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USE OF MEIJER'S G–FUNCTION AND BESSEL FUNCTIONS IN ELECTRICAL NETWORK THEORY

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ABSTRACT. In this paper we have obtained a solution of the classical time-domain synthesis problem occuring in electrical network theory in a series of Bessel functions. We have also obtained a particular solution of the problem involving Meijer's G-function and Bessel functions.

§1. Introduction.

The object of this paper is to derive a solution of the classical problem know as the, ***time-domain synthesis problem***, occuring in electrical network theory in terms of an infinite series involving Bessel functions and employ Meijer's G-fuction to obtain a very general particular solution of the problem with the help of an integral earlier evaluated by the author [1].

The Meijer's G-function [2,p.207,(1)] is a generalization of almost all special functions appearing in applied sciences and engineering [3,pp.434-444]. Therefore, the particular solution obtained in this is of a very general character and hence may encompass several cases of interest. The particular solution (3.1) and the $f(t)$ given in (3.2) become master of key formulae from which a large number of solutions can be obtained for MacRobert's E-function, hypergeometric functions, Bessel functions, Legendre functions, Whittakar functions, orthogonal polynomials and other related functions [2,pp.215-222]. The results so derived may be found of great use for computing different values of $f(t)$ for several special functions.

The following formulae are required in the proof:

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(i) The orthogonality property of Bessel functions [5,p.291,(6)]:

$$(1.1)\quad \int_0^\infty t^{-1} J_{\gamma+2n+1}(t) J_{\gamma+2m+1}(t)\,dt = \begin{cases} 0, & \text{if } m \neq n \\ (4n+2\gamma+2)^{-1}, & \text{if } m = n \text{ and,} \end{cases}$$

$\mathrm{Re}\,(\gamma) + m + n > -1.$

(ii) The following integral earlier established by the author [1,p.286,(2.3)]:

$$(1.2)\quad \int_0^\infty x^{-\rho} J_\mu(x) J_\gamma(x) G_{p,q}^{m,n}\left(z \cdot x^{2\delta}\Big|_{bq}^{ap}\right) dx =$$

$$= \frac{\delta^{-\rho-\frac{1}{2}}}{2\sqrt{\pi}} \cdot G_{p+4\delta,q+2\delta}^{m+2\delta,n+\delta}\left(z\delta^{2\delta}\Big|_{\Delta(2\delta,\rho),b_q}^{\Delta(\delta,\frac{1+\rho-\mu-\gamma}{2}),a_p,\Delta(\delta,\frac{\rho+\gamma+\mu+1}{2}),\Delta(\delta,\frac{\rho-\mu+\gamma+1}{2}),\Delta(\delta,\frac{\rho+\mu-\gamma+1}{2});}\right)$$

where $2(m+n) > p+q$, $|\arg z| < (m+n-\frac{1}{2}p-\frac{1}{2}q)\pi$, $Re(\mu+\gamma-p+2\cdot\delta\cdot b_j) > -1$, $j = 1,2,\cdots,m$, $\mathrm{Re}\,(2\delta a_j - p) < 2\delta$, $j = 1,2,\cdots,n$.

§2. The statement and solution of the problem.

The classical time domain synthesis problem ocurring in electical network theory is stated as follows [4,p.139]:

Given an electrical signal described by a real valued function $f(t)$ on $0 < t < \infty$, construct an electrical network consisting of finite number of components R,C,I which are all fixed liner and positive, such that the output of $f_N(t)$ resulting from a delta function $\delta(t)$ approximates $f(t)$ on $0 < t < \infty$ in some sense.

In order to obtain a solution of this problem, we expand the function $f(t)$ into a convergent series:

$$(2.1)\quad f(t) = \sum_{n=0}^{\infty} \psi_n(t),$$

of real value functions $\psi_n(t)$ such that every partial sum

$$(2.2)\quad f_N(t) = \sum_{n=0}^{N} \psi_n(t), \quad N = 0,1,2,\cdots,$$

follows the following conditions :

(i) $f_N(t) \equiv f(t)$, for $0 < t < \infty$.

(ii) $f_n(t) \equiv 0$, for $-\infty < t < 0$.

(iii) The Hankel transform $F_N(s)$ of $f_N(t)$ is a rational function having a zero at $s = \infty$, and all its poles in the left half s-plane ($\mathrm{Re}\,(s) < 0$) except possibly for a simple pole at the origin.

On taking N in (2.2) sufficiently large to satisfy whatever approximation criterion is being taken, an orthonormal series expansion may be obtained. The Bessel trnsformation yields a solution as given below :

$$f(t) = \sum_{n=0}^{\infty} C_n J_{\gamma+2n+1}(t). \tag{2.3}$$

From the above discussion it is important to note that $f(t)$ is continuos and of bounded variation in the open interval $(0, \infty)$.

In the preceding section, we will show that this case is an example of the use of hypergeometric integral in an orthonormal series expansion.

§3. Particular solution of the problem.

The solution of the problem to be obtained is ;

$$f(t) = \frac{\delta^{-\rho-\frac{1}{2}}}{\sqrt{\pi}} \cdot \sum_{n=0}^{\infty} (2n+\gamma+1) \cdot J_{\gamma+2n+1}(t) \tag{3.1}$$

$$\cdot G^{u+2\delta, v+\delta}_{p+4\cdot\delta, q+2\delta} \cdot \left(z \cdot \delta^{2\delta} \middle| \begin{matrix} \Delta(\delta, \frac{\rho-\mu-\gamma-2n}{2}), a_p, \Delta(\delta, \frac{\rho+\gamma+\mu+2m+2}{2}), \Delta(\delta, \frac{\rho+\mu-\gamma+2n+2}{2}), \Delta(\delta, \frac{\rho+\mu-\gamma-2n}{2}); \\ \Delta(2\delta, p), b_q \end{matrix} \right).$$

where $2(u+v) > p+q$, $|argz| < (u+v-\frac{1}{2}p-\frac{1}{2}q)\pi$, $\mathrm{Re}\,(\mu+\gamma-p+2\delta b_j) > -1$, $j = 1, 2, \cdots, m$, $\mathrm{Re}\,(2\delta a_j - \rho) < 2\delta+1$, $j = 1, 2, \cdots, n$.

Proof.

Let us consider

$$f(t) = t^{-\rho} J_\mu(t) G^{u,v}_{p,q}\left(zt^{2\delta} \middle|^{a_p}_{b_q} \right) =$$

$$= \sum_{n=0}^{\infty} C_n J_{\gamma+2n+1}(t), \quad 0 < t < \infty. \tag{3.2}$$

Equation (3.2) is valid, since $f(t)$ is continuos and of bounded variation in the open interval $(0, \infty)$. Multiplying both sides of (3.2) by $t^{-1} \cdot J_{\gamma+2m+1}(t)$ and integrating with respec to t from 0 to ∞, we get

$$\int_0^\infty t^{-\rho-1} J_\mu(t) J_{\gamma+2m+1}(t) G^{u,v}_{p,q}\left(zt^{2\delta} \middle|^{ap}_{bq} \right)$$

$$= \sum_{n=0}^{\infty} C_n \int_0^\infty t^{-1} J_{\gamma+2n+1}(t) J_{\gamma+2m+1}(t) dt.$$

Now with the help of (1.1) and (1.2) we get

$$C_m = \frac{\delta^{-\rho-\frac{1}{2}}}{\sqrt{\pi}} (2m+\gamma+1) \cdot \tag{3.3}$$

$$\cdot G^{u+2\delta, v+\delta}_{p+4\delta, q+2\delta} \left(z\delta^{2\delta} \middle| \begin{matrix} \Delta(\delta, \frac{\rho-\mu-\gamma-2m}{2}), a_p, \Delta(\delta, \frac{\rho+\gamma+\mu+2m+2}{2}), \Delta(\delta, \frac{\rho-\mu+\gamma+2m+2}{2}), \Delta(\delta, \frac{\rho+\mu-\gamma-2m}{2}); \\ \Delta(2\delta, \rho), b_q \end{matrix} \right)$$

Now, from (3.2) and (3.3), the solution (3.1) follows.

References

1. Bajpai, S.D., *Some expansion formulae for G-function involving Bessel functions.*, Proc. Indian. Acad.Scs **LXVIII Sec A** (1968).
2. Erdélyi, A, *Higher Transcendental Functions*, vol. 1, McGraw-Hill,, New York, 1953.
3. Erdélyi, A, *Tables of Integral Transforms*, vol. 2, McGraw-Hill ,, New York,, 1954.
4. Exton, H, *Handbook of Hypergeometric Integrals*, Ellis Horwood Ltd., 1978.
5. Luke, Y.L, *Integrals for Bessel Functions*, McGraw-Hill, New York, 1962.