An evolutionary algorithm for evolvable hardware

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1. INTRODUCTION

The evolution process experienced by a species in nature turns their members more efficient as regard survival in a changing and complex environment. These processes can be compared to the minimization (or maximization) of a cost function in a mathematical optimization problem. Numerous works on evolutionary algorithms have been presented that are based on the theory of natural evolution [6], such as Evolution Strategies (ES), Evolutionary Programming (EP) and Genetic Algorithm (GA). Although these Evolutionary Algorithms are widely applied to many fields, the mathematical foundation of evolutionary algorithms is very weak. There has been growing interest in a mathematical model to characterize the evolutionary algorithm. In [11], an abstract mathematical model is presented, where it uses an abstract selection and evolution operators.

Evolvable hardware [10] is an exciting and emerging field. It lies at the intersection of evolutionary computation and physical design. Through the use of evolutionary computation methods, the electrical and mechanical hardware systems are automatically designed, adapted, and reconfigured. There are many researches in this field. For example, in [12] a field programmable gate array was evolved to discriminate a tone, in [9] an evolutionary system to automatically design both the robot morphology and controller was developed, in [7] an autonomous evolution of a dynamic gait on Sony’s entertainment robot was developed, and in [2] a normalized steady state genetic algorithm is proposed and used to design by simulation a ST5 antenna (X-band antenna for NASA’s Space Technology 5 spacecraft).
II. EVOLUTION OF A SPECIES

A. Evolution in nature

The animals of a species in nature share similar characteristics. If these characteristics could be put in a vector space, a set of the animal characteristics vectors of the species would be grouped in a zone. In many cases, the evolution of a species in nature is gradual [3] and it would take many generations for a noticeable change to take place.

From the sexual selection studied in [3], the following can be concluded: the number of children in a group of parents will show greater probability of having a large value when those parents are more similar in characteristics to the fittest parent. Conversely, the number of children in a group of parents will have greater probability of having a low value when those parents differ too much in characteristics from the fittest parent, and/or differ too much in characteristics from each other. The last analysis considers the difficulty in bearing children between two biologically different animals (different with respect to their characteristics). Indeed, they could be incompatible to have descendants and, consequently, will have no descendant with probability 1. The mutants, deformed or mutilated children have extremely different features from the animals of their own species, which makes them most unlikely to have children. Even if these animals were capable of reproducing, they would most likely be rejected for reproduction by the members of their own species.

B. Evolution of a physical system

Let us consider three things: 1) an individual is a physical system (a robot, an industrial plant, etc.), 2) the species' individuals have similar characteristics, and 3) the evolution of this species is gradual. Then, the following analysis is done. If the physical system (individual) of a species works well, then the physical systems of the same species sharing similar characteristics will work well too with high probability. Hence, the risk that the physical system breaks down in an evolutionary process is very low or null. The eliminated individuals are those that do not work as well as the rest of the group; those individuals eliminated do not necessarily work badly. There is a great probability that some children will work better than their parents, consequently, the species will have a tendency to a minimum of the cost function.

C. Example of evolution

For example, let us consider an evolution process with a convex function $F(x) = \|x\|^2$, and a uniform random distribution in the mutation operator. In this example, if the children dispersion is small enough, the probability that the fittest individual creates a child with less cost than that of itself is in a range close to 0.5 (see Fig. 1).

III. DEFINITIONS

The following definitions are used in the mathematical analysis of the evolutionary algorithm proposed in the next section.

A. Theoretical definitions

1) Power or Parts of a Set: The set of all subsets belonging to $B \subseteq X$, where $X$ is a universal set, is called the power of $B$ and is denoted as $P(B)$ or $2^B$. $P(B) = \{ A \subseteq X : A \subseteq B \}$.
2) **Metric Space**: A metric space is a pair \((X,d)\), where \(X\) is a set and \(d\) is a metric in \(X\) (or a distance function on \(X\)), that is, a function defined on \(X \times X\) such that for all \(x, y, z \in X\) we have:

- **M1**: \(d(x,y)=0\) iff \(x=y\)
- **M2**: \(d(x,y)=d(y,x)\)
- **M3**: \(d(x,y)\leq d(x,z)+d(z,y)\).

3) **Vector Space**: A vector space (or linear space) over a field \(K\) is a nonempty set \(X\) of elements \(x, y, \ldots\) (called vectors) together with two algebraic operations. These operations are called vector addition (that is a mapping \(X \times X\) which is commutative and associative) and multiplication of vector by scalars (that is a mapping \(K \times X\) which is distributive). To delve into this subject refer to [8].

4) **Banach Space**: A Banach Space is a complete normed space (complete in the metric defined by the norm). A normed space is a vector space with a norm defined on it (to delve into this subject refer to [8]).

5) **Ball**: Given a point \(x \in X\), where \(X\) is a metric space, and a real number \(r>0\), two types of sets are defined:

- \(B(x;r) = \{y \in X : d(x,y)<r\}\) (Open ball)
- \(\overline{B}(x;r) = \{y \in X : d(x,y)\leq r\}\) (Closed ball)

6) **Neighborhood**: A neighborhood of \(x \in X\) is any subset of \(X\), where \(X\) is a metric space, that contains an open ball \((x,ε)\).

7) **Global and local minimum on a wide sense**: Let the function \(f:X \rightarrow R\), where \(X\) is a metric space and \(R\) is the real line. The global minimum, on a wide sense, of function \(f\) is the point \(x^* \in X\) such that \(f(x^*) \leq f(x)\) for all \(x \in X\) different from point \(x^*\). The local minimum, on an wide sense, of function \(f\) is the point \(x^* \in X\) such that \(f(x^*) \leq f(x)\) for all \(x \in E^*\) different from \(x^*\), where \(E^*\) is a neighborhood of \(x^*\).

B. Definitions formulated to be used in this work

1) **Individual**: It is a biological animal, or a physical system, or software, or any other system having a defined vector of characteristics. This vector has information on the features that differentiate the biological animals, or physical systems, or software, etc. The characteristics in a biological animal may be: size of adult animal parts, adult muscular mass, adult brain volume, etc. The characteristics in other systems may be: control gains, parameter values, etc. When a vector of characteristics of a physical system is changed, a new individual is obtained. When we are working in a vector space, the individuals are represented by its vector of characteristics.

2) **Creation of a Child**: It is the process of mutation and/or recombination of the vector of characteristics of one or several individuals to attain a new vector of characteristics. The child, then, is the physical system with this new vector of characteristics.

3) **Fitness**: The extent to which an individual is adapted to or able to produce children.

4) **Species**: It is a group of individuals that share a common ancestor (a parent or, recursively, a parent of an ancestor), a lineage (a common ancestor) that keeps its integrity with respect to other lineages, both in time and space. At some point in the progress of the group, the members may diverge into two branches. When such a divergence becomes clear enough, each new branch is considered a different species.

5) **Species dispersion**: A term that refers to the location, in a vector space, of species' individuals related to a central value.

6) **Space**: It is a Banach space, whose elements or points are characteristic vectors of individuals. These individuals possess an equal structure (same number of limbs, similar organs, with feathers or hair, same physical structure, same software structure, and the like).

7) **Cost functional \(F\) and cost of an individual**: cost functional \(F\) is a functional with domain in the space \(S\) and range in the positive real numbers; thus, \(F:S \rightarrow R^+\). This function assigns a cost to each individual. The costs functional \(F\) must be such that \(F(x) \rightarrow \infty\) with \(\|x\| \rightarrow \infty\), where \(\|x\|\) is a norm of \(x\) [8].

8) **Fittest Individual**: It is the individual which has less cost than the other individuals of the species, or which is the most adapted to or the ablest to produce children.

9) **Optimal Individual**: It is the individual which has a vector of characteristics equal to the global minimum, on a wide sense, of function \(F\).

10) **Evolution stage or generation**: It is the average time interval between the creation of parents and the creation of their children.

11) **Death by inefficiency**: It is the elimination, based on some elimination criteria, of an individual which has high costs with respect to other individuals of his same group.

IV. **EVOLUTIONARY ALGORITHM \((M,1+\lambda)-E1E\)**

In the evolutionary algorithm \((m,1+\lambda)-E1E\), the term \((m,1+\lambda)\) means that the \(m\) parents will have \(\lambda\) children. Then, the \(\lambda\) children and the fittest of the \(m\) parents will be evaluated in order to obtain \(m\) individuals with the smallest cost values to form the new population of the species. E1E indicates the evolution of only one Species.

In the algorithm \((m,1+\lambda)-E1E\), it should be complied with \((\lambda+1)\geq m, m \geq 2\) and \(\lambda \geq 2\). The first condition is aimed at ensuring that the new population has \(m\) individuals. The second and
third ones are necessary for self-adjustment of the species’ dispersion, which will be covered further below in this section.

A. Algorithm (m,1+1)-EIE

The procedure is as follows:

1. Initialize the first individual \( a^0 \), then initialize \( a^n \) such that \( a^n \in \bar{B}(\alpha^n; r / c_n) \); \( i = 2, \ldots, \mu \).

2. Determine the fittest individual \( v^\mu \) and initialize \( n = 1 \).

3. While the finalization criteria are not yet met, perform the following:
   a. Select \( \bar{a}^{n-1} \) parents. These selected parents are the elements of \( a^{n-1} \) contained by the closed ball \( \bar{B}(v^{n-1}, r) \).
   b. Modify the order of \( a^{n-1} \) such that:
      \[
      a^{n-1} = \bar{B}(v^{n-1}, r), \text{ for } i = 1, \ldots, \bar{a}^{n-1}; \text{ and } \]
      \[
      F(a^{n-1}) \leq F(a^{n-1}) \leq \ldots \leq F(a^{n-1})
      \]
   c. If \( i = 1 \), create the children in the following way:
      \[
      Z_{ik}^n = v^{n-1} + X_k^{n-1} r / c_n
      \]
      where \( 1 \leq k \leq \lambda \).
   
   If \( \bar{a}^{n-1} \geq 2 \), create the children in the following way:
      \[
      Z_{ik}^n = v^{n-1} + X_k^{n-1} d(a^{n-1}, v^{n-1}) / c_n
      \]
      where \( j \geq 2 \), and \( Z_{ik}^n \) is the \( k \)-th child created using the first and \( j \)-th selected parent. The number of children created by the first and \( j \)-th selected parent is calculated as follows:
      \[
      \bar{X}_{ik}^n = \begin{cases} 
      \lambda - f(x) \left( \frac{\lambda}{\bar{a}^{n-1} - 1} \right) / (\bar{a}^{n-1} - 2) & \text{if } j = 2 \\
      f(x) \left( \frac{\lambda}{\bar{a}^{n-1} - 1} \right) & \text{if } j = 3, 4, \ldots, \bar{a}^{n-1}
      \end{cases}
      \]
   d. Form the vector \( b^n \) with all the created children in the following way:
      \[
      b^n = (b_1^n, \ldots, b_{\mu}^n, \ldots, b_{\mu}^n) \in S; b_2^n = Z_{ik}^n \in S
      \]
   e. Calculate \( a^n \):
      \[
      a^n = T_2^n \left( a^{n-1}, b^n \right)
      \]
   f. Increment \( n = n + 1 \).

In the algorithm, the closed ball \( \bar{B}(\alpha, r) \) is \( \bar{B}(\alpha, r) = \{ y \in S : d(y, x) \leq r \} \); \( d(\alpha, r) \) is the metric induced by the norm of space \( S \); \( r \) and \( c_n \) are real constants (algorithm parameters); \( f(x) \) is a function that returns the integer part of its argument; \( X_{ik}^n \in \mu \) is a random variable having uniform probability on the closed ball \( \bar{B}(0,1) \); the individuals \( a^n, i = 1, 2, \ldots, \mu \) form the vector \( a^n = (a_1^n, a_2^n, \ldots, a_\mu^n) \in S^n \); and \( v^n \) is an element of \( \alpha^n \) such that
      \[
      F(v^n) = \min \left( F \left( \alpha_i^n \right) \right)
      \]

Mapping \( T^n : S^n \rightarrow S^{n+1} \) evaluates \( v^{n+1} \) and the elements from \( b^n \) selects \( m \) characteristics vectors which give the smallest cost values and forms a vector with the \( m \) selected vectors.

Possible spaces \( S \) can be: a real vector space with 1-norm; a real vector space with 2-norm, or a real vector space with \( \infty \)-norm (these three are Banach spaces).

B. Self-adaptation of the species dispersion

In the creation of children, the distance \( d(a^n, v^{n-1}) \) is used, which is the distance between \( v^{n-1} \) and another element of \( a^{n-1} \). Constant \( c_n \) can be taken as \( c_n = \frac{1}{\sqrt{F_{a^n}}} \) if considering \( S \) equal to the real vector space of dimension \( N \) with \( \infty \)-norm.

This constant makes \( (X_{ik}^{n+1} / c_n) \in \bar{B}(0,1) \) be compliant with a probability \( P_{a^n} \). Therefore, it is expected that 100\% of the children will be less dispersed than the parents, and the remaining children will be more dispersed than the parents. The dispersions are measured with respect to \( \alpha^n \). The self-adaptation of the species dispersion can be controlled by the parameter \( P_{a^n} \).

Choosing \( P_{a^n} = 0.5 \), the dispersion of the species tends to increase when such species is relatively far from a minimum of the costs function (far with respect to their distance on space \( S \)). The reason is that the most distant children to the fittest one of their parents will have greater probabilities of having the smallest cost of the species than the least distant children. Figure 2 shows this fact when using the same example of section 2.

Conversely, choosing again \( P_{a^n} = 0.5 \), the dispersion of the species tends to decrease when such species is relatively close to a minimum of the costs function, because, now, the least distant children to the fittest one of their parents will have
greater probabilities of having a smaller cost than the most
distanced children. Figure 3 shows this fact when using the
same example of section 2 again.

Only some special cases do not have these properties. One
element is illustrated in Fig. 4. That would happen in this example is a decrease of the species dispersion while it go through the
strait zone.

This self-adaptation of the dispersion of a species increases
their task (for example points that have costs equal or less than
50 in Fig. 5).

The dispersion of parents that will have children is limited
by the closed ball \( B(v^{*n}, r) \). Considering this fact, the vector
of characteristics of the children will be placed within the closed
ball \( B(v^{*n}, r/c_n) \), on account of the nature of the probability
density function with which the children are created.

The aim here is to avoid the characteristics vector of a child
getting into the Bad Zone. Algorithm \((m, 1+λ)-E1E\) accepts only
a vector \( v^* \) with equal or less cost than that of \( v^{*n-1} \). Let us
consider that algorithm \((m, 1+λ)-E1E\) begins with within the
Acceptable Zone. Every vector \( v^n \) with \( n ≥ 0 \) is equal to or
equal to a vector with a smaller cost; therefore, \( v^n \) is in the
acceptable zone \( v/n ≥ 0 \). Then, if \( r/c_n \) has low enough value, the
vector of characteristics of a child will never get into the bad
zone. For example, let us consider \( r/c_n = 0.1 \), then, the distance
between the characteristics vector of a child and \( v^{n+1} \) is smaller
than or equal to 0.1. If we consider Fig. 5 in this example, the
characteristics vector of any child will never get into the bad
zone; because, to happen, the distance between the vector of
characteristics of the child and \( v^{n+1} \) has to be equal or greater
than 0.2. Therefore, the constant \( r \) has to be defined with a value
that makes null the risk to break down a physical system. The

**Figure 2.** Best child when the species is relatively far from the
minimum point. The points \( v' \) and \( v^* \) are, respectively, the vector of
characteristics of the fittest individual and the optimal individual.

**Figure 3.** Best child when the species is relatively close to the
minimum point. The points \( v' \) and \( v^* \) are, respectively, the vector of
characteristics of the fittest individual and the optimal individual.

**C. Making null the risk to break down a physical system**

Let us consider two zones in space \( S \): The first one, the **Bad
Zone**, whose points are characteristics vectors of individuals
prone to collapse (for example points that have costs equal or
greater than 60 in Fig. 5). This means that the physical system
works very badly or that it has a high risk of breaking down.
The second, the **Acceptable Zone**, whose points are
characteristics vectors of individuals that acceptably perform

**Figure 4.** Special case of self-adaptation. The points \( v' \) and \( v^* \) are,
respectively, the vector of characteristics of the fittest individual and the
optimal individual.

**Figure 5.** Graph of Acceptable and Bad values of the characteristics
vectors of individuals. Here \( d \) is the minimum distance between points
of acceptable and bad zones.
value of \( r \) can be defined based on simulation results. If the mathematical model does not accurately describe the real system, a smaller value of \( r \) than that defined by simulations can be used.

In Sections 4.2 and 4.3, the properties of algorithm \((\mu,1+\lambda)\)-E1E are reviewed. These properties improve the performance of the evolutionary algorithm (i.e., good convergence rate and better chances of implementing it on evolvable hardware, as will be seen in the experiments). However, these properties can make the evolution converge to a sub-optimal solution. For numerous engineering problems, especially to evolve hardware with security, a sub-optimal solution will be an acceptable solution whenever that solution improves the performance of the system.

V. RESULTS FROM SIMULATIONS

The algorithm \((\mu,1+\lambda)\)-E1E was tested on the Ackley cost function:

\[
F_a(x) = -20 \exp \left(-0.2 \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2} \right) - \exp \left(\frac{1}{N} \sum_{i=1}^{N} \cos(2\pi x_i)\right) + 20 + \epsilon
\]

where \(x \in S\) is the characteristics vector of an individual; \(x_i\) is the \(i\)-th element of the vector \(x\); and \(N\) is the dimension of space \(S\). These functions have the global minimum at the origin. Space \(S\) is a real vector space with norm \(\infty\) and 30 dimensions. This is a Banach space. The initial vector \(a^e\) is such that \(\|a^e\| \leq 30\).

Before testing the algorithm \((\mu,1+\lambda)\)-E1E, some heuristics are given.

A. Some heuristics for algorithm \((m,1+\lambda)\)-E1E

The self-adaptation of the dispersion works well when a large population is used. When a small population (for example \(m=2\) and \(\lambda=4\)) and \(c_m=\sqrt{P_m}\) with \(P_m=0.5\) are used, the dispersion tends to decrease all the time. This causes that the algorithm \((\mu,1+\lambda)\)-E1E does not converge to a minimum. To avoid this problem, with \(P_m<0.5\) should be used, for example \(P_m=0.25\). The probability to increase the dispersion is larger when a smaller \(P_m\) is used but excessively small values of \(P_m\) affect the precision of the results.

To increase the probability of avoiding local minimums, small \(P_m\) should be used. For example \(P_m=0.16\).

To decrease the risk of breaking down a physical system in an evolvable hardware, a small value of \(r\) should be used. However, the value of \(r\) should not be excessively small because the convergence of the algorithm could be very slow. To increase the probability of avoiding local minimums a large value of \(r\) should be used.

The choice of the population of the species depends on the difficulty of the problem. More difficult problems need more population.

Figure 6. Cost of the fittest individual per evolution stage obtained from the test of \((m,1+\lambda)\)-E1E on the Ackley cost function.

Figure 7. Trajectory generated on the first and second dimension of space \(S\) by the fittest individual. Results obtained from the test of \((m,1+\lambda)\)-E1E on the Ackley cost function. An individual is represented by a point. The closed ball \(\tilde{B}(x^{m+1},r_{11})\) which contains the children at the \(n\)-th evolution stage is represented by a square.
The relation of $\lambda$ and $\mu$ can be $\lambda=2\mu$, with this equality a good performance is obtained in the dispersion’s self-adaptation.

B. Test of (m,1+l)-E1E on the Ackley cost function

The algorithm parameters are: $\mu=10$, $\lambda=20$, $c_s = \sqrt{p_m}$ with $p_m=0.16$. The result of the cost variation is shown in Fig. 6. The variation of the species dispersion and the trajectory generated by the fittest individual are shown, on the first and second dimension of space $S$, in Fig. 7. The number of function evaluations was 200000. The final cost, obtained after applying the algorithm $(\mu,1+\lambda)$-E1E, is $5.5 \times 10^3$. Additionally, ten independent tests were performed, in all of these tests the algorithm located the global minimum; the average final cost is $10.496 \times 10^3$. In [1] the same test was performed with ES and EP; results were: final cost $7.48 \times 10^4$ with $\mu=30$, $\lambda=200$ for ES; and final cost $1.39 \times 10^2$ with $\mu=200$ for EP.

VI. EXPERIMENT

Let us consider as an individual a mobile robot (see Fig. 8) with a tracking control. The mobile robot is a unicycle like vehicle which has two traction wheels and one castor wheel (see Fig. 9). The point

$$h = [x \ y]^T$$

is the point that is required to track a trajectory. The trajectory tracking control of the robot system is obtained from [5]. This tracking control considers the dynamic model of the mobile robot in its control law to compensate the non-linear part of the system. The tracking control is presented as follows:

$$v_{rr} = \hat{D}M(u - N) + T_s \dot{\theta}$$

$$u = h_x + K_x \ddot{h} + K \dot{h}, \quad \ddot{h} = h_x - h.$$

Figure 8. Mobile robot Pioneer 2DX

Figure 9. Schematic of the mobile robot

Figure 10. Experimental result: cost of the fittest individual per evolution stage

Figure 11. Experimental result: cost of all individuals of the species per evolution stage
reference velocity (these references are send to the robot),
\( \hat{\theta}_i \) is the \( i \)-th estimated parameter of the dynamic model, and
the rest of variables are shown in Fig. 9.

The elements of the characteristics vector of the individual
are the estimated parameters of the dynamic model of the robot.
These estimated parameters are used to compensate the
nonlinearities in the control system. Better estimated parameters
imply better control performance. The problem here can be
formulated as to obtain the estimated parameters that give the
least control errors.

In the experiment, the cost of an individual is calculated by
evaluating the performance of the individual (mobile robot)
executing a task. The task considered in this experiment is that
the robot must follow a circular trajectory of 0.5m of radius
with a velocity of 0.4m/s. The cost function that evaluates
the performance of the tracking control is:

\[
F(\hat{\theta}) = \int_{t_i}^{t_f} \sqrt{(\dot{h}^T \dot{h} + \ddot{h}^T \ddot{h})} \, dt
\]

where \( t_i \) and \( t_f \) are the initial and final time instant of the
evaluation of one individual. In the experiment \( t_f - t_i = 4 \)
seconds. Figure 14 shows an example of the trajectory that
the vehicle describes during the evaluation procedure.

The initial model parameters of the mobile robot, obtained by
an identification method, are [4]:

\[
\hat{\theta} = \begin{bmatrix} 0.3037 & 0.2768 & -0.0004 & 0.9835 & 0.0038 & 1.0725 \end{bmatrix}^T.
\]

One can observe that some parameters have greater values
than others. Therefore, a normalized vector of parameters is
considered as an individual. The space \( S \) is \( \mathbb{R}^8 \) with \( \infty \)-norm.
The parameters of the evolutionary algorithm \((\mu, \lambda+1)\)-E1E
are:

\[
r=0.2, \mu=6, \lambda=12, c_n = \sqrt{P_m} / P_m \approx 0.5.
\]

The value of \( r \) was chosen using the experiences that the
authors have in this kind of tracking control.

In the experiment 7 evolution stages were performed. In each
evolution stage, except the first, 13 evaluations were computed
that correspond to 12 children and the last fittest individual,
which is evaluated again. The first evolution stage evaluates
only the initial population that is 6 individuals. Every evolution
stage, except the first, lasts 52 seconds and every evaluation
lasts 4 seconds. In order to wait that the system stabilizes, the
first evaluation begins after 11.9 seconds since the control is
initiated.

The result of the fittest individual cost variation per evolution
stage obtained in the experiment is shown in Fig. 10. In this
figure at the 6-th evolution stage, one can see an increment of the cost of the fittest individual. This increment is caused by random perturbations in the cost function $F$. The function $F$ is perturbed because the task that the individual executes depends on the initial condition, noise in the velocity measurements, variations of the processing time in the operative system (Windows), irregularities in the ground, etc. Thus, the task never is exactly repeated. Therefore, when the fittest individual is evaluated again, the obtained cost is a little different to the previous result. In Fig. 11 is shown the cost of all individuals of the species per evolution stage. One can see that the worst cost per evolution stage tends to decrease. This result demonstrates that the physical system works better and better during the evolution. Figure 12 shows the position error of the tracking control during the evolution. After 100s the control error are smaller than before 100s; that is the consequence of the optimization of the tracking control when using the proposed evolutionary algorithm. In Fig. 13 is presented the variation of the estimated parameters in time. A tendency to some value can be observed specially in 3-th and 6-th parameters. Figures 14, 15 and 16 show comparisons of trajectories that the vehicle describes when some individuals are evaluated. One can see that the trajectories in the last evolution stage perform better (see figures 15 and 16) than the obtained using the initial individual (see Fig. 14).

It is important in the experiment that function $F$ must only depends on the estimated parameters. This is difficult to reach at, because $F$ depends on the initial position, the random uncertainties, and other factors. However, good results can be obtained if $F$ varies slowly through time and with small amplitude.

Remark: This experiment was performed only to demonstrate the functionality of the evolutionary algorithm ($\mu, 1+\lambda$)-E1E in a real situation. Better results (faster adaptation and less control errors) can be obtained using adaptive control and stability theories [5]. However, these theories can not be applied when a

simple and exact enough model of the real system is not available. In contrast, to apply the evolutionary algorithm presented in this work, only an approximated model of the real system is required.

VII. CONCLUSIÓN

The evolutionary algorithm ($\mu, 1+\lambda$)-E1E differs from the other algorithms developed so far on the fact that it shows good properties for hardware evolution. This is so because it can be guaranteed that the algorithm will not lead the system, when evaluating the costs function, to instability or to undesirable stances. Another property of the evolutionary algorithm ($\mu, 1+\lambda$)-E1E is that the self-adaptation of the species dispersion concentrates such species around the solution for tuning up the results. The convergence towards a minimum for the evolutionary algorithm ($\mu, 1+\lambda$)-E1E was proven by simulations. The experimental results validate the theoretical aspects of the evolutionary algorithm. Besides, the experiment shows that a real system can evolve without any risk of work badly while it is executing a repetitive task. It is important that the system executes a repetitive task because function $F(.)$ must depend only on the characteristics vector of an individual.

Future work will analyze evolving hardware with function $F(.)$ perturbed by random uncertainties. Besides, future work will analyze the use of multiple species in order to obtain a better search of the global minimum.
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