Transición energética en la 4ta revolución industrial
Small-Signal Stability and Sensitivity Analysis for Grid Following Converters

Authors:
Simon Sepulveda Garcia
Alejandro Garces Ruiz

Institution:
Universidad Tecnologica de Pereira
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I. Introduction

**Figure 1.** Active distribution network

**Figure 2.** Schematic diagram of the hierarchical control in active distribution systems
II. Theoretical aspects

Figure 3. Voltage source converter (VSC) with vector-oriented control (VOC)
II. Theoretical aspects

\[ x_a + x_b + x_c = 0 \]
\[
\begin{pmatrix}
  x_a \\
  x_b \\
  x_c
\end{pmatrix}
= k
\begin{pmatrix}
  1 & -\frac{1}{2} & -\frac{1}{2} \\
  0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix}
\begin{pmatrix}
  x_a \\
  x_b \\
  x_c
\end{pmatrix}
\]

\[ P_{\alpha\beta0} = [V_{\alpha/0}]^T[I_{\alpha/0}] \]

\[ P_{\alpha\beta0} = [V_{abc}]^T[TC]^T[TC][I_{abc}] \]

\[ P_{\alpha\beta0} = \frac{2}{3}[V_{abc}][I_{abc}] \]

\[ P_{\alpha\beta0} = \frac{2}{3}P_{abc} \]

SEC. 1 Clark transformation

SEC. 2 Park transformation

\[
\begin{pmatrix}
  x_d \\
  x_q
\end{pmatrix}
= \begin{pmatrix}
  \cos(\theta') & \sin(\theta') \\
  -\sin(\theta') & \cos(\theta')
\end{pmatrix}
\begin{pmatrix}
  x_\alpha \\
  x_\beta
\end{pmatrix}
\]

SEC. 3 Active and reactive power in the dq frame

\[ p = \frac{3}{2}(v_di_d + v_qi_q) \]
\[ q = \frac{3}{2}(v_qi_d - v_di_q) \]

Note: SEC mean set of equations
II. Theoretical aspects

The current dynamics of the system on the AC side can be obtained by applying Kirchoff's second law

\[
(L_f + L_g) \frac{di_d}{dt} = -w(L_g + L_f)i_q - (R_g + R_f)i_d - V_g^d + V_d^s \\
(L_f + L_g) \frac{di_q}{dt} = w(L_f + L_g)i_d - (R_f + R_g)i_q - V_q^g + V_q^s
\]

it is also possible to obtain the voltage dynamics of the DC side of the converter through the power balance.

\[
C \frac{dV_{dc}}{dt} = I_{dc} - \frac{3V_d^s i_d}{2V_{dc}}
\]

SEC. 4 Dynamics of the grid
II. Theoretical aspects

Figure 3. Inner Loop
II. Theoretical aspects

The inner loop can be described mathematically by adding a couple of auxiliary variables $\gamma_{d/q}$

\[ \frac{d\gamma_d}{dt} = \tilde{i}_d - i_d \]
\[ \frac{d\gamma_q}{dt} = \tilde{i}_q - i_q \]

\[ V_d^s = V_d^g + R_g i_d - R_f \frac{L_g}{L_f} i_d + \frac{L_f + L_g}{L_f} (w L_f i_q + k_{pi} (\tilde{i}_d - i_d) + k_{ii} \gamma_d) \]

\[ V_q^s = V_q^g + R_g i_q - R_f \frac{L_g}{L_f} i_q + \frac{L_f + L_g}{L_f} (-w L_f i_d + k_{pi} (\tilde{i}_q - i_q) + k_{ii} \gamma_q) \]

SEC. 5 Mathematical description of the Inner loop
II. Theoretical aspects

Figure 4. Outer Loop
II. Theoretical aspects

The outer loop can be described mathematically by adding a couple of auxiliary variables $\gamma_{v/q}$.

\[
\frac{d\gamma_v}{dt} = \bar{V}_{dc} - V_{dc}
\]

\[
\bar{I}_d = k_{pv}(\bar{V}_{dc} - V_{dc}) + k_{iv}\gamma_v
\]

\[
\frac{d\gamma_q}{dt} = \bar{Q} - Q
\]

\[
\bar{I}_q = K_{pq}(\bar{Q} - Q) + K_{iQ}\gamma_Q
\]

SEC. 6 Mathematical description of the Inner loop
II. Theoretical aspects

Figure 4. Dynamic model of the VSC
II. Theoretical aspects

The state-space model of the converter can be rewritten as

\[
\dot{x} = f(x, y, u) \\
y = g(x, u)
\]

Where \(x\) represents the state variables, \(y\) represents the control variables and \(u\) represents the input variables.

\[
\Delta x = A\Delta x + B\Delta u
\]

Where:

\[
A = J(X_0) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial g} \frac{\partial g}{\partial x}\bigg|_{x=x_0}
\]

\[
B = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial g} \frac{\partial g}{\partial u}\bigg|_{u=u_0}
\]
II. Theoretical aspects

The state, control and input variables of the VSC are respectively:

\[ x = [i_d, i_q, V_{dc}, \gamma_d, \gamma_q, \gamma_v]^T \quad u = [V_d^g, V_{dc}, I_{dc}, i_q^*]^T \quad y = [V_d^s, V_q^s]^T \]

Taking this into account, it is possible to obtain the operating point (or initial condition) of the system assuming that it is operating at steady state.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i_{do})</td>
<td>(i_{q} = 0)</td>
</tr>
<tr>
<td>(i_{qo})</td>
<td>(i_q = 0)</td>
</tr>
<tr>
<td>(v_{dco})</td>
<td>(V_{dc})</td>
</tr>
<tr>
<td>(\gamma_{do})</td>
<td>(\frac{i_{d0}R_f}{k_{tu}})</td>
</tr>
<tr>
<td>(\gamma_{qo})</td>
<td>(0)</td>
</tr>
<tr>
<td>(\gamma_{vo})</td>
<td>(\frac{i_{d0}}{k_{fv}})</td>
</tr>
</tbody>
</table>

**Table 1. Initial conditions**
II. Theoretical aspects

Where the state matrix is described in

\[
A = \begin{pmatrix}
\frac{-R_s-k_{pi}}{L_f} & 0 & k_{pi}k_{pv} & k_{ii} & 0 & -k_{pi}k_{iv} \\
0 & \frac{-R_s-k_{pi}}{L_f} & 0 & k_{ii} & 0 & 0 \\
J_{31} & J_{32} & J_{33} & J_{34} & J_{36} \\
-1 & 0 & k_{pv} & 0 & 0 & -k_{iv} \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}
\]

\[
J_{31} = \frac{3(V_{d}^2 + (R_f + R_g)i_{do})}{2CV_{dc}} - \frac{R_fL_f}{L_f}i_{do} - \frac{L_f}{L_f}k_{pi}i_{do} \\
J_{32} = \frac{-3(i_{do}w_{e}L_f)}{2CV_{dc}} \\
J_{33} = \frac{3i_{do}(V_{d}^2 + R_fi_{do}^2 - \frac{L_f}{L_f}k_{pi}k_{pv}V_{dc})}{2CV_{dc}^2} \\
J_{34} = \frac{3i_{do}k_{ii}}{2CV_{dc}} \\
J_{36} = \frac{3i_{do}k_{pi}k_{iv}}{2CV_{dc}^2}
\]

SEC. 7 State matrix of the VSC.
II. Theoretical aspects

The sensitive analysis aims to determine the influence of each element of the state matrix in the system’s eigenvalues

\[ p_{ikm} = \frac{\partial \lambda_i}{\partial a_{km}} = \varphi_i^k \phi_i^m \]

It is difficult to visualize the results in a tensor, therefore, it is more useful to analyze the influence of a specific parameter of the system in the position of the eigenvalues. Thus, by applying chain of rule to the above equation

\[ p_i^x = \sum_{k=1}^{n} \sum_{m=1}^{n} \frac{\partial \lambda_i}{\partial a_{km}} \frac{\partial a_{km}}{\partial x} \]
III. Numerical results

From the state matrix it is possible to obtain the eigenvalues of the system, which are as follows.

\[
\lambda = \begin{pmatrix}
-112.179 + 436.585j & -112.179 - 436.585j \\
-195.482 + 135.993j & -195.482 - 135.993j \\
-267.000 + 266.120j & -267.000 - 266.120j
\end{pmatrix}
\]

from the same state matrix, it is possible to obtain the real part of the participation factors, as illustrated in the following table

<table>
<thead>
<tr>
<th>Participation factor</th>
<th>$L_f$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>3796.68</td>
<td>-4171.90</td>
</tr>
<tr>
<td>$p_2$</td>
<td>3796.68</td>
<td>-4171.90</td>
</tr>
<tr>
<td>$p_3$</td>
<td>1057.86</td>
<td>2245.72</td>
</tr>
<tr>
<td>$p_4$</td>
<td>1057.86</td>
<td>2245.72</td>
</tr>
<tr>
<td>$p_5$</td>
<td>4854.54</td>
<td>$-1.14 \times 10^{-27}$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>4854.54</td>
<td>$-1.14 \times 10^{-27}$</td>
</tr>
</tbody>
</table>

*Table 2.* Participation factors (real part) system parameters.
III. Numerical results

However, it is important to know the influence on the imaginary part, because it directly influences the damping factors.

\[ r_D = -\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \times 100\% \]

<table>
<thead>
<tr>
<th>Participation factor</th>
<th>( L_f )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>-690.37</td>
<td>-96230.6</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>690.37</td>
<td>96230.6</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>-777.58</td>
<td>22583.16</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>777.58</td>
<td>-22583.16</td>
</tr>
<tr>
<td>( p_5 )</td>
<td>16.01</td>
<td>3.44 \times 10^{-27}</td>
</tr>
<tr>
<td>( p_6 )</td>
<td>-16.01</td>
<td>-3.44 \times 10^{-27}</td>
</tr>
</tbody>
</table>

**Table 3.** Participation factors (imaginary part) system parameters.

<table>
<thead>
<tr>
<th>Factor</th>
<th>( L_f )</th>
<th>( L_g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{1.2} )</td>
<td>3947.41 ± 288.29</td>
<td>475.8 ± 55.07</td>
</tr>
<tr>
<td>( p_{3.4} )</td>
<td>1180.67 ± 686.03</td>
<td>161.97 ± 84.83</td>
</tr>
<tr>
<td>( p_{5.6} )</td>
<td>4856.19 ± 509.26</td>
<td>-3.270 \times 10^{-30} ± 6.66 \times 10^{-31}</td>
</tr>
</tbody>
</table>

**Table 4.** Participation factors of the non ideal grid.
III. Numerical results

The corresponding participation factors of the parameters of the PI type controllers are as follows

<table>
<thead>
<tr>
<th>Participation factor</th>
<th>$k_{pi}$</th>
<th>$k_{pv}$</th>
<th>$k_{ii}$</th>
<th>$k_{iv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>-6.83</td>
<td>-91.424</td>
<td>-0.0035</td>
<td>0.371</td>
</tr>
<tr>
<td>$p_2$</td>
<td>-6.83</td>
<td>-91.424</td>
<td>-0.0035</td>
<td>0.371</td>
</tr>
<tr>
<td>$p_3$</td>
<td>3.65</td>
<td>56.75</td>
<td>0.0035</td>
<td>-0.37</td>
</tr>
<tr>
<td>$p_4$</td>
<td>3.65</td>
<td>56.75</td>
<td>0.0035</td>
<td>-0.37</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-9.09</td>
<td>$3.92 \times 10^{-30}$</td>
<td>$-1.911 \times 10^{-19}$</td>
<td>$-3.49 \times 10^{-32}$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>-9.09</td>
<td>$3.92 \times 10^{-30}$</td>
<td>$-1.911 \times 10^{-19}$</td>
<td>$-3.49 \times 10^{-32}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor</th>
<th>$k_{pi}$</th>
<th>$k_{pv}$</th>
<th>$k_{ii}$</th>
<th>$k_{iv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0.26</td>
<td>149.7</td>
<td>0.018</td>
<td>0.115</td>
</tr>
<tr>
<td>$p_2$</td>
<td>-0.26</td>
<td>-149.7</td>
<td>-0.018</td>
<td>-0.115</td>
</tr>
<tr>
<td>$p_3$</td>
<td>-3.14</td>
<td>-72.74</td>
<td>0.013</td>
<td>0.113</td>
</tr>
<tr>
<td>$p_4$</td>
<td>3.14</td>
<td>72.74</td>
<td>-0.013</td>
<td>-0.113</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-8.92</td>
<td>$3.86 \times 10^{-30}$</td>
<td>0.033</td>
<td>$-2.20 \times 10^{-32}$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>8.92</td>
<td>$-3.86 \times 10^{-30}$</td>
<td>-0.033</td>
<td>$2.20 \times 10^{-32}$</td>
</tr>
</tbody>
</table>

Table 5 and 6. Participation factors of the control parameters.
III. Numerical results

**Figure 5.** Bifurcation of the system for $L \ [50 - 150]\text{mH}$

**Figure 5.** Bifurcation of the system for $k_{ii} \ [5 - 100] \times 10^3$
IV. CONCLUSIONS

- In this paper, a sensitivity analysis is used to determine numerically how system parameters and controller constants influence the stability of a VSC operating as a grid feeder, demonstrating its usefulness in determining variables that can cause instability and those that are desirable to control in order to improve it.
- An increase in the proportional action increases the stability margins of all the eigenvalues, which is useful since it can be coordinated with a high inductance (or capacitance) value for filtering, which decreases these margins.
- A high value for the integral action is desired to ensure zero steady state error, however, as can be seen from the analysis, a high value results in a significant decrease in the damping ratios.
- The non-ideal network is a parameter that depends on the connection point; therefore, its impedance varies. It must be taken into account because, as demonstrated, it affects the stability margins and also worsens the sensitivity of the system.