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Universidad
Tecnológica
de Pereira



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Small-Signal Stability and Sensitivity Analysis for Grid Following Converters

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I. Introduction

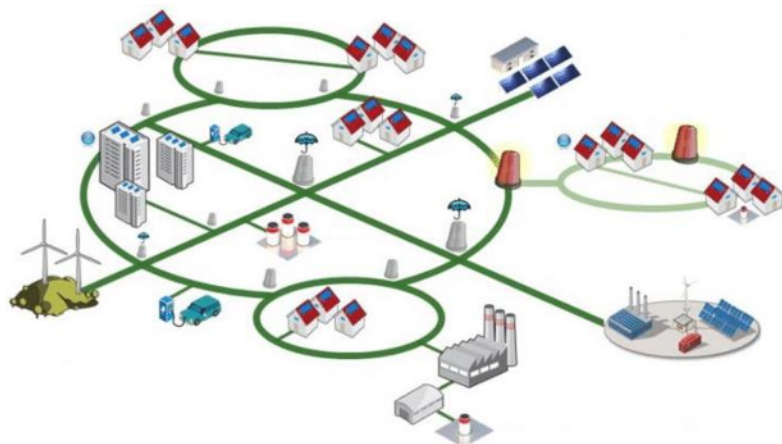


Figure 1. Active distribution network

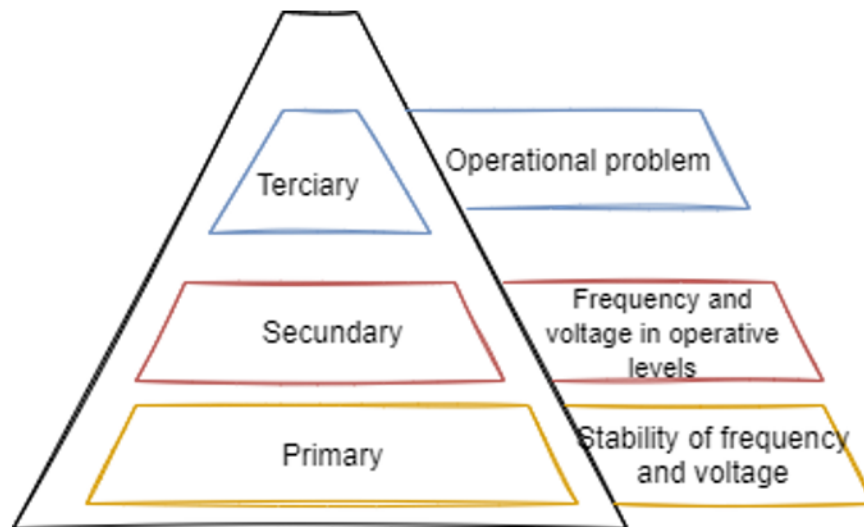


Figure 2. Schematic diagram of the hierarchical control in active distribution systems

II. Theoretical aspects

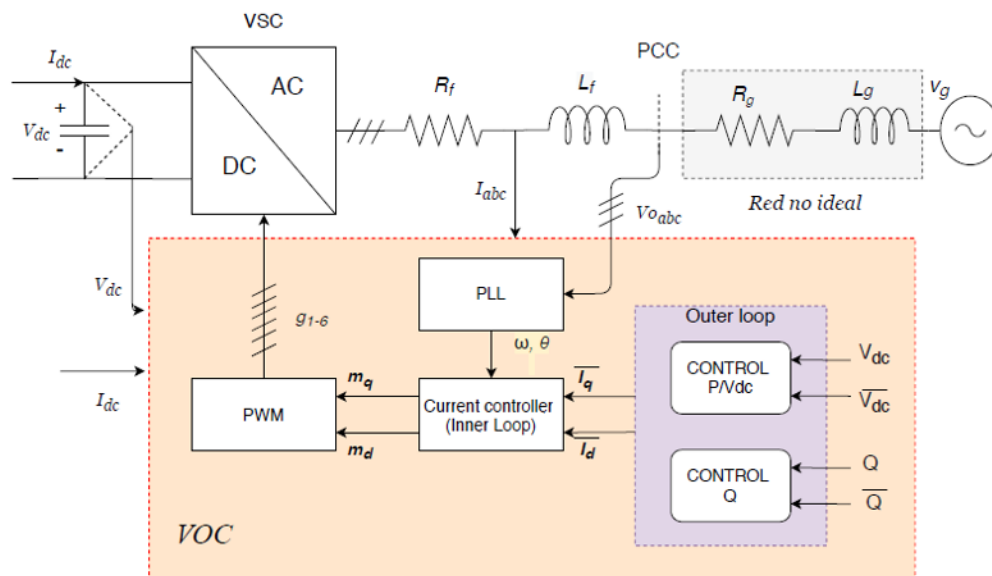


Figure 3. Voltage source converter (VSC) with vector-oriented control (VOC)

II. Theoretical aspects

$$x_a + x_b + x_c = 0$$

$$\begin{pmatrix} x_\alpha \\ x_\beta \end{pmatrix} = k \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} x_a \\ x_b \\ x_c \end{pmatrix}$$

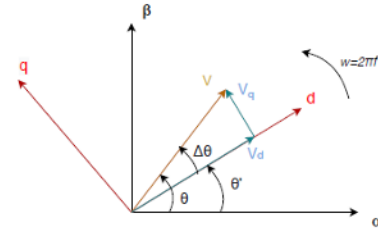
$$p_{\alpha\beta 0} = [V_{\alpha\beta 0}]^T [I_{\alpha\beta 0}]$$

$$p_{\alpha\beta 0} = [V_{abc}]^T [TC]^T [TC] [I_{abc}]$$

$$p_{\alpha\beta 0} = \frac{2}{3} [V_{abc}] [I_{abc}]$$

$$p_{\alpha\beta 0} = \frac{2}{3} p_{abc}$$

SEC. 1 Clark transformation



$$\begin{pmatrix} x_d \\ x_q \end{pmatrix} = \begin{pmatrix} \cos(\theta') & \sin(\theta') \\ -\sin(\theta') & \cos(\theta') \end{pmatrix} \begin{pmatrix} x_\alpha \\ x_\beta \end{pmatrix}$$

SEC. 2 Park transformation

$$p_{\alpha\beta 0} = p_{odq}$$

$$p_{abc} = \frac{3}{2} p_{odq}$$

$$p = \frac{3}{2} (v_d i_d + v_q i_q)$$

$$q = \frac{3}{2} (v_q i_d - v_d i_q)$$

SEC. 3 Active and reactive power in the dq frame

Note: SEC mean set of equations

II. Theoretical aspects

The current dynamics of the system on the AC side can be obtained by applying kirchoff's second law

$$(L_f + L_g) \frac{di_d}{dt} = -w(L_g + L_f)i_q - (R_g + R_f)i_d - V_g^d + V_d^s$$
$$(L_f + L_g) \frac{di_q}{dt} = w(L_f + L_g)i_d - (R_f + R_g)i_q - V_q^g + V_q^s$$

it is also possible to obtain the voltage dynamics of the DC side of the converter through the power balance.

$$C \frac{dV_{dc}}{dt} = I_{dc} - \frac{3V_d^s i_d}{2V_{dc}}$$

SEC. 4 Dynamics of the grid

II. Theoretical aspects

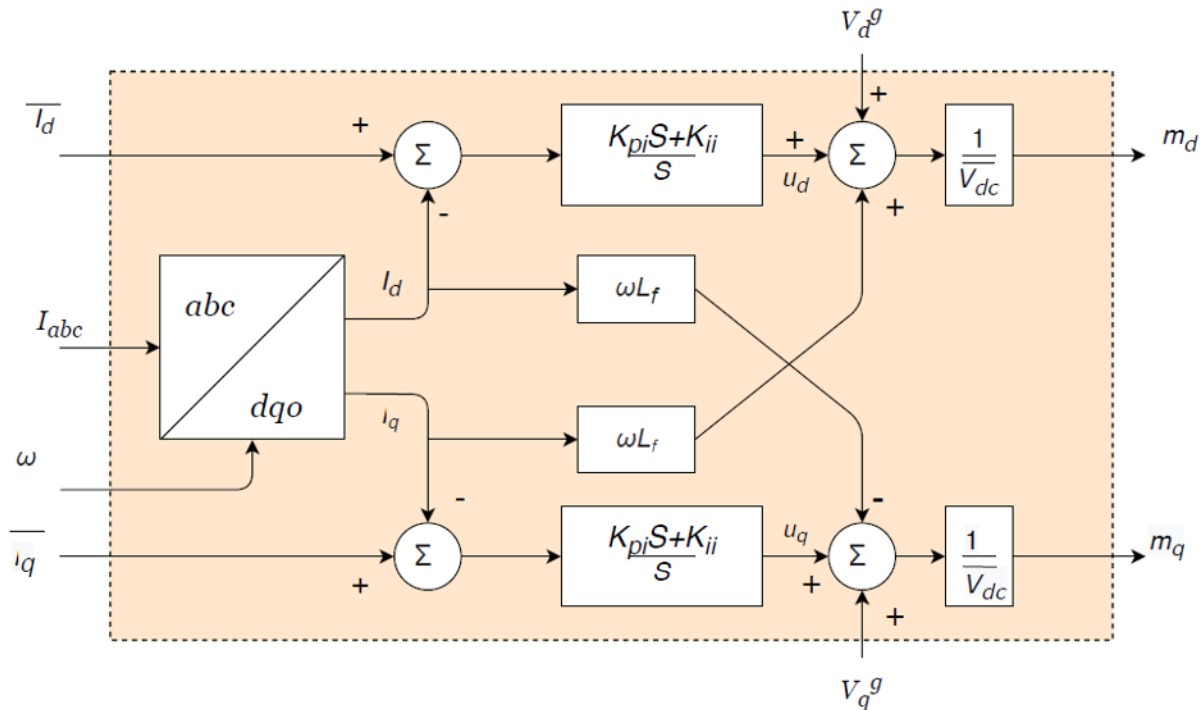


Figure 3. Inner Loop

II. Theoretical aspects

The inner loop can be described mathematically by adding a couple of auxiliary variables $\gamma_{d/q}$

$$\frac{d\gamma_d}{dt} = \bar{i}_d - i_d$$

$$\frac{d\gamma_q}{dt} = \bar{i}_q - i_q$$

$$V_d^s = V_d^g + R_g i_d - R_f \frac{L_g}{L_f} i_d + \frac{L_f + L_g}{L_f} (w L_f i_q + k_{pi} (\bar{i}_d - i_d) + k_{ii} \gamma_d)$$

$$V_q^s = V_q^g + R_g i_q - R_f \frac{L_g}{L_f} i_q + \frac{L_f + L_g}{L_f} (-w L_f i_d + k_{pi} (\bar{i}_q - i_q) + k_{ii} \gamma_q)$$

SEC. 5 Mathematical description of the Inner loop

II. Theoretical aspects

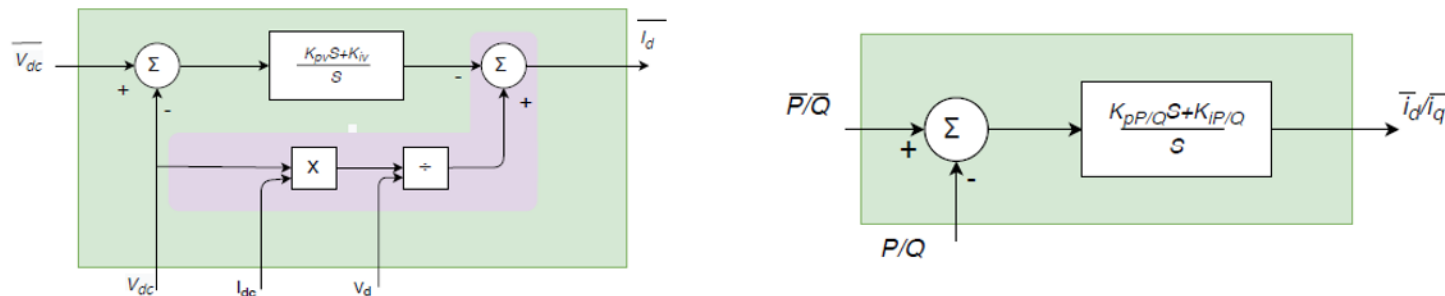


Figure 4. Outer Loop

II. Theoretical aspects

The outer loop can be described mathematically by adding a couple of auxiliary variables $\gamma_{v/q}$.

$$\frac{d\gamma_v}{dt} = \bar{V}_{dc} - V_{dc}$$
$$\bar{I}_d = k_{pv}(\bar{V}_{dc} - V_{dc}) + k_{iv}\gamma_v$$

$$\frac{d\gamma_q}{dt} = \bar{Q} - Q$$
$$\bar{I}_q = K_{pq}(\bar{Q} - Q) + K_{iq}\gamma_q$$

SEC. 6 Mathematical description of the Inner loop

II. Theoretical aspects

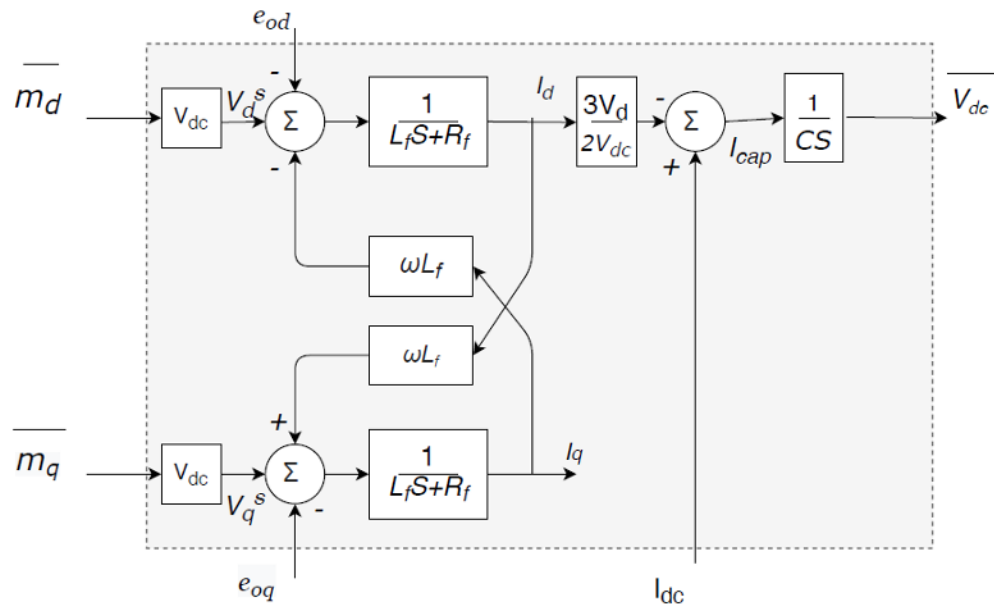


Figure 4. Dynamic model of the VSC

II. Theoretical aspects

The state-space model of the converter can be rewritten as

$$\begin{aligned}\dot{x} &= f(x, y, u) \\ y &= g(x, u)\end{aligned}$$

Where x represents the state variables, y represents the control variables and u represents the input variables.

$$\Delta \dot{x} = A \Delta x + B \Delta u$$

Where:

$$\begin{aligned}A &= J(X_0) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial g} \frac{\partial g}{\partial x} \Big|_{x=x_0} \\ B &= \frac{\partial f}{\partial u} + \frac{\partial f}{\partial g} \frac{\partial g}{\partial u} \Big|_{u=u_0}\end{aligned}$$

II. Theoretical aspects

The state, control and input variables of the VSC are respectively:

$$x = [i_d, i_q, V_{dc}, \gamma_d, \gamma_q, \gamma_v]^T \quad u = [V_d^g, V_{dc}, I_{dc}, i_q^*]^T \quad y = [V_d^s, V_q^s]^T$$

Taking this into account, it is possible to obtain the operating point (or initial condition) of the system assuming that it is operating at steady state

Variable	Initial condition
i_{do}	$\frac{-v_g^d + \sqrt{(v_d^g)^2 + \frac{8v_{dc}I_{dc}(R_g + R_f)}{3}}}{2(R_f + R_g)}$
i_{qo}	$i_q = 0$
v_{dco}	V_{dc}
γ_{do}	$\frac{i_{d0}R_f}{k_{\gamma d}}$
γ_{qo}	0
γ_{vo}	$-\frac{i_{d0}}{k_{\gamma v}}$

Table 1. Initial conditions

II. Theoretical aspects

Where the state matrix is described in

$$A = \begin{pmatrix} \frac{-R_s - k_{pi}}{L_f} & 0 & \frac{k_{pi}k_{pv}}{L_f} & \frac{k_{ii}}{L_f} & 0 & -\frac{k_{pi}k_{iv}}{L_f} \\ 0 & \frac{-R_s - k_{pi}}{L_f} & 0 & 0 & \frac{k_{ii}}{L_f} & 0 \\ J_{31} & J_{32} & J_{33} & J_{34} & 0 & J_{36} \\ -1 & 0 & k_{pv} & 0 & 0 & -k_{iv} \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}$$

$$J_{31} = -\frac{3(V_d^g + (R_f + R_g)i_{d0} - R_f \frac{L_g}{L_f} i_{do} - \frac{L_f + L_g}{L_f} k_{pi} i_{do})}{2CV_{dc}}$$

$$J_{32} = -\frac{3(i_{do} \omega_o L_f)}{2CV_{dc}} \frac{L_f + L_g}{L_f}$$

$$J_{33} = \frac{3i_{do}(V_d^g + R_f i_{do} - \frac{L_f + L_g}{L_f} k_{pi} k_{pv} V_{dc})}{2CV_{dc}^2}$$

$$J_{34} = -\frac{3(i_{do} k_{ii})}{2CV_{dc}} \frac{L_f + L_g}{L_f}$$

$$J_{36} = \frac{3i_{do} k_{pi} k_{iv}}{2CV_{dc}} \frac{L_f + L_g}{L_f}$$

SEC. 7 State matrix of the VSC.

II. Theoretical aspects

The sensitive analysis aims to determine the influence of each element of the state matrix in the system's eigenvalues

$$p_i^{km} = \frac{\partial \lambda_i}{\partial a_{km}} = \varphi_i^k \phi_i^m$$

It is difficult to visualize the results in a tensor, therefore, it is more useful to analyze the influence of a specific parameter of the system in the position of the eigenvalues. Thus, by applying chain of rule to the above equation

$$p_i^x = \sum_{k=1}^n \sum_{m=1}^n \frac{\partial \lambda_i}{\partial a_{km}} \frac{\partial a_{km}}{\partial x}$$

III. Numerical results

From the state matrix it is possible to obtain the eigenvalues of the system, which are as follows.

$$\lambda = \begin{pmatrix} -112.179 + 436.585j & -112.179 - 436.585j \\ -195.482 + 135.993j & -195.482 - 135.993j \\ -267.000 + 266.120j & -267.000 - 266.120j \end{pmatrix}$$

from the same state matrix, it is possible to obtain the real part of the participation factors, as illustrated in the following table

Participation factor	L_f	C
p_1	3796.68	-4171.90
p_2	3796.68	-4171.90
p_3	1057.86	2245.72
p_4	1057.86	2245.72
p_5	4854.54	-1.14×10^{-27}
p_6	4854.54	-1.14×10^{-27}

Table 2. Participation factors (real part) system parameters.

III. Numerical results

However, it is important to know the influence on the imaginary part, because it directly influences the damping factors.

$$r_D = -\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} 100\%$$

Participation factor	L_f	C
p_1	-690.37	-96230.6
p_2	690.37	96230.6
p_3	-777.58	22583.16
p_4	777.585	-22583.16
p_5	16.01	3.44×10^{-27}
p_6	-16.01	-3.44×10^{-27}

Table 3. Participation factors (imaginary part) system parameters.

Factor	L_f	L_g
$p_{1,2}$	3947.41 ± 288.29	475.8 ± 55.07
$p_{3,4}$	1180.67 ± 686.03	161.97 ± 84.83
$p_{5,6}$	4856.19 ± 509.26	$-3.270 \times 10^{-30} \pm 6.66 \times 10^{-31}$

Table 4. Participation factors of the non ideal grid.

III. Numerical results

The corresponding participation factors of the parameters of the PI type controllers are as follows

Participation factor	k_{pi}	k_{pv}	k_{ii}	k_{iv}
p_1	-6.83	-91.424	-0.0035	0.371
p_2	-6.83	-91.424	-0.0035	0.371
p_3	-3.65	56.75	0.0035	-0.37
p_4	-3.65	56.75	0.0035	-0.37
p_5	-9.09	3.92×10^{-30}	-1.911×10^{-19}	-3.49×10^{-32}
p_6	-9.09	3.92×10^{-30}	-1.911×10^{-19}	-3.49×10^{-32}

Factor	k_{pi}	k_{pv}	k_{ii}	k_{iv}
p_1	0.26	149.7	0.018	0.115
p_2	-0.26	-149.7	-0.018	-0.115
p_3	-3.14	-72.74	0.013	0.113
p_4	3.14	72.74	-0.013	-0.113
p_5	-8.92	3.86×10^{-30}	0.033	-2.20×10^{-32}
p_6	8.92	-3.86×10^{-30}	-0.033	2.20×10^{-32}

Table 5 and 6. Participation factors of the control parameters.

III. Numerical results

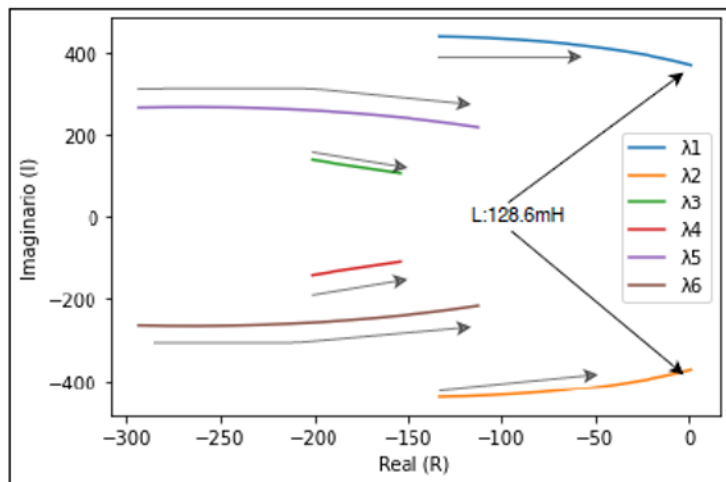


Figure 5. Bifurcation of the system for L [50 - 150]mH

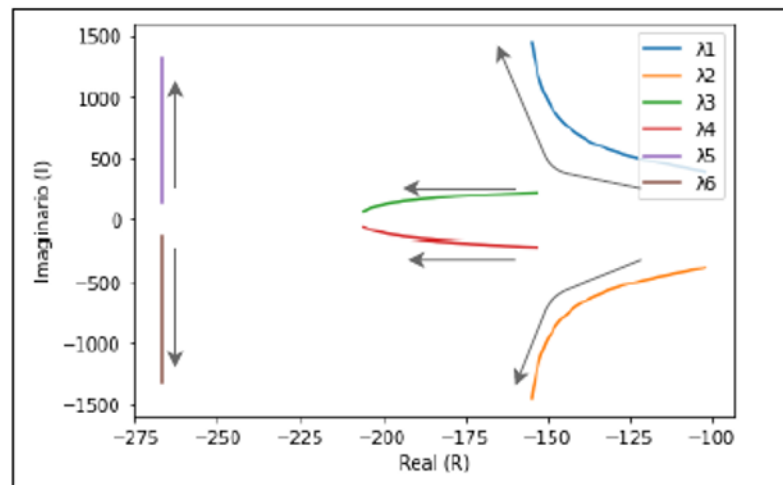


Figure 5. Bifurcation of the system for k_{ii} [5 - 100] $\times 10^3$

IV. CONCLUSIONS

- In this paper, a sensitivity analysis is used to determine numerically how system parameters and controller constants influence the stability of a VSC operating as a grid feeder, Demonstrating its usefulness in determining variables that can cause instability and those that are desirable to control in order to improve it.
- An increase in the proportional action increases the stability margins of all the eigenvalues, which is useful since it can be coordinated with a high inductance (or capacitance) value for filtering, which decreases these margins.
- A high value for the integral action is desired to ensure zero steady state error, however, as can be seen from the analysis, a high value results in a significant decrease in the damping ratios
- The non-ideal network is a parameter that depends on the connection point; therefore, its impedance varies. It must be taken into account because, as demonstrated, it affects the stability margins and also worsens the sensitivity of the system.