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Self and Mutual Transmission Line Impedance Estimation by Means of the Non-Linear Least Squares Method

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***Abstract*—This work presents two methods for estimating the self and mutual impedance of a tranmission line by means of the non-linear square method. The first method uses voltage magnitude, active power and reactive power measurement, whereas the second method uses synchronized phasor voltage and current measurements at both ends of the transmission line. The line impedances values were estimated simultaneously with the line voltages. No restriction was impose to the values of the impedances to be estimated. Both methods were tested in simulated conditions on a 2-bus test system. Parameter estimations with errors below $1[\%]$ were obtained.**

***Index Terms*—Parameter estimation, Phasorial measurement units, power systems, non-linear least squares, self and mutual impedance.**

I. Introduction

THE reliable operation of a power grid is higly dependent on the value of the parameters used to estimate the states of the system. Furthermore, exact tranmission line models are essential for determining the fault locations, short circuit analysis, fault analysis and relay settings.

The self and mutual impedance values can be calculated by solving Maxwell's equations for the boundary coinditions at the surface of the conductors, air and ground [14]. Nevertheless, the value of these impedances is a function of the frequency, specific resistance, magnetic permeability and dielectric permitivities [5]. Thus, usually approximations are used in order to simplify the calculations of the impedances. The main approximations can be spplitted into two groups: geometrical and electromagnetical. The geometrical approximations refer to assumming that the soil surface is a plane, that the line cables are horizontal and parallel among themselves and that the distance between any two conductors is much higher than the radious of those. Whereas the electromagnetical approximations refers to assuming that there is no electromagnetic field effects from near structures nor insulators and, that the electromagnetic field can be considered quasi-stationary [5], [14].

A tranmission line can cross simultaneously different regions, each of which could have different values for the soil resistivity. The value of the soil resistivity in a mountain region is usually different from the value in a plain region or in a coastline. Furthermore, the soil resistivity can have different values for the same region at different times of the year, depending on different conditions such as dryness and temperature, which are higly dependent on the current season. Therefore relying on constant parameters to calculate the line impedances can have several innacuracities [14].

The state estimators calculate the magnitude and the angle of the bus voltages using a set of redundant measurements, including the magnitude of those and the power fluxes through the transmission lines. A basic problem of the state estimators is the need to set values for all the parameters in the system. In order to solve the equations of a state estimator, exact values for all the parameters of the transmission lines must be assumed for each line in the power system. This condition implies that all the impedances are calculated under theoretical assumptions and not under working conditions [1], [2].

This paper proposes two methods to estimate the self and mutual impedance of a transmission line represented by lumped parameters. Both methods make no assumption about the value of the parameters to be estimated. All the parameters are calculated simultaneuosly with the states of the system in order to check its convergence. Since no assumption is made regarding the value of the parameters, the estimated values account for all the differences between a theoretical approach and the working conditions.

II. Line Parameters

The voltage drop along a transmission line per unit length can be described in the form of partial differential equations, e.g., for a single-phase line as [5]:

$$-\frac{\partial v(x,t)}{\partial x} = Ri(x,t) + L\frac{\partial i(x,t)}{\partial t} \tag{1}$$

Where R and L are the per unit length resistance and inductance, respectively. Since the parameters R and L are not constant but a function of frequency, it is not common to use (1); Instead, the voltage drops per unit length are expressed in the form of phasor equations for AC steady state conditions at a specified frequency as [5]:

$$-\begin{bmatrix} \frac{\partial V_1}{\partial x} \\ \frac{\partial V_2}{\partial x} \\ \vdots \\ \frac{\partial V_n}{\partial x} \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \dots & Z_{1,n} \\ Z_{2,1} & Z_{2,2} & \dots & Z_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n,1} & Z_{n,2} & \dots & Z_{n,n} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \quad (2)$$

Where:
V_i : Voltage phasor, measured from conductor i to ground.
I_i : current phasor in conductor i.
n : number of conductor in the transmission line.
In general:

$$-\frac{d\mathbf{V}(x)}{dx} = \mathbf{Z} \cdot \mathbf{I}(x) \quad (3)$$

$\mathbf{V}(x)$: Vector of phasor voltages,
$\mathbf{I}(x)$: Vector of phasor currents in the conductors.
Implied in (3) is the existence of ground as a return path, to which all voltages are refered. The matrix $\mathbf{Z} = \mathbf{R}(\omega) + j\omega\mathbf{L}(\omega)$ is called the series impedance matrix; it is complex and symmetric. The diagonal elements $Z_{ii} = R_{ii} + j\omega L_{ii}$ are the series self-impedances per unit length of the loop formed by conductor i and the ground return. The off-diagonal elements $Z_{ik} = Z_{ki} = R_{ki} + j\omega L_{ki}$ determine the longitudinally induced voltage in conductor k if a current flows in conductor i, or vice versa and, are called the series mutual impedances per unit length between conductors i and k [5].

A. Sequence Parameter of Balanced Lines

A "balanced" transmission line is a line where all diagonal elements of $\mathbf{Z}_{phase}$ and $\mathbf{C}_{phase}$ are equal among themselves, and all off-diagonal elements are equal among themselves [5]. Single-circuit three-phase lines become more or less balanced if the line is transposed, provided the length of the span is much less than the wavelength of the frequencies involved in the particular study [5]. If the span length is much shorter than the wavelength, then series impedances can be averaged by themselves through the three sections, as well as shunt capacitances [5].
Balanced single-circuit three-phase lines can be studied much easier with symmetrical components because the three coupled equations in the phase domain, become three decoupled equations. Symmetrical components for three-phase lines are calculated with [5]:

$$\mathbf{V}_{phase} = \mathbf{A}\mathbf{V}_{symm} \quad (4)$$

where

$$\mathbf{V}_{symm} = \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix}$$

is the vector of symmetrical voltage components, and

$$\mathbf{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$a = e^{j120^\circ}$$

Applying this transformation to the voltage and current vectors in (3) produces:

$$-\begin{bmatrix} \frac{dV_0}{dx} \\ \frac{dV_+}{dx} \\ \frac{dV_-}{dx} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \begin{bmatrix} I_0 \\ I_+ \\ I_- \end{bmatrix} \quad (5)$$

from which the zero, positive and negative impedance equations can be extracted by simple inspection as:

$$Z_0 = Z_s + 2Z_m \quad (6)$$

$$Z_+ = Z_s - Z_m \quad (7)$$

Using the three decoupled equations in (5) instead of the three coupled equations allow us to solve the line as if it consisted of three single-phase lines, which is much simpler than trying to solve the equations of a three-phase line.

III. Non-Linear Least Squares

Given a set of measurements $\mathbf{z}_{m\times1}$, a vector of states $\mathbf{x}_{n\times1}$, and, a vector to account for the measurement erros $\mathbf{e}_{m\times1}$, the measurements can be expressed as [1], [2]:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \quad (8)$$

Where $\mathbf{h}(\mathbf{x})_{m\times1}$ are the equations that relates the direct measurements with the states. The real values of the states variables $\mathbf{x}$ can not be found [6] but, the *estimations* $\hat{\mathbf{x}}$ can be calculated as:

$$\hat{\mathbf{e}} = \mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}) \quad (9)$$

The quantities with $\hat{\tau}$, where τ is any variable, are the estimations of the ones without it. In (9), the vector at the left side $\hat{\mathbf{e}}$ represents the difference between the actual measurements $\mathbf{z}$ and its estimated value, which can be written as:

$$\hat{\mathbf{e}} = \mathbf{z} - \hat{\mathbf{z}} = \mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}) = \mathbf{e} - \mathbf{h}(\hat{\mathbf{x}} - \mathbf{x}) \quad (10)$$

The least squares method is used to minimize the sum of the squares of the errors estimation shown in (9) given by:

$$f = \sum_{j=1}^{m} e_j^2 = \mathbf{e}^T\mathbf{e}$$

The best estimations of the state variables are the ones that minimize f. To minimize the function f, the estimations $\hat{\mathbf{x}}$ are the ones that satisfy the following equation [1], [2]

$$\left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}} = 0 = \mathbf{H}^T\Big|_{\hat{\mathbf{x}}} \cdot (\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})) \quad (11)$$

Where

$$\mathbf{H} = \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \quad (12)$$

is known as the Jacobian matrix.
To solve the problem of estimation of the states variables, $\mathbf{h}(\mathbf{x})$ is linearized near an initial point $\mathbf{x}_0$ in the following way

$$\mathbf{h}(\mathbf{x}) = \mathbf{h}(\mathbf{x}_0) + \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}_0} (\mathbf{x} - \mathbf{x}_0) \quad (13)$$

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Replacing (13) into (11) leads to

$$\mathbf{x} = \mathbf{x}_0 + (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T(\mathbf{z} - \mathbf{h}(\mathbf{x}_0)) \tag{14}$$

In an iterative way, the previous equation can be generalize as follows

$$\mathbf{x}_{k+1} = \mathbf{x}_k + (\mathbf{H}_k^T\mathbf{H}_k)^{-1}\mathbf{H}_k^T(\mathbf{z} - \mathbf{h}(\mathbf{x}_k)) \tag{15}$$

The state estimation x_k is calculated until two successive solutions converge inside an accuracy previously determined; that is $|\mathbf{x}_{k+1} - \mathbf{x}_k| < \varepsilon$. The parameter estimation consist of appending the parameter suspected of being wrong to the state vector $\mathbf{x}$. The Jacobian $\mathbf{H}$ is then calculated and (15) is executed until two states converge.

IV. Line Impedance Estimations

Two methods are proposed; the diference between the them is mainly the type of measurement that are used. The first method uses the typical measurements in a power system, i.e. voltage magnitude, active and reactive power per phase of the transmission line, whereas the second method uses Phasorial Measurement Units (PMU) measurements, i.e. positive, negative and zero sequence voltage and current per phase of the transmission line.

On both methods the full set of measurements were taken from a test circuit simulated on ATP [3]. The circuit represents the behavior of a transmission line in steady-state conditions. These measurement were later disturbed with white noise in order to simulate the behavior of an actual measurement device. Both the noise and, the estimation of the states and parameters were done using Matlab [8].

Both methods calculate the parameters by appending those to the state vector and solving it by the non-linear least squares method. Allowing the full set of parameters to be estimated simultaneously with the states of the system, ensures that no assumption is made regarding the values of them.

A. Parameter Estimation in the Phase Domain

The proposed set of equations consist of the phase-voltages, phase active power and phase reactive power in a transmission line of the power network. The mathematical model, in the phase domain, of a 3-phase transmission line obeys the following equations.

$$\mathbf{Y}_{3\times3} = \begin{pmatrix} Y_s & Y_m & Y_m \\ Y_m & Y_s & Y_m \\ Y_m & Y_m & Y_s \end{pmatrix} \tag{16}$$

$$\mathbf{I}_{phase} = -\mathbf{Y}_{3\times3} \cdot \triangle\mathbf{V}_{phase} \tag{17}$$

$$S_i^* = V_i^* \cdot I_i \tag{18}$$

Where the subscript i represents the phase whose power is been calculated, i.e. a, b, c. The Active and Reactive power are determined by:

$$P_i = Re\{S_i^*\} \tag{19}$$

$$Q_i = -Im\{S_i^*\} \tag{20}$$

With (19) and (20) and asssuming the full set of quantities are being measured, the complete set of equations has: 6 Voltages Magnitudes, 6 Active Power and 6 Reactive Power. The number of unknown variables range from 11 to 15: 6 Voltages magnitudes, 5 angles and 4 parameters (if included). Thus, the redundancy factor range from 1.2 to 1.63.

Note that the previous equations do not take into account the capacitance of the transmission line, this is because the capacitance of the line is split into two parts, i.e. one at the beginning and one at the end of the line, also the capacitances in the phase domain connect each phase with the other phases and with ground return, making the circuit model too complicated for a proper parameter estimation model.

B. Parameter Estimation in the Sequence Domain

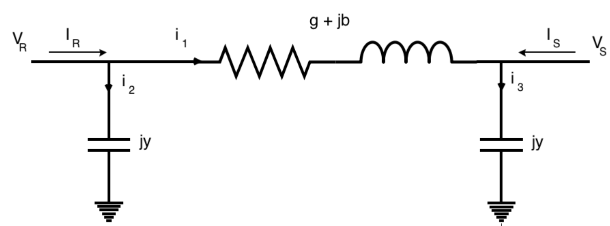

Fig. 1. Pi model for the three decoupled sequences

As previously discussed, a transposed single circuit three-phase transmission line can be studied more easily in the sequence domain using three decoupled equations. If the three sequence parameters Z_0, Z_+, Z_- are estimated, then the self and mutual impedances can be calculated by:

$$\mathbf{Z}_{phase} = \mathbf{A}\mathbf{Z}_{symm}\mathbf{A}^{-1} \tag{21}$$

Since it is not possible to measure the value of the Active or Reactive Power in the zero or negative sequence, it is necessary to use phasor measurements to estimate the value of the sequence parameters. The mathematical model used for the simultaneous estimation of states and parameters uses the π sequence-model of a transmission line. In rectangular coordinates, the equations that relate the state variables and the parameters with the measurements for the zero sequence are as follows.

$$I_{o_R} = i_{o_1} + i_{o_2} = (V_{o_R} - V_{o_S})(g_o + jb_o) + V_{o_R}(jy_o) \tag{22}$$

$$I_{o_S} = i_{o_3} - i_{o_1} = (V_{o_S} - V_{o_R})(g_o + jb_o) + V_{o_S}(jy_o) \tag{23}$$

For the positive sequence:

$$I_{p_R} = i_{p_1} + i_{p_2} = (V_{p_R} - V_{p_S})(g_p + jb_p) + V_{p_R}(jy_p) \tag{24}$$

$$I_{p_S} = i_{p_3} - i_{p_1} = (V_{p_S} - V_{p_R})(g_p + jb_p) + V_{p_S}(jy_p) \tag{25}$$

And for the negative sequence:

$$In_R = i_{n_1} + i_{n_2} = (V_{n_R} - V_{n_S})(g_p + jb_p) + V_{n_R}(jy_p) \tag{26}$$

$$I_{n_S} = in_3 - i_{n_1} = (V_{n_S} - V_{n_R})(g_p + jb_p) + V_{n_S}(jy_p) \tag{27}$$

Both the positive and negative sequence share the same parameter values, due to the fact that the line is assumed to

be transposed and single circuit.
Since there are two current equations and two voltages equations for each sequence and both the real and imaginary part have to be expressed, a total of 24 equations conform the complete set of equations.
The currents are a function of the voltages, therefore 6 voltages have to be estimated, resulting in 12 states (imaginary and real part). If the parameter of the system are also estimated, then the number of unknown variables is 18.
Base on the aforementioned, the redundancy factor range from 1.33 to 2, which is higher than for the first set of proposed equations.

V. Results

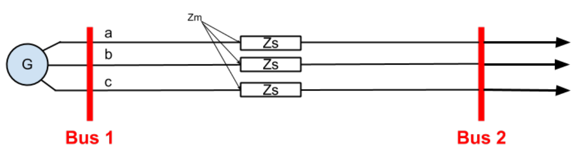

Fig. 2. Test system.

The test system is a radial two-bus system with an ideal generator connected to one sending end of the tranmission line, while the load is connected to the receiving end of it. The test system is shown in Fig. 2 with each phase named by the letters a, b, c.
The tower configuration is shown in Fig. 3. The earth resistivity is $100[\Omega m]$ and, the line has a length of $30[km]$. The generator is an ideal 3-phase $60[Hz]$ source of $230[kV]$ line-to-line. All the errors were set to have a Gaussian distribution with mean zero and standard deviation equal to $1/3$ of the error class thus ensuring that at 3 times the standard deviation, 95% of all the generated error values will be within the class specification [10].
In the following tables, the symbol μ refers to the relative error, whereas σ refers to the standard deviation of the total dataset. This section only shows the results of the algorithms; the analysis of those is done in VI.

A. Parameter Estimation in the Phase Domain

In order to get a good statistical sample, one thousand simulations were made. Given the failures in convergence for an unconstrained optimization problem, the method was switched to a constrained optimization problem. For each dataset, an estimation is made and the relative error is calculated. Finally the standard deviation is calculated using the error vector obtained from all the estimations. For this method, two cases were simulated: softly unbalanced load and very unbalanced load. The softly unbalanced load was assumed as: $Z_a = 30 + j12[\Omega], Z_b = 32 + j14[\Omega], Z_c = 28 + j10[\Omega]$, while the very unbalanced load was assumed as: $Z_a = 10 + j10[\Omega], Z_b = 20 + j5[\Omega], Z_c = 30 + j5[\Omega]$. Table I shows the results for class 3 instruments with softly and very unbalanced load.
In Table I the variables d are the relative angle with respect

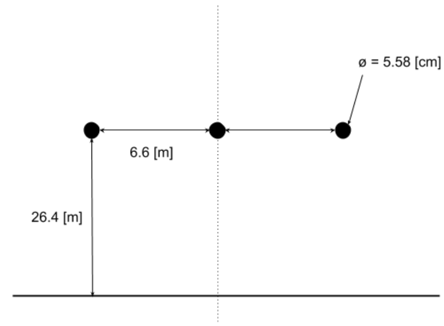

Fig. 3. Transmission line configuration

to the reference angle d_{a_1}, g and b are the conductance and suceptance that result of expanding the admittances shown in (16). The sub-index 1 means the bus where the generator is connected, whereas 2 means the bus where the load is connected.

TABLE I
Mean and Standard deviation of the error in the state and parameter estimations in the phase domain with softly and very unbalanced load for class 3 measurement devices

	Very		Softly	
Var	$\mu[\%]$	$\sigma[\%]$	$\mu[\%]$	$\sigma[\%]$
V_{a1}	-0.1239	1.1542	0.2255	1.569
V_{b1}	-0.0608	1.2209	-0.3463	1.732
V_{c1}	0.0541	1.0769	0.3990	2.022
V_{a2}	-0.1609	1.4082	0.2354	2
V_{b2}	-0.0548	1.4208	-0.3918	2.004
V_{c2}	0.059	1.1828	0.4143	2.125
d_{b1}	0.6181	6.4707	-1.715	12.28
d_{c1}	-0.3767	3.6347	-10.05	17.68
d_{a2}	-0.3885	6.239	-0.1022	2.75
d_{b2}	0.4577	4.9349	-1.519	10.87
d_{c2}	-0.3597	3.6326	-11.90	20.94
g_s	-0.129	9.9481	-12.24	34.69
g_m	0.9721	30.75	57.19	115.07
b_s	-0.1997	5.1801	1.151	25.28
b_m	0.9615	2.9024	-3.848	81.30

B. Parameter estimation in the sequence domain

For this method, one thousand simulations were also made. The noise and error calculations were done in the same manner as for the previous case. During the implementation some errors were too large, therefore the simulated cases were changed to a softly unbalanced source and a softly unbalanced load. The unbalanced source has: $V_a = 187794\angle 0°[V_{peak}], V_b = 183551\angle -110°[V_{peak}]$, $V_c = 192036\angle 135°[V_{peak}]$. For the softly unbalanced source case, the load was set as balanced, whereas the softly unbalanced load is the same as in the previous method.
In Table II the sub-indexes mean sequence zero (o), positive (p) and negative (n). The capacitance of the transmission line was set to zero inside the algorithm in order to be able to

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compare the results with the ones from the previous algorithm. Table II shows the results obtained for these conditions.

TABLE II
MEAN AND STANDARD DEVIATION OF THE ERROR IN PARAMETER AND STATE ESTIMATIONS IN THE THREE SEQUENCES DOMAIN FOR SOFTLY UNBALANCED SOURCE

	Source		Load	
Var	$\mu[\%]$	$\sigma[\%]$	$\mu[\%]$	$\sigma[\%]$
Re(V_{o1})	-0.0088	1.0153	0.0082	0.9821
Im(V_{o1})	-0.0072	1.037	0.0536	1.0454
Re(V_{o2})	0.0229	0.9980	-0.0151	1.0033
Im(V_{o2})	0.0420	1.0066	0.0395	1.0593
Re(V_{p1})	0.0055	1.0017	-0.0256	0.9923
Im(V_{p1})	0.0010	0.9411	-2e+07	2e+08
Re(V_{p2})	0.0037	1.0375	-0.0507	0.9814
Im(V_{p2})	0.0211	1.0371	0.0372	0.9818
Re(V_{n1})	0.0498	0.9782	-2e+07	3e+09
Im(V_{n1})	-0.0304	0.988	-1e+09	2e+10
Re(V_{n2})	0.0218	1.0009	0.0336	0.9870
Im(V_{n2})	0.0411	0.972	0.0464	0.9876
g_o	-0.0556	4.8141	0.1444	5.1608
g_p	-0.96574	19.833	-1.0412	16.367
b_o	0.0543	1.6925	-0.0341	1.0136
b_p	-0.0107	2.897	-0.0240	3.0874

VI. Discussion

As seen in Table I the first proposed method sucessfully estimates the values of the states of the transmission line, nevertheless the standard deviation for the softly unbalanced case can be very large, therefore this method is not recommended for online parameter estimations under these conditions.

For the very unbalanced load case, the method is able to estimate the value of the parameters without significant errors. However the standard deviation of the g_m is very large due to this parameter closeness to zero. Nevertheless, according to the obtained results, if an offline parameter estimation is done under these conditions and a big sample is taken, the mean value of the parameter estimations could be very close to the real value.

As shown in Table II, the results obtained for the unbalanced load case can have large errors. This is due to the fact that the generator was assumed as an ideal one, therefore the value of the zero and negative sequence value is zero. In real operating conditions, this case is not possible due to the inherent unbalance of both the source and the load.

For the softly unbalanced source case, the method sucessfully estimates the states and parameter of the transmission line. As can be seen, all the mean errors are under $1[\%]$ and besides the standard deviation of the g_p parameter, all the other tandard deviations are under $5[\%]$. This method estimate the full set of states and parameter simultaneously. Thus, no restriction was impose on those values. This method gave the best results for a parameter estimation.

Taking into account the mean and standard deviation of the softly unbalanced source case, this method could be use for both online and offline parameter estimation.

VII. Conclusions

The results here shown are obtained only for some basic cases. A more throughly revision of the methods and equations is necessary to fully implement them in a given system.

These methods can be improved if the redundancy factor of the equation system is increased. Such an increase can be made by appending sets of measurements from different times into the equation system. These measurement relate to the same line parameters while providing different data sets. More information can be found in [1] [2].

If both the self and mutual impedance of a transmission line are calculated and, assuming that all geometrical characteristic of the line are known, the mean resistivity of the transmission line can also be estimated using the Deri equations [5].

In this work a mutual conductance was estimated due the use of Deri's equations, nevertheless if real ground-return-path equations are used, then the mutual conductance can be assumed as zero and only three parameters should be estimated, thus improving the redundancy factor.

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