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Transición energética en la 4ta revolución industrial

Universidad Tecnológica de Pereira

UNIVERSIDAD NACIONAL DE COLOMBIA

Branch Optimal Power Flow Model for DC Networks with Radial Structure: A Conic Relaxation

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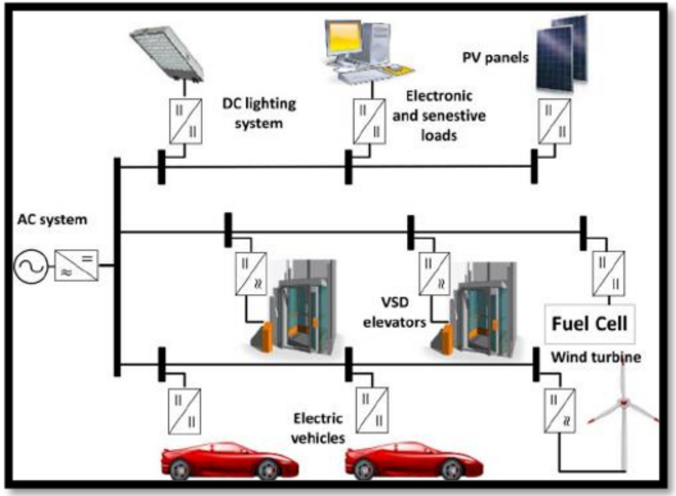
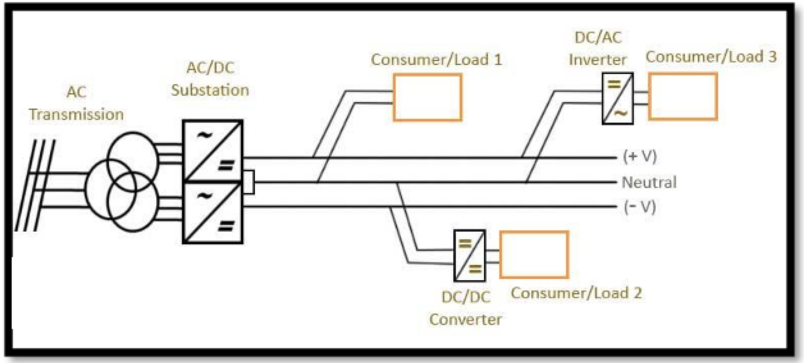
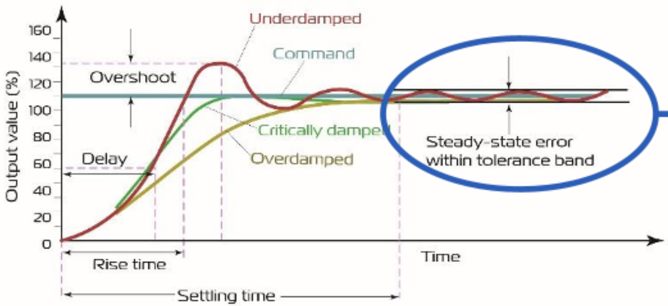
I. Introduction



Monopolar DC distribution grid

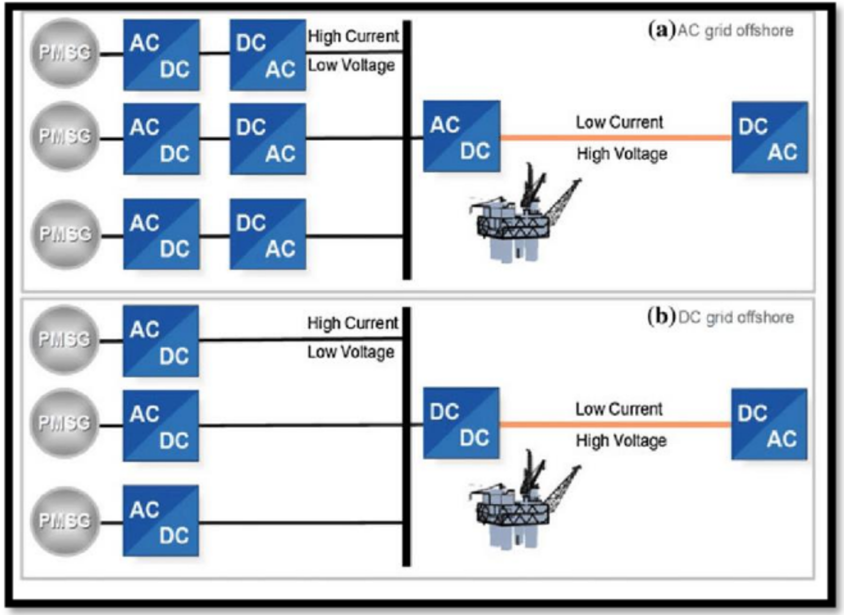


Bipolar DC distribution grid



How model and solve the OPF analysis in steady-state conditions?

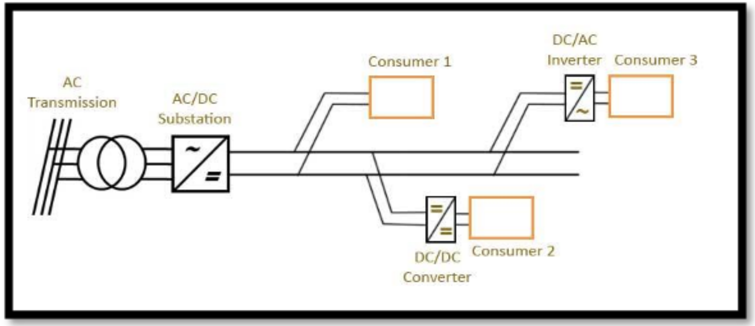
I. Introduction



Advantages of DC grids

1. Reduction of the number of power electronic converters for renewable generation and variable load applications.
2. Easy control implementations since the variable under control are the voltage in the DC bus or the (active) power injection.
3. Reactive or frequency concepts are not present in DC grids.
4. High efficiency regarding energy losses when compared with AC networks.

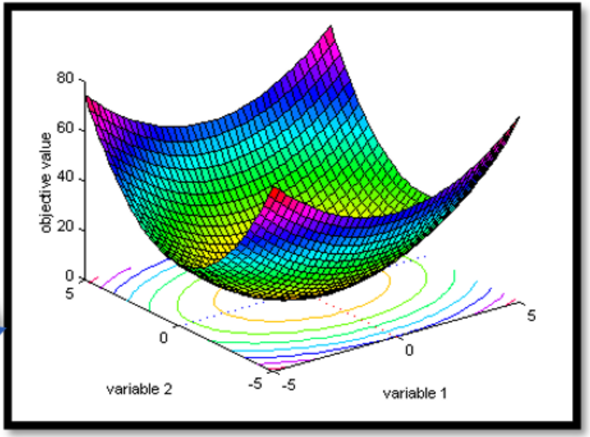
II. Exact NLP formulation



Objective function:

$$\min p_{loss} = \sum_{(j,k)\in \mathrm{E}} R_{jk}\, i_{jk}^2 \quad (1a)$$

Convex function



Set of constraints:

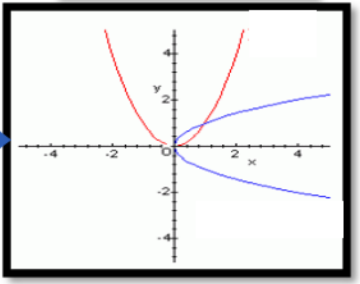
$$p_{jk} - R_{jk} i_{jk}^2 - \sum_{m:(k,m)\in \mathrm{E}} p_{km} = p_k, \forall (j,k) \in \mathrm{E} \quad (4b)$$

$$i_{jk} = \frac{v_j - v_k}{R_{jk}}, \forall (j,k) \in \mathrm{E} \quad (4c)$$

$$p_{jk} = v_j i_{jk}, \forall (j,k) \in \mathrm{E} \quad (4d)$$

Linear (convex) constraint

Non-convex constraints



II. Exact NLP formulation

Note that an equivalent optimization model can be obtained from (4) by making some algebraic manipulations. To do so, let us pre-multiply (4c) by v_j, which produces:

$$v_j^2 - v_j v_k = R_{jk} v_j i_{jk}, \forall (j,k) \in \mathrm{E} \tag{5a}$$

$$v_j^2 - R_{jk} p_{jk} = v_j v_k, \forall (j,k) \in \mathrm{E} \tag{5b}$$

Now if we raise (4c) to the square in both sides, then, we have:

$$R_{jk}^2 i_{jk}^2 = \left(v_j - v_k\right)^2, \forall (j,k) \in \mathrm{E} \tag{6a}$$

$$R_{jk}^2 i_{jk}^2 = v_j^2 - 2 v_j v_k + v_k^2, \forall (j,k) \in \mathrm{E} \tag{6b}$$

Note that if Equation (6b) is substituted into Equation (5b) and some algebraic manipulations are made, then, the following result yields:

$$v_k^2 = v_j^2 - 2 R_{jk} p_{jk} + R_{jk}^2 i_{jk}^2, \forall (j,k) \in \mathrm{E} \tag{7}$$

To obtain an equivalent model, then, let us define two auxiliary variables, which are $l_{jk} = i_{jk}^2$ and $u_j = v_j^2$. With these new variables, the optimal branch power flow model (4) can be rewritten as follows.

Objective function:

$$\min p_{loss} = \sum_{(j,k) \in \mathrm{E}} R_{jk}\, l_{jk} \tag{8a}$$

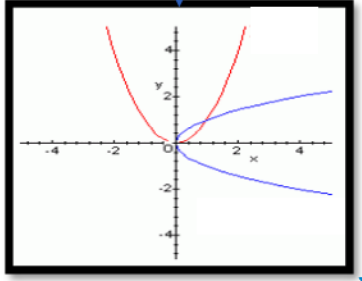
Set of constraints:

$$p_{jk} - R_{jk} l_{jk} - \sum_{m:(k,m) \in \mathrm{E}} p_{km} = p_k, \forall (j,k) \in \mathrm{E} \tag{8b}$$

$$u_k = u_j - 2 R_{jk} p_{jk} + R_{jk}^2 l_{jk}, \forall (j,k) \in \mathrm{E} \tag{8c}$$

$$p_{jk}^2 = u_j l_{jk}, \forall (j,k) \in \mathrm{E} \tag{8d}$$

Non-convex constraint



III. Proposed SOCP reformulation

Objective function:

$$\min p_{loss} = \sum_{(j,k)\in E} R_{jk}\, l_{jk} \quad (8a)$$

Set of constraints:

$$p_{jk} - R_{jk} l_{jk} - \sum_{m:(k,m)\in E} p_{km} = p_k, \forall (j,k) \in E \quad (8b)$$

$$u_k = u_j - 2R_{jk} p_{jk} + R_{jk}^2 l_{jk}, \forall (j,k) \in E \quad (8c)$$

$$p_{jk}^2 = u_j l_{jk}, \forall (j,k) \in E \quad (8d)$$

Hyperbolic equivalent

$$u_j l_{jk} = \frac{1}{4}\left(u_j + l_{jk}\right)^2 - \frac{1}{4}\left(u_j - l_{jk}\right)^2, \forall (j,k) \in E \quad (9)$$

Now, if we substitute (9) into (8d), then, we have

$$\left(2p_{jk}\right)^2 = \left(u_j + l_{jk}\right)^2 - \left(u_j - l_{jk}\right)^2, \forall (j,k) \in E \quad (10a)$$

$$\left(2p_{jk}\right)^2 + \left(u_j - l_{jk}\right)^2 = \left(u_j + l_{jk}\right)^2, \forall (j,k) \in E \quad (10b)$$

Observe that (10) can be rewritten using the Euclidean norm as follows

$$\left\| \begin{matrix} 2p_{jk} \\ u_j - l_{jk} \end{matrix} \right\| = u_j + l_{jk}, \forall (j,k) \in E \quad (11)$$

Objective function:

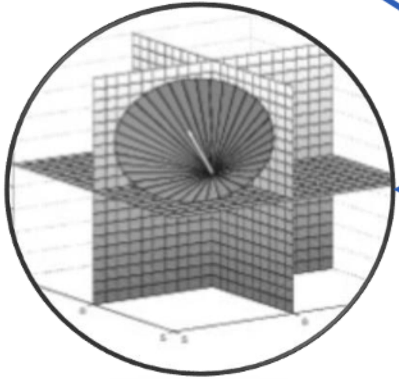
$$\min p_{loss} = \sum_{(j,k)\in E} R_{jk}\, l_{jk} \quad (13a)$$

Set of constraints:

$$p_{jk} - R_{jk} l_{jk} - \sum_{m:(k,m)\in E} p_{km} = p_k, \forall (j,k) \in E \quad (13b)$$

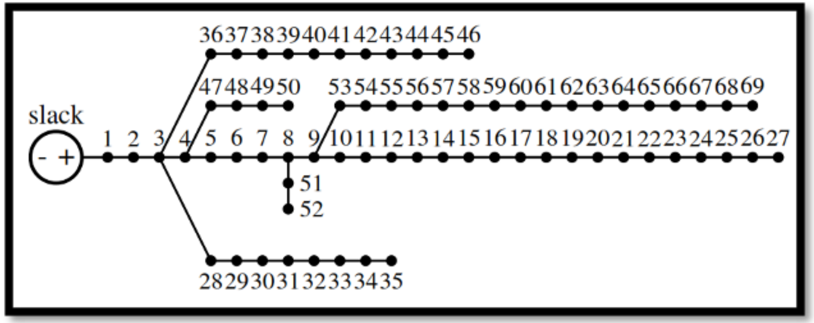
$$u_k = u_j - 2R_{jk} p_{jk} + R_{jk}^2 l_{jk}, \forall (j,k) \in E \quad (13c)$$

$$\left\| \begin{matrix} 2p_{jk} \\ u_j - l_{jk} \end{matrix} \right\| \leq u_j + l_{jk}, \forall (j,k) \in E \quad (13d)$$



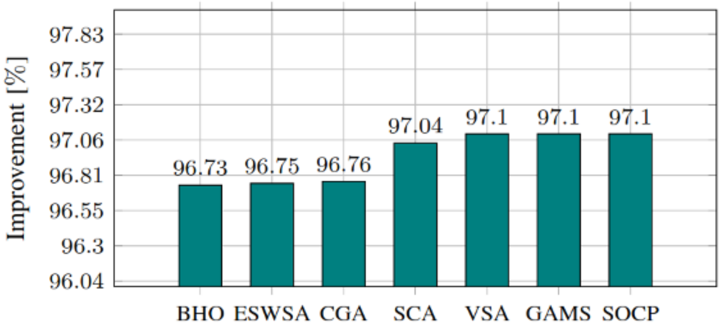
Convex solution space

IV. Results



Equivalent IEEE 69-bus system for DC OPF studies

Distributed generation in nodes 21, 61 and 64



Benchmark case 153.847557 kW

Table 1. Numerical results in the OPF problem with different comparative approaches.

Method	**Generation** (kW)	**Power losses** (kW)
BHO	[460.21, 1170.28, 639.88]	5.025771
ESWSA	[495.80, 1049.55, 699.62]	5.005033
CGA	[401.31, 1191.55, 584.05]	4.982861
SCA	[499.86, 1199.90, 564.26]	4.557555
VSA	[455.47, 1200.00, 584.85]	4.454562
GAMS	[453.21, 1200.00, 585.16]	4.454342
SOCP	[453.22, 1200.00, 585.17]	4.454342

Metaheuristic methods

V. Conclusions

A SOCP model for OPF analysis in DC distribution networks have been presented in this paper, which combines nodal and branch variables. The exact nonlinear programming model is convexified via relaxation of the power sent from node j to node k, i.e., $p_{jk} = v_j i_{jk}$, with its conic equivalent of hyperbolic representation.

Numerical results in the 69-nodes test feeder demonstrate that the proposed SOCP model allows to reach the global optimal solution for the optimal power flow problem in DC distribution networks with distributed generators, since its results are better than metaheuristic methods such as BHO, ESWSA, CGA, and SCA, respectively.

In addition, numerically speaking, the only one metaheuristic method that can reach a near-optimal solution is the VSA approach, which is compared with the proposed SOPC model and the interior point methods available in the GAMS software.

VI. Questions