 and 12 and their comparison with the real load responses are shown in Fig. 11 and 12.

![Comparison of estimated and measured active and reactive power at bus 10 after a line outage near to the study buses.](image)
The adjustment errors between the model response and the real response of the load are shown in Table V.

<table>
<thead>
<tr>
<th>Loads</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 10</td>
<td>1,4169E-06</td>
</tr>
<tr>
<td>Bus 12</td>
<td>1,4698E-04</td>
</tr>
</tbody>
</table>

Fig. 10. Voltage measurements at buses 10 and 12 after a fault at line 2-5.

Fig. 11. Comparison of estimated and measured active and reactive power at bus 10 after a fault far from the study buses.

In this case, it can be seen that the estimated load models have good capabilities for the representation of load behavior under several disturbances, including disturbances near and far from the analyzed buses. For the parameter estimation, it is important to notice that variations of the measured variables should be perceptible, in order to obtain a better approximation of load dynamic behavior.

From the obtained results, it is marked that load models may be developed using one set of measurements and the estimated models are adequate for several conditions. Extensive studies are required to analyze the generalization capabilities of the developed models under this strategy.

VII. CONCLUDING REMARKS

A methodology for the development of dynamic load models for small disturbances was presented. One of the major contributions of this work is to demonstrate that an exponential recovery load model could be proposed when the load in a specific bus has considerable number of induction motors and the voltage variations remains near of the rated operating point. This represents an advantage under the CL model commonly used, which has a high number of parameters to be calculated.

The models of two composite loads were developed using a data set of measurements from the power system, and posteriorly, the obtained load models were validated using different datasets of new disturbances. One of the advantages of the proposed methodology is that load models can be developed using one set of measurements and can be applied...
to other operational conditions with satisfactory results. These measurements can be obtained from normal operational disturbances, and then additional disturbances are not required for the development of accurate load models.

Finally, the approach here proposed helps to provide a simplified way to represent the dynamic characteristic of the load, which is useful in voltage stability studies.

REFERENCES


