

Recent Developments on the Lightning Performance of Transmission Lines

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Abstract— This work aims to present recent developments in the topic lightning performance of transmission lines, highlighting the relevance of the soil impulsive behavior, especially the variation of soil electric parameters with frequency, the use of realistic representations of the waveform of the lightning current, and the application of models designed to accurately represent the electromagnetic couplings among the components of the system.

Index Terms—Backflashover, Lightning performance of transmission lines, Soil resistivity and permittivity, Impact of frequency-dependent electric parameters of soil on backflashover.

I. INTRODUCTION

LIGHTNING is a frequent cause of transmission line outages. For those lines below 500 kV installed in regions whose soil present moderate or high resistivity values, the backflashover prevails as the main mechanism of lightning-related outages [1-2].

The evaluation of the lightning performance of transmission lines is based on the outage rate per 100 km of the line per year. The specification of such rate is fundamental to determine the more convenient protective practice to be adopted. These may be classified as conventional such as the reduction of tower-footing grounding impedance or the installation of surge arresters to prevent flashovers, or non-conventional, such as the use of underbuilt wires connected to the tower [3].

Several methodologies to estimate the backflashover rate of transmission lines are presented in the literature [4-7]. However, such traditional methodologies adopt several simplifications, whose generality of application is not deeply verified. The Brazilian experience has shown that differences between real and estimated lightning performance of transmission lines are common. Such differences may be related to simplifications adopted by such methodologies for modeling the impulsive behavior of grounding system, the representation of lightning currents, flashover model, etc.

The proposed work aims to present recent developments in the topic lightning performance of transmission lines, highlighting the relevance of the soil impulsive behavior, especially the variation of soil electric parameters with

frequency [8-10], the use of realistic representations of the waveform of the lightning current [11], mainly relating to the peculiar characteristics of its wavefront and the application of models designed to accurately represent the electromagnetic couplings among the components of the system (overhead cables, tower and grounding).

In this work, the fundamental aspects of the aforementioned developments are evaluated in terms of the impact to the lightning performance of transmission lines. Following, a case study is developed to denote the application of these contributions, considering a real 138-kV transmission line.

II. DEVELOPMENTS TO IMPROVE THE EVALUATION OF LIGHTNING PERFORMANCE OF TRANSMISSION LINES

A. Representation of the lightning current waveform

Investigations of the lightning performance of transmission lines traditionally adopt simplified representations of the lightning current waveform. Some studies employ double exponential or triangular waves, and more recently, waves obtained by means of the sum of Heidler functions have been applied.

However, all these representations differ significantly from the individual registers of current waves commonly measured at instrumented towers [12-13], implying on overvoltages across line insulators very different (in terms of shape and amplitude) from those developed in response to real lightning currents.

The authors' research group developed in [11] the characterization of the typical waveform of real lightning currents and proposed in [14] analytical formulae of such currents along with the parameters to describe their amplitude and associated time. Fig. 1 illustrates impulsive current waves that closely reproduce the median peak currents (I_p) and front time parameters of first- (FST) and subsequent-strokes (SUB) measured at Mount San Salvatore [13].

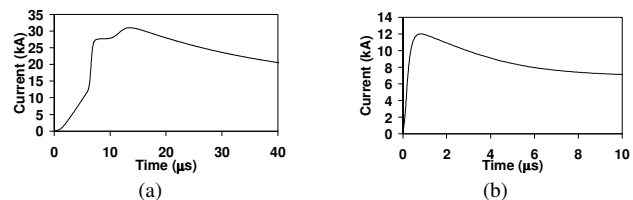


Fig.1. Representative current waveforms with median peak currents and front times of first (a) and subsequent (b) strokes. FST: $I_p = 31.1$ kA, $Td30 = 3.83$ μ s, SUB: $I_p = 11.8$ kA, $Td30 = 0.67$ μ s.

The first stroke current representation includes the typical

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initial concavity followed by an abrupt rise around the half-peak value and the second peak usually higher than the first peak usually indicated in measured data. The waveform of the subsequent stroke current present a relatively smooth shape and only one peak.

Similar waveforms, but considering median current parameters measured at Morro do Cachimbo station, Brazil (MCS) were also developed [14]. The use of this kind of current representation allows obtaining realistic results in terms of the resulting overvoltages across insulators of transmission lines.

B. Frequency-dependent electric parameters of soil

Generally, the evaluations of lightning performance of transmission lines do not take into account the effect of the frequency dependence of soil parameters on the resulting overvoltages, mainly to the lack of formulations able to compute such effect. The resistivity used to be assumed as the value measured by commercial instruments that employ low frequencies and the relative permittivity of soil used to be considered to vary from 4 to 81 [8].

However, Visacro and Alipio developed accurate expressions to consider such effect [9] based on a methodology experimentally validated for measuring the frequency variation of soil resistivity and permittivity in the range of frequency content of lightning currents [8]. Such equations are reproduced as follows:

$$\rho = \rho_0 \{1 + [1.2 \cdot 10^{-6} \cdot \rho_0^{0.73}] \cdot [(f - 100)^{0.65}]\}^{-1} \quad (1)$$

$$\epsilon_r = 7.6 \cdot 10^3 f^{-0.4} + 1.3 \quad (2)$$

where ρ is the soil resistivity at frequency f (Hz), ρ_0 is the soil resistivity at 100 Hz and ϵ_r is the soil relative permittivity at frequency f (Hz). Expressions (1) and (2) are valid in the ranges of 100-Hz to 4-MHz.

The relevance of the frequency dependence effect on the lightning response of grounding electrodes was evaluated in [10]. It was found a general decrease on the resulting grounding impedance. Also, in [15,16], it was verified this effect on the lightning performance of transmission lines. The decrease of tower-footing grounding impedance for frequency-dependent soil parameters contributes to reduce the overvoltage across insulator strings, and consequently may diminish the resulting line outage rate.

C. Elaborate computational model for calculating overvoltage across transmission line insulator strings

The evaluation of the condition needed for flashover occurrence across insulator strings for a given lightning current depends on the resulting overvoltage wave across insulators and the insulation withstand. The quality of such evaluation is related to the accuracy of the overvoltage representation.

Traditionally, the calculation of such overvoltage is performed by analytical approach or by means of computational models such as EMTP/ATP based on distributed circuit parameters. In both cases, the estimated overvoltage wave is not accurate enough due to the difficulty

of such approaches in representing precisely physical aspects involved in the problem, particularly, the electromagnetic coupling among the components of the system (overhead cables, tower and grounding).

The use of electromagnetic models allows performing accurate calculation of the resulting overvoltages, although the long processing time involved. Since such models are developed from Maxwell's equations, they are able to provide general solutions without the need of checking the validity for each kind of application, as required in modeling that use analytical approaches or distributed circuit parameters. Furthermore, the physical system is represented directly from the geometry of the components of the system and the constants of the involved medium. Also, the solution automatically contemplates the complex electromagnetic coupling and the propagation effects.

The authors have been using an advanced computational model named HEM (Hybrid Electromagnetic Model) to simulate the resulting overvoltages across insulator strings due to direct strikes to transmission lines [17-18]. It is formulated by means of fundamental electromagnetic equations and provides results in terms of circuit quantities that are more useful in several engineering applications. This frequency domain model uses Fast Fourier Transform to provide results in time domain. The details of the HEM model application to evaluations of lightning performance of transmission lines are noted in [18], including its validation by means of the comparison with experimental results.

D. Criterion for checking backflashover occurrence

The volt-time (v-t) curve is generally adopted in literature to verify the flashover condition across insulators subjected to lightning overvoltages [19]. However, more elaborated methods are indicated such as the leader progression model (LPM) [20] and the disruptive effect model (DE) [21] that have physical appeal. The authors have adopted the DE model to verify the backflashover occurrence considering the overvoltages associated with representative lightning current waveforms simulated by the HEM model to determine the critical currents for each condition of the simulated transmission line.

E. Including subsequent strokes on the computation of transmission line outage rate

Only first-stroke currents are considered as potential source of backflashover in regular evaluations of the lightning performance of high-voltage transmission lines [1]. It is generally believed that subsequent strokes are not likely to cause backflashovers since the value of the median peak current of subsequent strokes is typically around three times lower than that of first strokes [12].

Recent works of the authors show that subsequent strokes may contribute significantly to the outage rate of 69-kV and 138-kV transmission lines [22-23]. The physical explanation for this effect is presented in [22] that shows this kind of outage is more frequent in high towers with low tower-footing grounding resistance.

III. RESULTS AND ANALYSIS

A. Methodology

A case study considering a real transmission line configuration was developed in order to illustrate the inclusion of the aforementioned contributions on the evaluation of the lightning performance of transmission lines. It is focused on the impact of the frequency dependence of soil parameters on the lightning performance of the line. The evaluations presented herein are based on results of systematic simulations using the HEM model.

First, the ground potential rise (GPR) of typical configurations of transmission lines grounding electrodes (Fig.2) was determined considering the impression of representative current waves of first and subsequent strokes (Fig.1) on the grounding electrodes for constant and frequency-dependent soil parameters. Following, the overvoltages developed across insulator strings of the 138-kV line (Fig. 3) due to the impression at tower top of the same current waves of first and subsequent strokes were calculated.

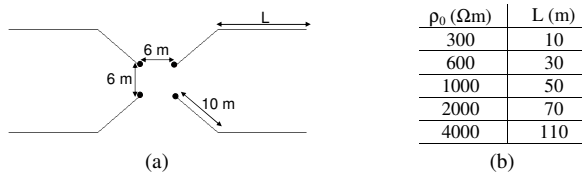


Fig. 2. Simulated grounding electrode arrangement (a) and electrode length as a function of soil resistivity (b).

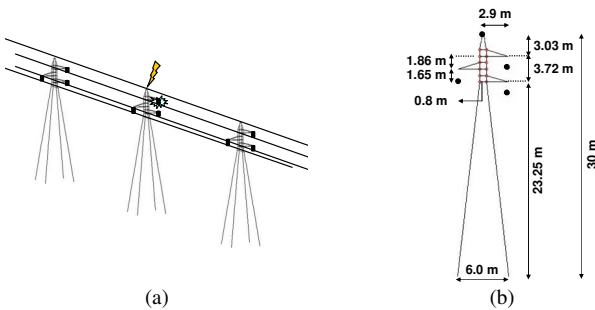


Fig. 3. Representation of direct strike to the tower (a) and the simulated 138-kV transmission line tower configuration (b).

B. Impulse behaviour of grounding

Fig. 4 illustrates the simulated grounding potential rise developed in response to the impression of the representative current waves of first and subsequent strokes of Fig. 1 directly on the tower-footing electrodes of the 138-kV line, represented in Fig. 2(a).

The impulse grounding impedance Z_p , given by the ratio of the peak values of developed GPR and impressed current, was calculated from each simulated tower-footing grounding potential rise, considering both assumptions of soil parameters. The results obtained for different values of low-frequency soil resistivity are summarized in Table I, which comprises the arrangement of electrodes, the low-frequency resistance R_g and the impulse impedance.

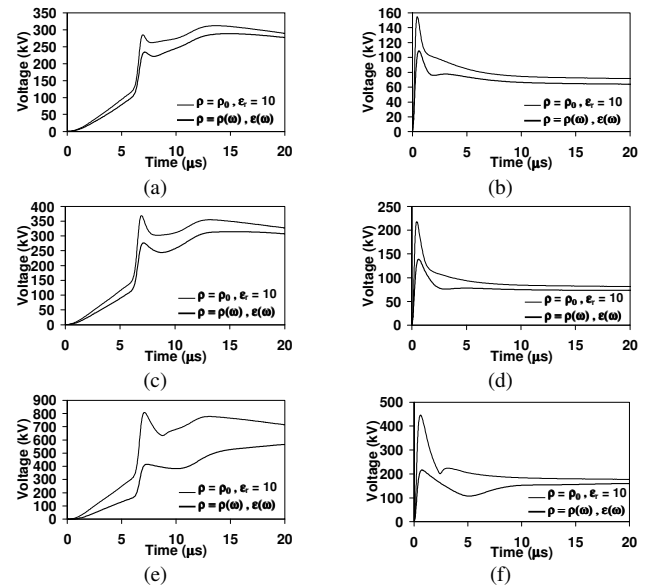


Fig. 4. Simulated GPR of the 138-kV-line tower footing under the assumption of constant and frequency-dependent soil parameters for different values of soil resistivity ρ_0 . (Left column: First stroke; Right column: Subsequent stroke). (a),(b) 600 Ωm , (c),(d) 1000 Ωm , (e),(f) 4000 Ωm .

TABLE I
IMPULSE IMPEDANCE OF FIRST- AND SUBSEQUENT-STROKE CURRENTS UNDER THE ASSUMPTION OF CONSTANT AND FREQUENCY-DEPENDENT ELECTRICAL PARAMETERS OF SOIL

ρ_0 (Ωm)	L (m)	R_g (Ω)	Impulse impedance Z_p					
			FST			SUB		
			$\rho=\rho_0, \epsilon_r=10$ (Ω)	$\rho=\rho(\omega), \epsilon_r(\omega)$ (Ω)	$\Delta\%$	$\rho=\rho_0, \epsilon_r=10$ (Ω)	$\rho=\rho(\omega), \epsilon_r(\omega)$ (Ω)	$\Delta\%$
300	10	11.1	10.4	9.9	-4.4	9.3	7.8	-15.9
600	30	11.0	10.0	9.3	-7.5	12.9	9.1	-29.6
1000	50	12.5	11.8	10.1	-14.5	18.1	11.6	-36
2000	70	19.5	18.2	14.7	-19.2	27.2	15.0	-44.7
4000	110	27.3	26.0	18.1	-30.1	37.2	18.0	-51.6

The results show that the frequency dependence of soil parameters is less significant for low resistivity soils, but becomes important for soils of resistivity larger than 600 Ωm . Also, this effect results in the decrease of the impulse impedance, being more evident for increasing low-frequency soil and considering subsequent strokes, due to their frequency content that includes higher frequency components. Reductions of grounding impedance around 16% and 52% are observed while soil resistivity varies from 300 to 4000 Ωm for subsequent strokes. Such decreases are around 5% and 30%, respectively, for first strokes.

C. Overvoltages across insulator strings

Fig. 5 shows the resulting overvoltages across the upper insulator string of the 138-kV line for both assumptions of grounding resistivity, considering 600 Ωm , 1000 Ωm and 4000 Ωm low-frequency soil resistivity. The peak voltages for ρ_0 varying from 300 to 4000 are indicated Ωm in Table II.

The results indicate that the frequency dependence of soil parameters does not influence significantly the overvoltage amplitude for resistivity soils of 600 Ωm and below. For larger soil resistivity, this effect becomes important as indicated in

Table II. The peak overvoltage decrease is important only for overvoltages due to first strokes (reductions of 5% and 22% for ρ_0 varying from 600 to 4000 Ωm). However, this reduction is much less significant for subsequent strokes, in spite of the much stronger reduction of impulse grounding impedance it yields for such currents. The slight impact for the overvoltages due to subsequent strokes can be partially attributed to the weak effect of decreasing grounding impedance to diminish the amplitude of overvoltages yielded by this event [22].

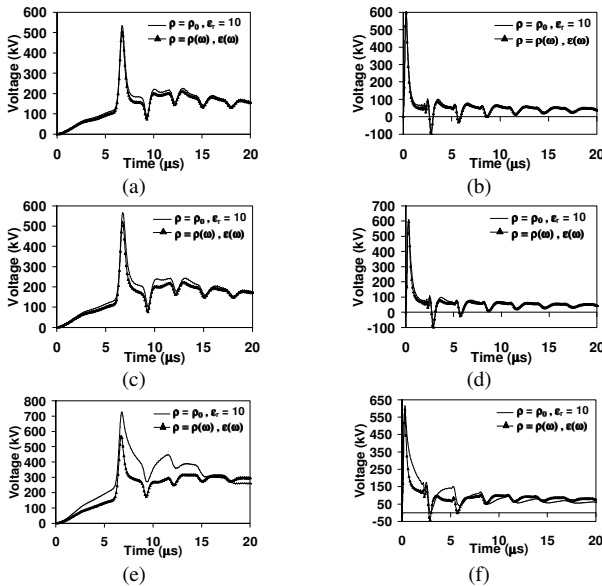


Fig. 5. Simulated overvoltage across upper insulator string of the 138-kV line for constant and frequency-dependent soil parameters for different values of soil resistivity ρ_0 . (Left column: First stroke; Right column: Subsequent stroke). (a),(b) 600 Ωm , (c),(d) 1000 Ωm , (e),(f) 4000 Ωm .

ρ_0 (Ωm)	L (m)	Overvoltage (kV)					
		FST			SUB		
		$\rho=\rho_0$, $\epsilon_r=10$	$\rho=\rho(\omega)$, $\epsilon_r(\omega)$	$\Delta\%$	$\rho=\rho_0$, $\epsilon_r=10$	$\rho=\rho(\omega)$, $\epsilon_r(\omega)$	$\Delta\%$
300	10	523.5	509.7	-2.6	598.7	594.0	-0.8
600	30	533.0	504.9	-5.3	604.6	595.8	-1.5
1000	50	564.5	519.3	-8.0	608.9	597.3	-1.9
2000	70	645.3	549.3	-14.9	612.8	597.4	-2.5
4000	110	727.3	567.2	-22.0	615.0	595.4	-3.2

It is worth mentioning that, for resistivity soils larger than 1000 Ωm , the frequency dependence effect causes the amplitude of voltage wave to remain decreased during a period along the wave after the peak, suggesting the reduction of potential to flashover across insulators.

D. Impact on the critical currents

Considering the overvoltages developed across insulator string for both constant and frequency-dependent soil parameters, the DE model was applied to determine the peak voltage (and therefore the peak current) required to flashover. Then, considering the cumulative distribution of first-stroke peak currents measured at Mount San Salvatore [19], the percentages of currents expected to overpass the value (critical peak current) required to flashover were determined for both

assumptions of soil parameters. The results are indicated in Table III and shows that the frequency dependent soil parameters cause the reduction of the expectation of backflashover, even considering low resistivity soils. The decrease is more pronounced for high resistivity soils: reductions around 24% and 43% for ρ_0 varying from 1000 to 4000 Ωm .

TABLE III
INFLUENCE OF FREQUENCY DEPENDENCE OF SOIL PARAMETERS ON CRITICAL FIRST-STROKE PEAK CURRENTS. LINE: 138-kV.

ρ_0 (Ωm)	L (m)	$(\rho = \rho_0, \epsilon_r = 10)$		$(\rho = \rho(\omega), \epsilon_r(\omega))$		$\Delta^*(\%)$
		I_c (kA)	$I > I_c$ A(%)	I_c (kA)	$I > I_c$ B(%)	
300	10	84.6	6.8	90.5	5.8	15.1
600	30	86.2	6.6	92.7	5.5	16.3
1000	50	77.8	8.4	87.1	6.4	23.8
2000	70	58.3	16.2	69.4	10.9	32.7
4000	110	46.5	25.8	60.8	14.8	42.6

$$*\Delta = |B - A| / A \cdot 100\%$$

E. Rehearsal to evaluate the impact on the outage rate

The evaluation considered a lightning flash density of 5 flashes/km²/year for the region the line is installed, leading to 30 strikes to the line per 100 km per year assuming the line with 30-m high towers.

The expected outage rates were calculated assuming the critical currents of Table III considering constant and frequency dependent parameters of soil for distinct low frequency soil resistivity distributions along the line, corresponding to ρ_0 from 300 to 4000 Ωm .

TABLE IV
EFFECT OF THE FREQUENCY DEPENDENCE OF SOIL PARAMETERS ON THE EXPECTED OUTAGE RATE OF A 138-kV LINE

Hypotheses for the distribution of ρ (%)					Outages/100 km/year		Δ (%)
300 (Ωm)	600 (Ωm)	1000 (Ωm)	2000 (Ωm)	4000 (Ωm)	$(\rho=\rho_0, \epsilon_r=10)$	$(\rho=\rho(\omega), \epsilon_r(\omega))$	
0	0	100	0	0	2.51	1.91	23.8
0	0	0	100	0	4.87	3.22	32.6
0	0	0	0	100	7.75	4.43	42.7
25	45	30	0	0	2.15	1.75	18.6
10	40	30	10	10	3.00	2.18	27.3
0	30	30	25	15	3.72	2.55	31.4

The results indicate the outage rate markedly increase with increasing low frequency soil resistivity. However, the decrease of the outage rates is noted when considering frequency dependent soil parameters (16% to 43% for resistivity soils of 300 and 4000 Ωm , respectively).

These results denote the significant effect of considering the frequency dependence of electrical parameters of soil on the outage rates of transmission lines, mainly for those characterized for grounding distributions of high resistivity soils. Even for the hypothesis of nonuniform distribution of soil resistivity, this effect is important, indicating reductions about 19% to 31%.

F. Effect of subsequent strokes on line outage rate

Using the same procedure previously adopted for first strokes, the critical currents of subsequent strokes able to

cause backflashover were calculated considering the frequency dependence of soil. The methodology described in [22-23] was applied. It was assumed a percentage of 80% for multiple stroke flashes with average multiplicity of 3 subsequent strokes [24]. The obtained results are indicated in Tables V and VI.

TABLE V
CRITICAL SUBSEQUENT-STROKE PEAK CURRENTS CONSIDERING THE
FREQUENCY DEPENDENCE OF SOIL PARAMETERS. LINE: 138-kV.

ρ_0 ($\Omega \cdot m$)	L (m)	$\rho = \rho(\omega), \epsilon(\omega)$	
		I_c (kA)	$I > I_c$ (%)
600	30	48.36	2.27
1000	50	47.40	2.39
2000	70	46.20	2.56
4000	110	45.12	2.72

TABLE VI
PERCENTAGE INCREASE ON THE OUTAGE OF THE 138-kV LINE DUE TO THE
INCLUSION OF SUBSEQUENT STROKES (30 FLASHES STRIKING THE LINE PER 100
KM PER YEAR).

ρ_0 ($\Omega \cdot m$)	L (m)	Outages/100 km/year		Δ %
		FST	FST + SUB	
600	30	1.64	3.15	91.8
1000	50	1.91	3.49	82.2
2000	70	3.28	4.88	48.7
4000	110	4.44	6.07	36.6

A substantial increase is observed on the resulting outage rate (between 92% and 37%) in comparison to the case that only considers first strokes. Note that contribution of subsequent strokes on the outage rate is much more significant for low tower footing grounding resistances. This result corroborates the analyses indicated in [22-23] for 69-kV and 138-kV transmission lines, respectively. However, further analysis shows this contribution is less important for 230 kV lines and above.

IV. CONCLUSIONS

This work considered the impact of recent developments on the calculation of the lightning performance of transmission lines. Such developments were applied to a real 138-kV line, mainly the use of realistic representations of the waveform of the lightning current, the application of an elaborated model to calculate the overvoltage across insulator strings and the definition of a consistent criterion to evaluate the backflashover occurrence.

The impact of the frequency-dependent soil parameters and the effect of subsequent strokes on the estimation of backflashover rate of transmission lines were evaluated. It was shown that considering frequency-dependent soil parameters implies on a significant decrease on the line outage rate, mainly for high resistivity soils. Also, subsequent strokes may have a relevant contribution on the outage rate of 138-kV lines. This effect is larger for 69-kV lines and less important for lines of higher voltage level.

REFERENCES

[1] IEEE Working Group on Lightning Performance of Transmission Lines, "A Simplified Method for Estimating the Lightning Performance of

Transmission Lines," IEEE Trans. Power App. Syst., vol.104, no.4, pp. 919-932, Apr. 1985.

[2] S. Visacro, "Direct strokes to transmission lines: Considerations on the mechanisms of overvoltage formation and their influence on the lightning performance of lines," J. Light. Res., vol. 1, pp. 60-68, 2007.

[3] S. Visacro, F.H. Silveira, and A. De Conti, "The use of underbuilt wires to improve the lightning performance of transmission lines," IEEE Trans. Power Del., vol. 27, no. 1, pp. 205-213, Jan. 2012.

[4] IEEE Guide for Improving the Lightning Performance of Transmission Lines, IEEE Standard 1243-1997, Dec. 1997.

[5] CIGRE Guide to Procedures for estimating the lightning Performance of Transmission Lines, WG 01 (Lightning), Study Committee 33, 1991.

[6] I.S. Grant, J.G. Anderson, A.R. Hileman, "A Simplified Method For Estimating Lightning Performance of Transmission Lines", IEEE Transactions on Power Apparatus and systems, vol. PAS-104, no. 4, pp. 919-932, Apr. 1985.

[7] J.A. Martinez, F. Castro-Aranda, "Lightning Performance Analysis of Overhead Transmission Lines Using the EMTP", IEEE Trans. Power Del., vol. 20, No. 3, Jul. 2005.

[8] S. Visacro, R. Alipio, M. H. Murta Vale, and C. Pereira, "The response of grounding electrodes to lightning currents: the effect of frequency-dependent soil resistivity and permittivity," IEEE Trans. Electromagn. Compat., vol. 53, no. 2, pp. 401-406, May 2011.

[9] S. Visacro and R. Alipio, "Frequency dependence of soil parameters: experimental results, predicting formula and influence on the lightning response of grounding electrodes," IEEE Trans. Power Del., vol. 27, no. 2, pp. 927-935, Apr. 2012.

[10] R. Alipio and S. Visacro, "Frequency Dependence of Soil Parameters: Effect on the Lightning Response of Grounding Electrodes," IEEE Trans. Electromagn. Compat., vol. 55, no. 1, pp. 132-139, Feb. 2013.

[11] S. Visacro, "A representative curve for lightning current waveshape of first negative stroke," Geophys. Res. Lett., vol. 31, L07112, Apr. 2004.

[12] S. Visacro, M.A.O. Schroeder, A. Soares Jr., L.C.L. Cherchiglia, V.J. Sousa, "Statistical analysis of lightning current parameters: Measurements at Morro do Cachimbo station," J. Geophys. Res., vol. 109, No. D01105, 1-11, doi:10.1029/2003JD003662, 2004.

[13] R. B. Anderson and A. J. Eriksson, "Lightning parameters for engineering application," Electra, vol.69, pp. 65-102, 1980.

[14] A. De Conti and S. Visacro, "Analytical representation of single- and double-peaked lightning current waveforms," IEEE Trans. Electromagn. Compat., vol.49, No.2, pp.448-451, May 2007.

[15] S. Visacro, F.H. Silveira, S. Xavier, H.B. Ferreira, "Frequency Dependence of Soil Parameters: The Influence on the Lightning Performance of Transmission Lines," in Proc. of 2012 International Conference on Lightning Protection (ICLP), Vienna, Austria, 2012.

[16] F.H. Silveira, S. Visacro, A. De Conti, Lightning performance of 138-kV lines: The influence of the frequency dependence of soil parameters and subsequent stroke currents - A case study, in Proc. of GROUND'2012 & 5th LPE, Bonito, Brazil, pp.204-209, Nov. 2012.

[17] S. Visacro, A. Soares J., "HEM: A Model for Simulation of Lightning-Related Engineering Problems", IEEE Trans. Power Del., vol.20, no.2, pp. 1026-1208, Apr. 2005.

[18] A. Soares J., M.A.O. Schroeder, and S. Visacro, "Transient voltages in transmission lines caused by direct lightning strikes", IEEE Trans. Power Del., vol. 20, no.2, pp. 1447-1452, Apr. 2005.

[19] Guide for Improving the Lightning Performance of Transmission Lines, IEEE Std 1243-1997, Dec. 1997.

[20] A. Pignini, G. Rizzi, E. Garbagnati, A. Porrino, G. Baldo, and G. Pesavento, "Performance of large air gaps under lightning overvoltages: experimental study and analysis of accuracy predetermination methods," IEEE Trans. Power Del., vol. 4, no. 2, pp. 1379-1392, Apr. 1989.

[21] A. H. Hileman, "Insulation Coordination for Power Systems". Boca Raton, FL: CRC, 1999, pp. 627-640.

[22] F.H. Silveira, S. Visacro, A. De Conti, C.R. Mesquita, "Backflashovers of Transmission Lines Due to Subsequent Lightning Strokes", IEEE Trans. Electromagn. Compat., vol.54, no.2, pp. 316-322, Apr. 2012. doi:10.1109/TEMC.2011.2181851.

[23] F.H. Silveira, S. Visacro, A. De Conti, "Lightning Performance of 138-kV Transmission Lines: The Relevance of Subsequent Strokes," IEEE Trans. Electromagn. Compat., doi: 10.1109/TEMC.2013.2246168, 2013.

[24] V. Rakov, M.A. Uman, "Lightning - Physics and Effects," Cambridge University Press, 2003.