Application of the Fractional Fourier Transform for Detection and Analysis of Partial Discharge Signatures in Distribution Transformers

Aplicación de la Transformación Fraccional de Fourier para la Detección y el Análisis de Registros de Descargas Parciales en Transformadores de Distribución

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ABSTRACT

Several works have been presented about the use of time-frequency representations (TFRs) and signal processing techniques on the study of partial discharges (PD). The fractional Fourier transform (FRFT) is a linear TFR that generalizes conventional Fourier transform (TF). This paper, presents a study related with the application of the FRFT for the analysis and processing of PD signals generated in distribution transformers. PD signals were acquired using an experimental setup designed and built in the High Voltage Laboratory at the Universidad Distrital Francisco José de Caldas. A total of 26 PD signatures were analyzed in the experiment. Simulation results show that spectrum of PD signatures, computed with FRFT, have a main peak, one or two internal peaks and a set of secondary oscillations (side lobes) associated with high frequency components, which are distinctive of this type of electromagnetic transients. In addition, a statistical analysis of several parameters of the PD signal transform in the FRFT domain is presented.

KEYWORDS: electrical methods, fractional Fourier transform, on-site PD measurements, partial discharges, signal processing

RESUMEN

Varios trabajos han sido presentados sobre el uso de representaciones tiempo-frecuencia (TFRs) y técnicas de procesamiento de señales en el estudio de descargas parciales (PD). La transformación fraccional de Fourier (FRFT) es una TFR lineal que generaliza la transformación de Fourier convencional (TF). Este artículo presenta un estudio relacionado con la aplicación de la FRFT para el análisis y procesamiento de señales de PD generadas en transformadores de distribución. Las señales de PD fueron adquiridas empleando un montaje experimental diseñado y construido en el Laboratorio de Alta Tensión de la Universidad Distrital Francisco José de Caldas. En el experimento se analizaron un total de 26 registros de PD. Los resultados de las simulaciones muestran que el espectro de las señales de PD, calculado con FRFT, tiene un pico principal, uno o dos picos internos y un conjunto de oscilaciones secundarias (lóbulos laterales) asociadas con las componentes de alta frecuencia, que son distintivos de este tipo de transitorios electromagnéticos. Adicionalmente, se presenta un análisis estadístico de varios parámetros de la transformada de las señales de PD en el dominio de la FRFT.

PALABRAS CLAVE: descargas parciales, mediciones de PD en el sitio, métodos eléctricos, procesamiento de señales, transformación fraccional de Fourier
1. INTRODUCTION

The use of time-frequency representations (TFRs) for the analysis and characterization of electromagnetic disturbances is a research topic that has had great growth in the last three decades. In this context, the development and application of novel techniques for signal processing and the use of efficient acquisition systems have allowed the application of different mathematical tools for analyzing partial discharges (PD), which are one of the most important factors that affect the insulation of power and distribution transformers.

Several papers have been presented about the analysis of PD on distribution transformers using TFRs, such as: fast Fourier transform (FFT) [1], short time Fourier transform (STFT) [2], [3], the Wavelet transform (WT) [1], [3], [4], the Wigner distribution (WD) [3], and recently, the local polynomial Fourier transform (LPFT) [5], [6]. Inside the linear TFR, a group of transformations deals with the rotation of the time-frequency plane (TFp). These techniques can be used to obtain a better energy concentration and extract information about the signal on a hybrid domain between the time and frequency domain [7]. Some examples of these transformations are the Wigner-Radon transform (WRT), the polynomial-time Fourier transform (PTFT), the local polynomial Fourier transform (LPFT) and the fractional Fourier transform (FRFT).

The FRFT is a generalization of the classic Fourier transform (FT) which provides better energy concentration and higher resolution rotating the signal on the TFp. The FRFT has been used in applications such as time-varying signal processing, noise reduction, information encryption, singularity detection, optics, radar and sonar [8], [9]. This variety of uses has shown the FRFT as a useful tool for working with non-stationary, chirps and transient signals [9]. In fact, in recent years the FRFT has been applied in the study of electric and magnetic fields generated by lightning flashes [10], [11].

This paper presents a first approximation about the use and application of the fractional Fourier transform (FRFT) in the detection, analysis and processing of PD signals generated in distribution transformers. The results present here show that the FRFT can be an alternative technique and with potential to analyze PD pulses that are currently studied using other time-frequency techniques. In addition, the FRFT offers some degrees of freedom in terms of its parameters and it can work efficiently in environments contaminated by noise. Such parameters can be exploited without increasing computational complexity.

2. THE FRACTIONAL FOURIER TRANSFORM

The FRFT is a mathematical tool that generalizes the classic FT [12]. Defined by V. Namias [13], it has been considered as a useful technique to manipulate signals in the time-frequency domain (TFd). The \(\alpha\)-th order of FRFT for a signal \(x(t)\), denoted as \(X_\alpha(u)\), can be interpreted as a rotation of the signal in an angle \(\alpha\) on the conventional time-frequency plane \((t - \omega)\). A graphical interpretation of the FRFT is shown in Figure 1. Mathematically, the FRFT of the signal \(x(t)\) is defined as follows:

\[
\mathcal{F}^\alpha[x(t)](u) \equiv X_\alpha(u) = \int_{-\infty}^{\infty} x(t)B_\alpha(t,u) \, dt \tag{1}
\]

where \(0 < |\alpha| < 2\), and \(B_\alpha(t,u)\) is the transformation kernel, which is defined by:

\[
B_\alpha(t,u) = K_\alpha \exp \left[ \frac{(t^2 + u^2)}{2} \cot \alpha - jut \csc \alpha \right] \tag{2}
\]

where, \(K_\alpha = \sqrt{(1 - j \cot \alpha)/2\pi}\) and \(\alpha = \pi \alpha / 2\) [12]. Some properties of the FRFT are: (a) it is a linear and unitary transformation; (b) the first order \((\alpha = 1)\) of FRFT, corresponds to the classic FT; (c) it has index additivity \(\mathcal{F}^a \mathcal{F}^b = \mathcal{F}^{a+b}\); and (d) the inverse FRFT operator (IFRFT) is expressed as \((\mathcal{F}^a)^{-1} = \mathcal{F}^{-a}\). The FRFT existence conditions, its properties and its integration kernel are explained with detail in [12], [13].

Figure 1. Graphical definition of the FRFT

Source: Authors

Since it exists in a domain between time and frequency, known as fractional Fourier domain (FRFd), the FRFT has relations with other mathematical tools such as the Wigner-Ville distribution (WVD), the Wavelets transform (WT), the linear canonical transformation (CLT) and other operations related to chirp signals [14].
3. EXPERIMENTAL SETUP

During this study, PD signals were acquired using an experimental setup designed and built in the High Voltage Laboratory at the Universidad Distrital Francisco José de Caldas. The experimental tests were set up using a modified transformer tank specially designed and constructed for research purposes. The tank was made from 8mm thick steel and its dimensions were 530 mm (H) x 550 mm (W) x 510 mm (D). During tests, the core and windings of the transformer were not included. In addition, one high voltage bushing was installed and a 12 mm thick acrylic window was included to observe the PD formation process [6]. The main characteristics of the transformer tank for PD generation are shown in Figure 2.

![Figure 2. Transformer tank used in the experimental tests](source: Adapted from [6])

The complete circuit used to acquire the PD signals is shown in Figure 3. A switch-relay arrangement is connected in series with the low voltage source in order to disconnect the high voltage power supply after a spark has occurred, which is necessary to ensure that there are no multiple sparks [6]. In the PD generation stage, a gap with a point-to-plane electrode configuration, suspended in the oil, was chosen as the discharge source. To acquire electrical signals, a fast current transformer Bergoz CT E-0.5-B with a bandwidth of 48 Hz-200 MHz and a sensitivity of 0.25V/A was used. To record the signals a digital oscilloscope (Tektronix DPO7054C) with a maximum sampling rate of 5 GSa/s was connect to the current transformer through a 8m coaxial cable RG58-U with a characteristic impedance of 50 Ω. The cable was shielded using a cylindrical metallic cover with 1-inch diameter. This shield was connected to ground.

The digital oscilloscope was triggered by the PD current directly. The average value of the applied voltage to obtain the PD signals was 19 kVrms. The high voltage source is connected 25 cm above the bottom of the transformer tank and the dielectric oil level was 45 cm deep. The distance between electrodes and the position of the PD source was fixed for all measurements.

4. DATA SET

During the measurements stage, all signatures were recorded with a sampling time \(T_s = 1\) ns using an observation window of 1 µs. At this way, each signature had 1000 samples. In this study, 26 PD transient pulses were acquired, from which two waveforms (time domain) with different temporal characteristics are shown in Figure 4 and Figure 5. These signatures illustrate the behavior of the experimental setup described in the section 3. The characteristics of the PD waveform in the time domain are defined as follows:
• Peak value (PK): maximum value of the PD pulse.
• Secondary peaks (SP): quantity of positive or negative peaks with amplitudes between the peak value and above 5% of the peak value.
• Duration (D): period when the PD pulse occurs.
• Zero crossing time (ZC): first point where the signal changes from positive to negative (or vice versa) and the signal value is zero.

**Figure 4.** PD signature in time domain. Code PD008  
Source: Authors

**Figure 5.** PD signature in time domain. Code: PD019  
Source: Authors

To extract reliable information from PD signals, during the acquisition all data were adjusted using the same time point reference. For this reason, all acquired signals start at \( t = 0 \) [s]. As can be seen, the signatures studied in this research come from the same phenomenon and they present two stages. The initial stage begins at zero time and finishes when signal reaches the peak value. The second stage consists of several secondary peaks (oscillations with positive and negative polarity) with a damping factor following the initial stage. All waveforms reached their maximum value close to the beginning of the pulse [6].

**5. SIGNAL PROCESSING OF PD SIGNATURES USING THE FRFT**

With exception of the data obtained during experimental tests, all simulations have been carried out using MATLAB®. The PD signals were recorded with amplitudes between 0.1 p.u. and 0.9 p.u. of rated voltage. In addition, the fractional order was varied from \( \alpha = 0.1 \) to \( \alpha = 1 \) (FT). In total, 260 simulations were performed. In this work, to evaluate the effectiveness of the FRFT in the detection and identification of PD pulses two types of analysis have been performed. The first analysis is based on visual inspection of the signal waveform and its transform. The second analysis provides quantitative information about the most relevant characteristics of the signal and its transform.

**5.1. Detection of PD using the FRFT**

Simulation results show that fractional spectrum of PD signatures, computed with FRFT, have a main peak (MP), one or two internal peaks (IP) and a set of small oscillations associated with high frequency components (side lobes; SL), which are distinctive of this type of electromagnetic transients. The energy distributions on the FRF for the signatures presented in Figure 4 and Figure 5 are depicted in Figure 6. These transforms were computed using a fractional order \( \alpha = 0.40 \).

**Figure 6.** FRFT of the PD signatures. (a) Transform of signature PD008; (b) Transform of signature PD019  
Source: Authors

When comparing the results it is possible to observe certain changes in the spectrum of the transforms for signatures PD008 and PD019. In particular, it is possible...
to evidence the appearance of more than one side lobe and the magnitude of the main and internal peaks. The differences in the shape of the FRFT spectrum for several PD signatures, added to the property of energy concentration that possess the FRFT, allow a rapid detection of PD pulses. During the simulations, the characteristics of the parameters MP, IP and SL are analyzed according to the fractional order, the magnitude and the duration of the signal. 

5.2. Influence of the fractional order on the magnitude and separation of main peaks

For the 26 PD signatures analyzed, all parameters were calculated with a fractional order between 0.1 and 1.0 using an interval of 0.1. In this way, for each fractional order (or FRFd) there is a greater volume of information. For this reason, in order to take advantage of the concentration and increase of energy provided by small fractional orders, both the maximum value of the FRFT and the separation of the main peaks where compared. The results of this analysis are presented in Figure 7. In this figure, the points are related with the peak value of the FRFT, while the "X" represents the separation between main peaks.

Figure 7 shows that at lower fractional orders a larger magnitudes of FRFT occur while compressing the distance between the main peaks. Thus, the maximum value resulting from the transformation is inversely proportional to the order of the FRFT. In addition, the separation between the main peaks is directly proportional to the fractional order. For this reason, to analyze and characterize the signal data set a fractional order \( a = 0.3 \) was adopted. For this value, there is a balance between the separation and the magnitude of the main peaks (see Figure 7). Under this condition, the characterization process of the PD waveforms presented in the following section was developed using this fractional order. These results are presented by statistical distributions or histograms.

6. STATISTICAL RESULTS AND DISCUSSION

In this section, several parameters of the PD signals in the FRFd are presented. The selected parameters are maximum value and separation distance of the main peaks, internal peaks magnitude and ratio between peaks and side lobes.

6.1. Maximum value and separation distance of the main peaks

In all cases, the maximum value of the FRFT is related to the magnitude of the main peaks. Figure 8 shows the distribution of this parameter. It is possible to observe that the major part of the signatures have a peak value greater than \( 3 \times 10^{-11} \). In addition, more than 75% of the analyzed samples are between \( 4.5 \times 10^{-11} \) and \( 6 \times 10^{-11} \). This distribution shows a concentration of data in a specific range, which confirms the existence of a phenomenon with a defined trend in the FRFd. This results show the advantage of analyzing PD pulses using the FRFT.

![Figure 8. Distribution of the maximum value of the FRFT](image)

Source: Authors

![Figure 7. Peak value of the FRFT and separation between main peaks (MP) with respect to the fractional order \( \alpha \)](image)

Source: Authors
On the other hand, the separation between main peaks was defined as the measured distance among the maximum values of the FRFT on the horizontal axis (independent fractional domain). In this case, all signatures present only two values of separation between peaks. Thus, 65.4% of the records (17/26) showed a minimum separation of $1.83 \times 10^8$, while for the remaining 34.6% (9/26) the distance between main peaks was $1.83 \times 10^8$. These results are important due to this parameter reflects the constant behavior of the signals spectrum in the FRFd, which facilitates the identification of the transient pulses associated with PD.

6.2. Internal peaks magnitude

This parameter is related with the peaks of the transform that are located between the main peaks. The histogram of the internal peaks magnitude is shown in Figure 9. From results, it was found that 92.3% of the spectrum in the FRFd have internal peaks with a magnitude between $15 \times 10^{-10}$ and $30 \times 10^{-10}$.

On the other hand, the separation between main peaks is shown in Figure 12. From results, more than 90% of the signsl es examined have between 3 and 5 side lobes. In addition, the ratio between the maximum lobe and the main peaks presents a similar distribution to the reported for the internal peaks, varying from 0.3 up to 0.6 with an average value of 0.426.

6.3. Ratio between peaks and side lobes

The histogram of the ratio between the internal peaks and the main peaks of the FRFT is illustrated in Figure 10. In this case, 24 signatures (92.3%) present an internal peak with a magnitude between 0.3 and 0.6 with respect to the maximum value of the FRFT. This parameter reflects the low frequency components of the transform and its concentration shows that all signatures analyzed were produced by the same type of phenomenon.

On the other hand, as can be seen in Figure 6, the fractional spectrum of the PD waveforms may have at least one side lobe. During the analysis, the number of side lobes for all signatures was quantified. Figure 11 presents the signatures grouped according to the number side lobes that exceeded a threshold of 5% of the maximum value of their respective transform. In addition, the statistical distribution of the ratio between the maximum of the side lobes and the magnitude of the main peaks is shown in Figure 12. From results, more than 90% of the signals examined have between 3 and 5 side lobes. In addition, the ratio between the maximum lobe and the main peaks presents a similar distribution to the reported for the internal peaks, varying from 0.3 up to 0.6 with an average value of 0.426.

7. CONCLUSIONS

The results presented in this paper show that the FRFT can be an alternative processing technique with potential to analyze PD single-pulses that are currently studied using other time-frequency representations. According to simulations, a reduction of the fractional order produces an increase in the magnitude of the FRFT spectrum and
a better energy concentration. This process highlights characteristics of the transform such as main (MP) and internal peaks (IP), both associated to the occurrence of PD pulse. On the other hand, it was observed that the separation between main peaks and the ratios IP/MP and SL/MP remain within a similar interval in the fractional domain. This behavior allows an adequate identification of the transients related with the occurrence of PD in distribution transformers. This as long as the disturbance is contained entirely within an observation window with a fixed width, since this factor directly affects the magnitude of the spectrum in the FRFd.

The processing showed that more than 80% of the PD signatures examined present the following characteristics in the FRFd: a maximum value between $4.5 \times 10^{-11}$ and $6 \times 10^{-11}$, an internal peaks magnitude from $15 \times 10^{-10}$ up to $30 \times 10^{-10}$, a ratio between the magnitude of internal peaks or side lobes with respect to main peaks from 0.3 up to 0.6 and a number of side lobes between 3 and 5. These parameters can be used to identify and classify this type of electrical disturbances.

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9. REFERENCIAS


