ABOUT THE RADIUS OF BENDING THAT MUST BE GIVEN TO CONCRETE MEMBERS PRINCIPAL REINFORCEMENT AT MAXIMUM TENSION STRESS POINTS

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Nota: El contenido de este artículo fue concebido como explicación de un grave accidente ocurrido por el diseño de una estructura de hormigón, hacia fines de 1970. Su destinatario era el American Concrete Institute, y esta es la razón fundamental de que haya sido escrito en inglés.

La reacción del A.C.I. no fue concluyente, pues al parecer exigían comprobación experimental del fenómeno. En la Facultad de Minas llevamos a cabo, en estos momentos, una investigación, financiada por Colciencias, que aunque no intenta corroborar la distribución de tensiones dada en la fórmula (9) del artículo, con inclusión de los fenómenos de fluencia y microfisuración, se relaciona íntimamente con aquél, pues tratamos de encontrar una forma racional de diseño de las piezas curvas, codos, etc., de hormigón armado, con resultados hasta la fecha muy alentadores.

En el mes de Noviembre de 1974, entramos en contacto con una publicación francesa (Revue No. 51, Septiembre 1974: "Pathologie des Constructions en Betón Armé" de la "Association Française du Betón") en la cual se estudian algunos desastres, totalmente similares al ocurrido en Medellín y acaecidos en Francia y Europa en los últimos años, a los cuales les dan exactamente la misma explicación encontrada por nosotros en 1971. En consecuencia creemos un deber moral el hacer público este estudio, aún antes de que podamos verificarlo completamente, en forma experimental, lo cual esperamos efectuar en un corto plazo.

SYNOPSIS:

A Method is developed to calculate the radius of bending of the principal reinforcing bars, at maximum tension stress points in curved members, corners of rigid frames, etc. A formula is found wich permits computation of lateral forces tending to split the concrete, caused by the curvature of the principal steel reinforcement. The solution is found by means of simple methods of static equilibrium, and has been confirmed by other solutions using the "Theory of Elasticity". Some comparisons with the 318–63, and 318–71 and other national norms and recommendation about stirrup utilization are made.

KEYWORDS

Radius of bending; bending of reinforcement,curved members, crushing; crushing of concrete; splitting of concrete; lateral forces in reinforcement; curvature of principal reinforcement; details of reinforcement; high stresses in the bars; lateral ties; stirrups on curved members.

INTRODUCTION

It is generally accepted that the curvature of the tension steel reinforcement at points of maximum tension stress, produces stresses in the concrete, named crushing stresses, that eventually may make the concrete member fail (1), (2).

This event is contemplated on the 318–63 code, (801 Norm, part C–2°) (3), but it does not give any rule to calculate such stresses. The study which is presented here, intends to give a method of calculation, based on simple considerations of static equilibrium. It seems that it can be confirmed on previous solutions of similar problems in the theory of elasticity (5). Although concrete is not an elastic solid, it is necessary to accept, in the absence of experimental confirmation, some elaborated theory which, founded on solid bases, let us control the phenomenae that could eventually produce a disaster.

ANALYSIS

Let us suppose a bar element of length, ds, diameter, D, curved with a radius, R, in equilibrium with a force, per unit of length, p, produced by the concrete, and a tensile force F. Fig. 1

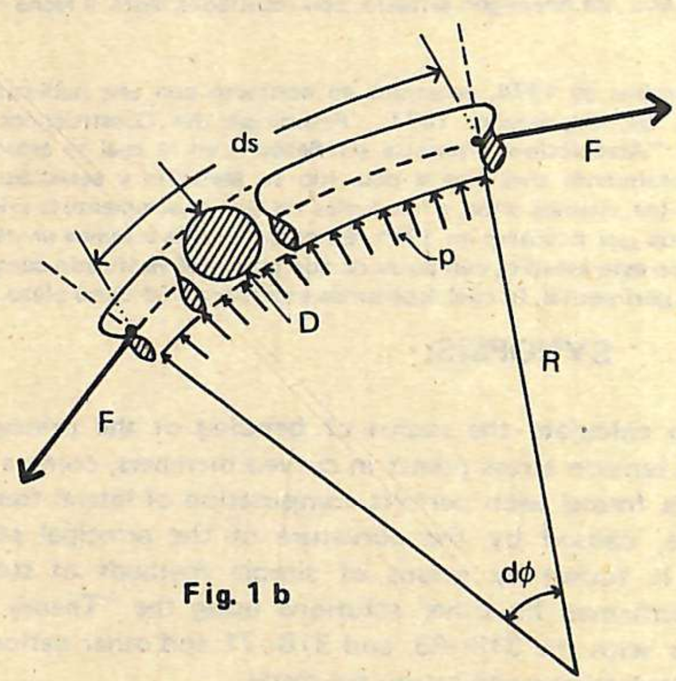

Fig. 1 b

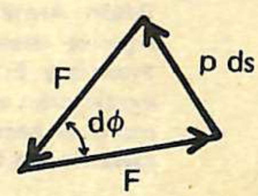

Fig. 1 a.

FIGURE 1

From the consideration of the static equilibrium, Fig. 1–a, we can write:

$$p\,ds = F\,d\phi \tag{1}$$

$$\therefore\ p = F\frac{d\phi}{ds} \tag{2}$$

but since, $ds = R\,d\phi$, eq. 2 may be written:

$$p = \frac{F}{R} \tag{3}$$

This unit force, p, must be distributed, now, conveniently, on the transversal section of the bar. In absence of friction forces, and considering that the concrete can not pull the bar on the upperside, the unit force, p, must be equilibrated by the stress distribution, a b a', which acts on the low side, a c a', of the bar. Calling θ the angle measured from the vertical, that gives the position of any point, and $f_c(\theta)$ the corresponding normal stress we can write in reference to Fig. 2

$$f_c(\theta) = f_o F(\theta) \tag{4}$$

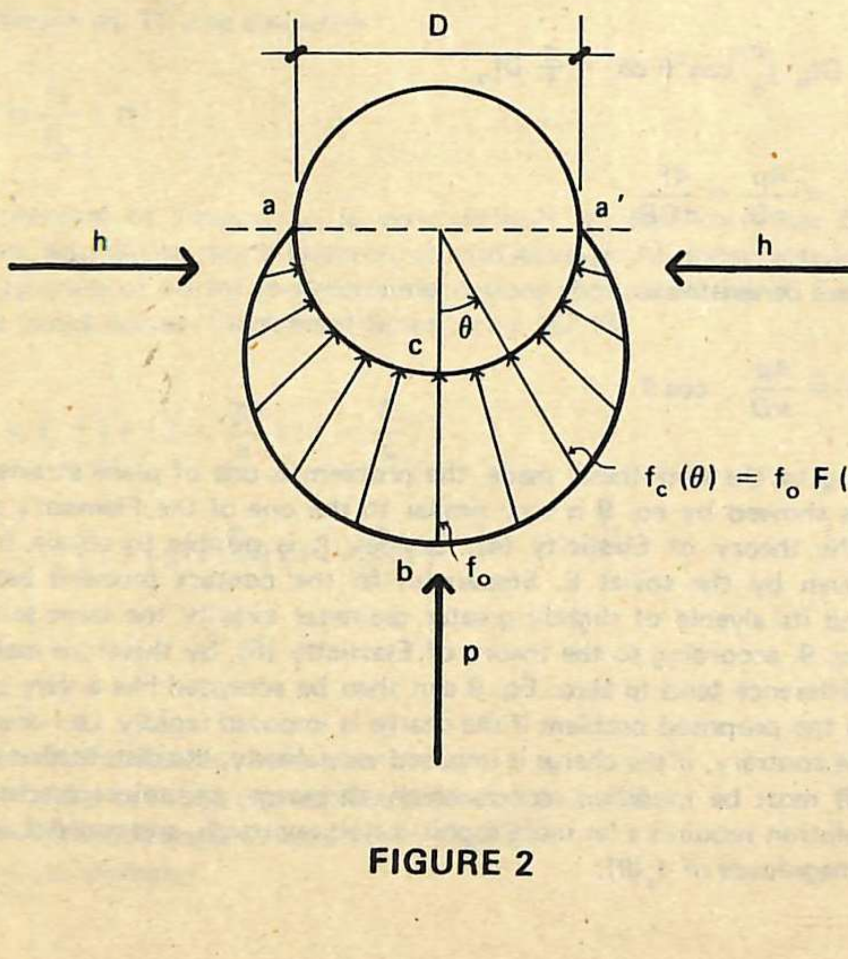

FIGURE 2

Where

$$f_c(0) = f_o$$

and

$$f_c\left(\frac{\pi}{2}\right) = 0$$

imply respectively

$$F(0) = 1$$
$$F\left(\frac{\pi}{2}\right) = 0 \qquad (5)$$

It can be supposed that the function $F(\theta)$ can be written as

$$F(\theta) = \cos\theta \qquad (6)$$

which satisfies conditions 5

From the static equilibrium of the section

$$p = Df_o \int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{\pi}{4} Df_o \qquad (7)$$

$$\therefore \; f_o = \frac{4p}{\pi D} = \frac{4F}{\pi DR} \qquad (8)$$

Eq. 4 can then be written as

$$f_c(\theta) = \frac{4p}{\pi D} \cdot \cos\theta \qquad (9)$$

According to the hypothesis made, the problem is one of plane strains, whose solution, as showed by eq. 9 is very similar to the one of the Flamant's problem given on the theory of Elasticity (4). Besides, it is posible to obtain from the solution given by the soviet E. Steuerman to the contact problem between a cylinder and its alveole of slightly greater diameter exactly the same solution as given by eq. 9. according to the theory of Elasticity (5), by therefore making the diameter difference tend to zero. Eq. 9 can then be accepted like a very probable solution to the proposed problem if the charge is imposed rapidly. i.e.: one loading test. On the contrary, if the charge is imposed very slowly, the distribution given by formula (9) must be modified, due to creep, shrincage, and minor cracks. In this case the solution requires a far more sophisticated approach, and could change very much the magnitude of $f_c(\theta)$.

The stress distribution given by these equation produces also two horizontal forces, called h, per unity of length along the bar, which equilibrate one another, and which could sometimes induce transverse splitting. Such forces, h, are given by the expression

$$h = \frac{2p}{\pi} \int_0^{\pi/2} \cos\theta \,\mathrm{sen}\,\theta \, d\theta = \frac{p}{\pi} \qquad (10)$$

The problem is now reduced to finding the crushing strength of the concrete in such a way that the radius of bending, R, of the bar can be calculated. Calling this strength f_b and equating it to the maximum stress, f_o, given by eq. 8, one can obtain:

$$R = \frac{4F}{\pi D f_b} \qquad (11)$$

Which gives the minimum radius of bending, R. If the stress of the tension bar is, f_s, the force will be

$$F = f_s \frac{\pi D^2}{4} \qquad (12)$$

substituting in eq. 11, one can write

$$R = \frac{f_s}{f_b} \cdot D \qquad (13)$$

The problem of finding f_b is very difficult. In fact there has been much discussion, and not so much research, around its value. All these facts are reflected in the vagueness of norms and recommendations about the subject. For example: using the french norms "Réglement Beton Armé 45" (2)

$$f_b \leqslant f_c \left[1 + \left(3 - \frac{2D}{a}\right)\left(1 - \frac{D}{a}\right)\right] \qquad (14)$$

$$f_b \leqslant f_c \left[1 + \left(3 - \frac{D}{e}\right)\left(1 - \frac{D}{2e}\right)\right] \qquad (15)$$

Where:

a = distance to the nearest bar
e = distance to the nearest lateral face of the member
f_c = allowable stress in concrete = $f'_c / 2,5$
D = bar diameter

and taking the usual values a = 1,5 D; e = 4 D eqs – 14 and 15 will give respectively

$$f_b = 1{,}6\, f_c = 0{,}64\, f'_c \tag{16}$$

$$f_b = 3{,}4\, f_c = 1{,}36\, f'_c \tag{17}$$

Taking the lower limiting equation i.e. eq. 16, we can compute from eq. 13 the radii, R, for different values, f_s and f'_c. On table I some of these values are presented as computed for the parameters, a = 1,5 D, e ⩾ 0,75 D.

TABLE I *

Radius of bending R for a = 1,5 D, e ⩾ 0,75 D

f_s \ f'_c	210 Kg/cm^2 (3.000 p.s.i.)	280 Kg/cm^2 (4,000 p.s.i.)	350 Kg/cm^2 (5.000 p.s.i.)
1410 Kg/cm^2 (20.000 p.s.i.)	10,4 D	7,8 D	6,3 D
2115 Kg/cm^2 (30.000 p.s.i.)	15,6 D	11,7 D	9,4 D
2640 Kg/cm^2 (37.500 p.s.i.)	19,5 D	14,6 D	11,7 D

* Calculated from french norms R. B. A. 45

If we use now the German norms DIN 1045, VI, § 29:

$$f_b = f_c \sqrt[3]{\frac{A_2}{A_1}} \tag{18}$$

in which we can take, approximately

$$f_c = \frac{f'_c}{3} \tag{19}$$

and

A_2 = rectangular distribution area

A_1 = bearing area

We can take unity length of beam, and so,

$A_2 = a + D = 1{,}5\,D + D = 2{,}5\,D$

$A_1 = D$

and then, eq. (18) becomes

$$f_b = \frac{f'_c}{3}\sqrt[3]{2{,}5} = 0{,}454\,f'_c \tag{20}$$

Which is, indeed, more conservative than formula 16. This has been underlined by Y. Guyon (7), who uses formulae 14 and 15 and shows an experimental curve (Fig. 35, pag. 205, reference 7), that gives values differing not very much from those of these formulae. He advices too a value $f_c = \frac{f'_c}{2{,}5}$ Using this value in lieu of the given by the formula 19, we obtain

$$f_b = 0{,}545\,f'_c \tag{21}$$

which is still lower than the value given by formula 16.
A.C.I. 318–71 does not give any rule to calculate f_b in confined conditions, but advices the use of the following formula (see commentary to A.C.I.–318–71, article 18.11)

$$f_b = 0{,}6\,f'_{ci}\sqrt[3]{\frac{A_2}{A_1}} \tag{22}$$

in which

f'_{ci} = compressive strength of concrete at time of initial prestress.

If we take $f'_{ci} = f'_c$, formula 22, becomes

$$f_b = 0{,}6\,f'_c\sqrt[3]{\frac{A_2}{A_1}} \tag{23}$$

and for the conditions

$A_2 = 2{,}5\,D$

$A_1 = D$

we can write

$$f_b = 1,36 \times 0,6 f'_c = 0,816 f'_c \qquad (24)$$

Taking this value in lieu of 0,64 f_c, as given from formula 16, we have recalculated table I, by means of eq. 13, and we have written table I–A.

TABLE I–A *

Radius of bending R for $a = 1,5$ D

f_s \ f'_c	210 Kg/cm² (3.000 p.s.i.)	280 Kg/cm² (4.000 p.s.i.)	350 Kg/cm² (5.000 p.s.i.)
1410 Kg/cm² (20.000 p.s.i.)	8,2 D	6,1 D	4,9 D
2115 Kg/cm² (30,000 p.s.i.)	12,2 D	9,2 D	7,4 D
2640 Kg/cm² (37.500 p.s.i.)	15,3 D	11,4 D	9,2 D

* Calculated from ACI–71 recommendation.

As can be seen, most of these values are not covered by the A.C.I. 318–63 code, table 801 (b), pag. 28, and it is demanded only (part C–2, pag. 28) that an adequate radii of bending be provided at the points of maximum tension stress of the bars, without establishing any explicit form to compute such radii. On the 318–71 (6) such provision has been deleted, and the demanded values on table 7.1.2 do not cover either, the minimum radii shown on table I, or in table I–A.

Going back, eq. 10 and taking account of eqs. 3, 12 and 13 one can write:

$$h = \frac{f_b}{4} D \qquad (25)$$

Here we observe that if we choose f_b in such a way that we obtain the minimum radius of bending, the maximum lateral forces will be obtained. For example, taking the f_b value given by eq. 16, we obtain

$$h = 0,16 f'_c D \qquad (26)$$

and taking these of eq. 24

$$h = 0{,}20\, f'_c\, D \qquad (27)$$

Usually, the unit forces, h_l, which are those of the internal bars, are equilibrated between two consecutives bars of the same layer, Figs. 3–b, but the external forces, h_e, must be absorbed either by the concrete, or by stirrups. Note that, according to this theory, the h value could be very large. Consider, for instance, a concrete whose $f'_c = 210$ Kgs/cm^2, and a curved bar No. 10. Eq. 26 gives

$$h = 0{,}16 \times 211 \times 3{,}18 = 107 \text{ Kg/cm. (aprox. 600 pounds/inch)} \qquad (28)$$

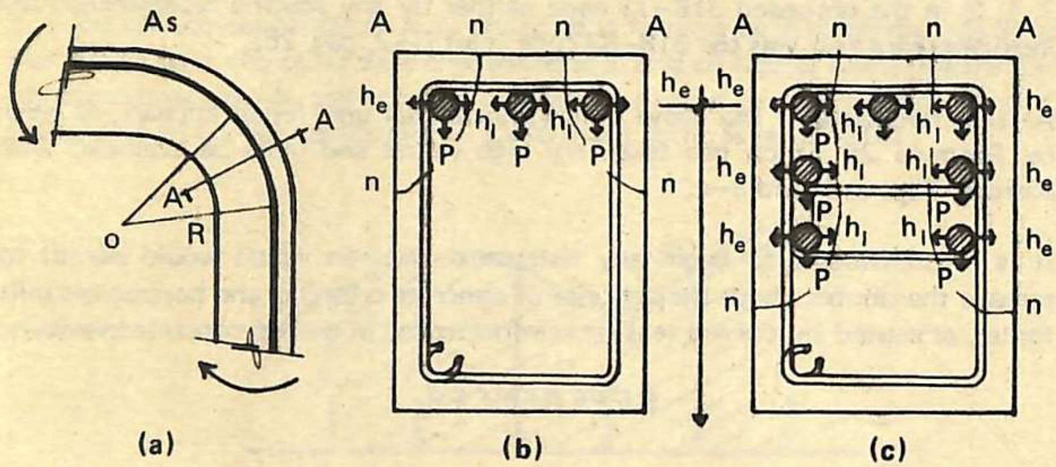

FIGURE 3

The concrete alone doesn't resist this force, h which tends to separate the corner, n–A–n, Fig. 3–b, from the member. Then we are forced to use stirrups. A No. 3 bar with an $f_s = 1410$ Kgs/cm^2 resists a force $Fl = 0{,}71 \times 1410 \approx 1.000$ Kgs. In consequence one could use No. 3 stirrups with a separation:

$$s = \frac{Fl}{h} = \frac{1.000}{107} = 9{,}3 \approx 10 \text{ cms. (aprox. 4")} \qquad (29)$$

If one wants to use No. 4 stirrups, the separation would be, s = 17 cms. (aprox. 7")

In the case that multiple bar layers are to be used, Fig. 3–c, the unit forces p, and h_e, would add, and the tendency to split the member by the n–n line, would be greater. It is then necessary to tie each bar laterally with stirrups as showed in Fig. 3–c.

CONCLUSIONS

1. In joints, curved members, etc. the radii of bending of the principal reinforcement at points of maximum tensions stress, must be calculated. In absence of experimental data Formula 13, based on the hypothesis of a plane strain problem is proposed.

2. Table I and Table I–A, give some radii as computed over the range as given by the french code R. B. A. 45 and A. C. I. code 318–71 for minimum bar separations, a = 1,5 D, and minimum distance from the bar to the lateral face e ⩾ 0,75 D. The values of these tables are not covered by those given by table 7. 1. 2. in the proposed 318–71 code neither by any specific recommendation included there as it was the 318–63 code (part C–2, pag. 28).

3. As a consequence of the above theory, horizontal unit forces appears, as given by Formula 25 which can take very high values and must be attended with stirrups. Figs. 3–b and 3–c.

4. It is recommended to begin any sistematic research which would permit to remove the doubts about the behavior of concrete crushing, and horizontals unit forces, as caused by curved tension reinforcement in curved concrete members.

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