ABSTRACT: The conjugated cooling of apples in tandem arrangements on trays was studied in this work by numerical simulation using a CFD code. The governing equations were discretized and solved using the finite volume method on unstructured triangular meshes; and a coupling scheme for the fluid and solid heat transfer problems was proposed. The computational model was validated using two test cases in which local and average Nusselt numbers are reported in the literature. The local Nusselt number for each apple was obtained during one hour of air cooling as a function of the Reynolds number. Nusselt number and temperature distribution were found to vary in the apples due to the conjugate behavior of the problem. The conjugate approach showed itself to be a powerful tool for optimizing the convective heat transfer correlations reported in the literature.

KEYWORDS: Cooling of fruits; conjugate problem; unstructured finite-volume; triangular mesh; numerical simulation.

1. INTRODUCTION

Agricultural products should be cooled from ambient temperature conditions to their optimal storage temperature in order to minimize post-harvest deterioration [1]. Significant loss of fresh fruits is observed due to decay and shriveling as a result of improper storage and handling conditions employed after harvest [2]. In practice, storage facility managers often establish and apply storage conditions based on experience thus allowing for some product losses due to the application of non-optimal conditions. Global heat and mass transfer rates through product layers are assumed and local transfer intensities are frequently neglected. Thus, the complex interactions between products in the same layer and between layers are not taken into account.

Computational fluid dynamics and computer modeling have been used to study problems in the agricultural and food industries [1, 3, 4, 5]. Conjugate heat transfer and fluid flow in a channel containing heated elements has been studied for several decades, using both numerical and experimental methods [6, 7, 8, 9]. Although such problems were mainly focused on the cooling of electronic components, some post-harvest processing techniques utilize the same problem configuration. The conjugate heat transfer problem for laminar flow over an array of three heated obstacles
The aim of the present study was to employ a numerical conjugate approach for analyzing the cooling of Fuji apples placed in a tandem arrangement in a cooling channel and to verify its applicability as a tool for understanding conjugate heat transfer. The proposed numerical conjugate approach was also used to design a new correlation for the heat transfer rate, expressed by the global Nusselt number. This dimensionless number is the major parameter used to propose suitable control strategies for refrigerated storage facilities. The study was carried out by computer simulation in two dimensions considering apples confined to a channel.

2. MATERIALS AND METHODS

In the study of the conjugate cooling of Fuji apples, air flow was assumed to be incompressible, and the buoyancy effect was neglected. The channel dimension was defined as a function of the apple diameter (D = 0.065 m) and the origin of the coordinate system was at the first apple, as shown in Figure 1.

The non-dimensional governing mass, moment, and energy conservation equations for the air domain are:

\[ \nabla \cdot \mathbf{u} = 0 \quad (1a) \]

\[ \frac{\partial \mathbf{u}}{\partial \tau} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}_H} \nabla^2 \mathbf{u} \quad (1b) \]

\[ \frac{\partial \theta_f}{\partial \tau} + \mathbf{u} \cdot \nabla \theta_f = \frac{1}{\text{Re}_H \text{Pr}} \nabla^2 \theta_f \quad (1c) \]

For the apple domain, the governing non-dimensional equation for energy conservation is

\[ \frac{\partial \theta_s}{\partial \tau} = \frac{\alpha_s}{\alpha_f} \nabla^2 \theta_s \quad (2) \]

The dimensionless variables were defined as

\[ \tau \equiv \frac{u_m t}{H}, \quad \mathbf{u} \equiv \frac{\mathbf{u}^*}{u_m}, \quad p \equiv \frac{p^*}{\rho u_m^2}, \quad \theta \equiv \frac{T - T_f}{T_i - T_f} \quad (3) \]

Initial temperatures of the air flow and the apples were set as \( T_i = 0 \) °C and \( T_f = 25 \) °C, respectively. At the inlet, the air flow is considered to be fully developed, as depicted in Figure 1, with a parabolic profile \( \mathbf{u}(y) = 6y(1 - y) \hat{y} \) and constant temperature \( T_f = 0 \) °C. Adiabatic and no-slip boundary conditions are set at the channel walls (\( y = 0 \) and \( y = 1 \)). At the outlet, all gradients are assumed to be zero. The no-slip boundary
condition is also set at the apple surfaces. At the apple-air interface, the continuity of both temperature and heat fluxes is assumed. Fluid and thermal properties are presented in Table 1.

**Table 1.** Apple and air physical properties [13]

<table>
<thead>
<tr>
<th>Property</th>
<th>Apple</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg m⁻³)</td>
<td>489.2</td>
<td>1.292</td>
</tr>
<tr>
<td>Heat capacity (J kg⁻¹ K⁻¹)</td>
<td>3650</td>
<td>1.0044×10³</td>
</tr>
<tr>
<td>Thermal conductivity (W m⁻¹ K⁻¹)</td>
<td>0.256</td>
<td>2.61×10⁻²</td>
</tr>
<tr>
<td>Viscosity (kg m⁻¹ s⁻¹)</td>
<td>-</td>
<td>1.721×10⁻⁵</td>
</tr>
</tbody>
</table>

The numerical solution consists of two major steps: discretization of the solution domains and of the equations. The first step was carried out by means of a multi-purpose unstructured triangular mesh generator in two-dimensions. This mesh generator was implemented using the Delaunay triangulation technique [13]; and an unstructured triangular mesh was generated for the calculation domain, as shown in Figure 2.

Since stepper temperature and velocity gradients are expected at the apple surface, mesh refinement is employed in order to reduce errors associated with numerical discretization of the conservation equations.

The conservation equations (Eqs. 1 and 2) can be rewritten in a general form as

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u} \phi) - \nabla \cdot (\Gamma \nabla \phi) = S_\phi(\phi) \]  \hspace{1cm} (4)

The finite volume method uses an integral over a control volume or cell (triangle) with the volume \( V \) and an interior point \( P \):

\[ \int_V \frac{\partial \phi}{\partial t} dV + \oint_{\partial V} \mathbf{u} \phi \cdot d\mathbf{s} - \oint_{\partial V} (\Gamma \nabla \phi) \cdot d\mathbf{s} = \oint_{\partial V} S_\phi(\phi) dV \]  \hspace{1cm} (5)

The second and the third terms in equation 5 represent the convective and diffusive flux, respectively, across the cell boundary \( \partial V \) with the normal surface vector \( s \). Point \( P \) is defined as the mass center inside a triangular cell and each non-boundary cell has three neighbors \( N \).

The time discretization scheme is implicit with first order accuracy. For the spatial discretization, the central second-order differencing scheme was used for the diffusive flux, applying a linear variation of \( \phi \) around \( P \) with iterative non-orthogonal correction [13]. The weighted upwind differencing scheme (WUDS) was used to discretize the convective flux. The resulting algebraic system of equations was solved by the bi-conjugated gradient method and the energy and momentum equations are coupled using the SIMPLE algorithm [14].

Regarding the conjugate approach, the flow and solid domains are modeled individually, and each is treated as an independent problem: the fluid problem, and two individual apple problems. Mass, energy and momentum equations are used solely in the fluid domain. For the solids domains, only the energy equation is used, neglecting the convective term. The individual problems are mathematically coupled by the following boundary conditions [15]:

\[ k_s (\mathbf{n} \cdot \nabla T_s) = k_f (\mathbf{n} \cdot \nabla T_f) \]  \hspace{1cm} (6a)

\[ T_s = T_f \]  \hspace{1cm} (6b)

**Figure 2.** Unstructured triangular mesh for the cooling process of two apples
3. RESULTS AND DISCUSSION

Represented by reference [16] is widely regarded as being the most accurate. The algebraic representation of this correlation is:

\[ Nu_D = 2 + \left( 0.4 R_e_D^{1/2} + 0.06 R_e_D^{3/4} \right) Pr^{0.4} \left( \frac{\mu}{\mu_w} \right)^{1/4} \]

\[ 1 \leq R_e_D \leq 10^5 \]

(7)

Computational simulations were performed and the Nusselt numbers were compared to those predicted using the Whitaker correlation. The Nusselt number was calculated according to its definition (i.e. the normal dimensionless temperature gradient at the wall) as:

\[ Nu_D = \frac{\sum (n_i \cdot \nabla \theta)}{\sum \Delta S} \]

(8)

From Figure 3, good agreement with the Whitaker correlation could be verified. The relative root mean square error was 2.88% so the implemented computational code was able to predict the Nusselt number for non-conjugated problems.

Figure 4 shows good correlation between the Nusselt numbers calculated in the present work and the results of [12]. The relative root mean square error was 4.53%.

In order to propose a new correlation for conjugate heat transfer, a simulation study was carried out using a time step of 0.05 s and a final time of one hour. The Reynolds number was varied from 200 to 30000, which correspond to the flow velocity of interest for industrial cooling processes. Streamlines and temperature contours at the final time for \( R_e = 300 \), \( R_e = 3000 \) and \( R_e = 30000 \) are presented in Figure 5.

Figure 5(a-c) shows two clockwise flow recirculation zones. The first recirculation is observed between the apples and an even larger one is verified downstream from the second apple. The structures of all vortices are affected by the Reynolds number. As the Reynolds number increases, downstream recirculation zones expand axially and gain strength. It can be verified in Figure 5(a) that a large thermal boundary layer occurs due to the low Reynolds number. Because significant heat transfer is verified between the apples, a lower temperature gradient is observed at the surface of the second apple. For higher Reynolds numbers, the velocity of the first recirculation zone increases, so smaller differences in the Nusselt number between apples is expected.

The average temperatures for the apples are presented in Figure 6 and the fact that the differences in average temperature for the individual apples increased with time and with the Reynolds number can be verified.
Figure 5. Streamlines contour for (a) Re = 300, (b) Re = 3000 and (c) Re = 30000

Figure 6. Average temperature of the apples as a function of time and Reynolds number

The local Nusselt numbers are presented in Figure 7 for Re = 300, Re = 3000 and Re = 30000 as a function of angular position. Near-stagnant flow at the top surface of each apple (90°) is observed for all Reynolds numbers. A cavity exists at this position and the low air velocity leads to a stagnant flow, so the Nusselt number decreases significantly for both apples. The convective heat transfer (i.e. Nusselt number) is more intensive in the upper left portion of the apples (70° and 100°) where both thermal and velocity boundary layers are thinnest. At the lower front and rear portions of each apple, the Nusselt was minimal. It can also be observed that the difference in the local Nusselt number for the apples decreases as the Reynolds number increases due to the growing velocity of the recirculation zone.

For engineering applications, global Nusselt number correlations are used to calculate heat transfer rates as well as cooling times. Although experimental data is often used to design empirical correlations, numerical simulations can also be used to propose new global Nusselt number correlations. These correlations are usually written as:
The Reynolds number is based on the apple diameter. Simulation simulations were performed for $200 \leq Re \leq 30000$ and the global Nusselt number was calculated for each apple. The global Nusselt number as a function of the Reynolds number is presented in Figure 8. The global Nusselt number correlations for the first and the second apple were respectively evaluated as:

$$Nu_D = 0.300 Re_D^{0.568} Pr^{1/3} \quad (10a)$$
$$R^2 = 0.9956$$

$$Nu_D = 0.154 Re_D^{0.637} Pr^{1/3} \quad (10b)$$
$$R^2 = 0.9997$$

A good fit was verified with the numerical results, as verified by $R^2$ values in Equation 10. Figure 8 shows that the global Nusselt numbers for both apples converged to a similar value as the Reynolds number increases. The proposed correlation considers the conjugate behavior of the problem, so the Nusselt number depends not only on the fluid flow but also on the ratio of fluid and solid thermal conductivities.

4. CONCLUSION

A finite volume formulation for the discretization of transport equations was presented together with a multi-problem approach for the solution of a conjugate problem for tandem arranged apple cooling. Streamlines and temperature distributions were presented and compared using the numerical simulation for three Reynolds numbers. It was verified that the local Nusselt number varies drastically over the apple surface and they tend to be closed as the Reynolds number increases. Simulation studies were
carried out in order to propose new correlations for the global conjugated Nusselt number. The proposed conjugated approach has shown itself to be a powerful tool for the understanding of the heat transfer phenomena and can assist in the design of refrigerated storage units.

5. ACKNOWLEDGEMENTS

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