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A methodology to obtain an analytical formula for the elastic modulus of lightweight aggregate concrete

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Abstract
This work proposes a methodology to predict the elastic modulus of lightweight aggregate concretes. To this end an analytical formula is achieved by curve fitting experimental results from 135 concrete samples made of 45 different mixes. The validation of the proposed methodology is carried out by applying the obtained analytical formula to a set of 90 concrete samples made of 30 different mixes. Comparisons with other methods applied to predicting the elastic modulus of lightweight aggregate concretes indicate that the results show good agreement and suggest that the proposed methodology could be applied in practical situations.

Keywords: lightweight aggregate concrete; elastic modulus.

Una metodología para obtener una fórmula analítica para el módulo de elasticidad del hormigón de áridos ligeros

Resumen
Este trabajo propone una metodología para evaluar el módulo elástico de los hormigones de agregados livianos. Para ello una fórmula analítica se logra mediante el ajuste de la curva de los resultados experimentales de 135 muestras de hormigón hechas de 45 mezclas diferentes. La validación de la metodología propuesta se lleva a cabo mediante la aplicación de la fórmula analítica obtenida a otro conjunto de 90 muestras de hormigón hecha de 30 mezclas diferentes. Las comparaciones con otros métodos utilizados para predecir el módulo de elasticidad de hormigones de agregados livianos muestran que los resultados sean justos y sugieren que la metodología propuesta podría aplicarse en situaciones prácticas.

Palabras clave: hormigón de áridos ligeros; módulo elástico.

1. Introduction

The structural application of Lightweight Aggregate Concrete (LWAC) is increasing around the world for economic and environmental reasons. The material leads to smaller dead loads, allowing lighter structural members and less amounts of reinforced steel, with no harm to safety. Due to this relatively recent tendency, many works have been dedicated to evaluate the long-term behavior of LWAC [1,2]. Another advantage of this kind of concrete is the fact that its thermal characteristics are normally attached to high levels of insulation [3].

On the other hand, mechanical properties of LWAC are frequently lower than those of ordinary concrete. For these reason, papers addressing the study of the elastic modulus, for instance, may be easily found in the literature [4-7]. Cui *et al* [5], for example, propose analytical formulas to evaluate LWAC elastic modulus based on a multiple linear regression analysis.

The present work aims to contribute to the practical application of LWAC, by proposing a methodology to

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achieve a simple analytical equation to evaluate the elastic modulus (E_c) of LWACs made of varied formulations.

It is well known that the elastic modulus plays a paramount role in structural design, since most of the practical applications adopt the theory of elasticity in the material modeling. Thus, it is very convenient for a structural engineer to dispose of a formula that supplies a reliable prevision of the elastic modulus of concrete. To this end a number of codes based on empirical formulas regarding LWAC are available in the literature - in which E_c is given in terms of two quantities: LWAC's characteristic compressive strength (f_{ck}) and oven-dry density of the LWAC (ρ_s).

The American Concrete Institute – ACI [8] adopts expression 1:

$$E_c = 0.043\rho_s^{1.5} f_{ck}^{0.5} \quad (1)$$

The Eurocode 2 (EN 1992-1-1) [9] indicates equation 2:

$$E_c = 22000\ (f_{cm}/10)^{0.3} + (\rho_s/2200)^2 \quad (2)$$

Where f_{cm} (MPa) is the mean value of concrete compressive strength. In Eqs. 1-2, E_c and f_{ck} are given in MPa and ρ_s in kg/m^3.

Another approach for predicting the elastic modulus of a LWAC consists of expressions in terms of the lightweight aggregate (LWA) properties. For instance, Cui et al [5] suggest the analytical equation presented in Eq. (5), where the LWAC's Young modulus is evaluated as a function of the volumetric fraction of the aggregate (V_a); the oven-dried density of the LWA (ρ_a) and aggregate shape factor (I_s).

$$E_c = -0.267\ V_a + 0.005508\ \rho_a + 8.096\ I_s + 14.221 \quad (3)$$

Where, ρ_a is given in kg/m^3; V_a and I_s are dimensionless.

It is also possible to predict concrete's mechanical properties by applying computational intelligence technics, such as Artificial Neural Network, Fuzzy Logic or Genetic Algorithms. Those kinds of methods require a set of experimental data in order to calibrate a computational based predictor and another set of laboratory results is applied to validate the quality of the adjusted numerical model. Several works in the literature deal with this strategy in order to predict concrete's mechanical properties [10,11].

The present work proposes an approach in which the evaluation of E_c is accomplished by using the oven-dry density (ρ_a) of the lightweight aggregate (LWA); volumetric fraction

Table 1.
Summary of Ke's [12] results.

Concrete #	V_a (%)	ρ_a (kg/m^3)	$E_{c.exp}$ (GPa)	f_{ck} (*) (MPa)
1a	0.0	n/a	28.59	32.18
2a	12.5	737 *A*	24.90	29.17
3a	25.0	737 *A*	21.39	24.18
4a	37.5	737 *A*	17.29	19.80
5a	45.0	737 *A*	15.70	17.79
6a	0.0	n/a	28.59	32.18
7a	12.5	921 *A*	26.16	31.03
8a	25.0	921 *A*	21.68	25.99
9a	37.5	921 *A*	17.90	20.63
10a	45.0	921 *A*	16.61	19.79
11a	0.0	n/a	28.59	32.18
12a	12.5	1577 *B*	27.37	34.31
13a	25.0	1577 *B*	26.26	34.02
14a	37.5	1577 *B*	25.28	35.12
15a	45.0	1577 *B*	24.32	34.63
16b	0.0	n/a	33.18	56.18
17b	12.5	737 *A*	27.57	37.55
18b	25.0	737 *A*	23.78	28.52
19b	37.5	737 *A*	20.82	21.65
20b	45.0	737 *A*	18.94	22.79
21b	0.0	n/a	33.18	56.18
22b	12.5	921 *A*	29.16	42.90
23b	25.0	921 *A*	24.93	28.37
24b	37.5	921 *A*	21.36	24.80
25b	45.0	921 *A*	19.70	25.56
26b	0.0	n/a	33.18	56.18
27b	12.5	1577 *B*	31.93	51.44
28b	25.0	1577 *B*	30.99	49.69
29b	37.5	1577 *B*	30.15	48.20
30b	45.0	1577 *B*	29.31	42.01
31c	0.0	n/a	35.40	77.96
32c	12.5	737 *A*	30.22	54.84
33c	25.0	737 *A*	26.03	38.41
34c	37.5	737 *A*	22.30	31.39
35c	45.0	737 *A*	20.08	25.95
36c	0.0	n/a	35.40	77.96
37c	12.5	921 *A*	32.09	56.63
38c	25.0	921 *A*	27.99	42.30
39c	37.5	921 *A*	23.68	30.92
40c	45.0	921 *A*	21.72	31.51
41c	0.0	n/a	35.40	77.96
42c	12.5	1577 *B*	34.21	73.71
43c	25.0	1577 *B*	33.85	69.95
44c	37.5	1577 *B*	32.94	67.40
45c	45.0	1577 *B*	33.00	65.20
46a	0.0	n/a	28.59	32.18
47a	12.5	900 *B*	25.13	30.07
48a	25.0	900 *B*	22.47	27.19
49a	37.5	900 *B*	19.43	22.46
50a	45.0	900 *B*	18.29	20.84
51a	0.0	n/a	28.59	32.18
52a	12.5	927 *A*	23.54	28.46
53a	25.0	927 *A*	20.67	22.81
54a	37.5	927 *A*	16.74	19.16
55a	45.0	927 *A*	15.67	16.91
56b	0.0	n/a	33.18	56.18
57b	12.5	900 *B*	29.48	47.67
58b	25.0	900 *B*	26.52	42.53
59b	37.5	900 *B*	22.19	33.72
60b	45.0	900 *B*	20.18	30.35
61b	0.0	n/a	33.18	56.18
62b	12.5	927 *A*	29.40	48.35
63b	25.0	927 *A*	23.71	36.21
64b	37.5	927 *A*	19.87	29.25
65b	45.0	927 *A*	17.18	25.15
66c	0.0	n/a	35.40	77.96
67c	12.5	900 *B*	32.78	62.72
68c	25.0	900 *B*	28.00	51.48
69c	37.5	900 *B*	24.34	39.59
70c	45.0	900 *B*	22.02	34.27
71c	0.0	n/a	35.40	77.96
72c	12.5	927 *A*	31.15	62.23
73c	25.0	927 *A*	26.75	47.62
74c	37.5	927 *A*	22.43	37.15
75c	45.0	927 *A*	20.35	34.44

n/a: non applicable
(*) Ke [12] presents mean values for compressive strength (f_m). f_{ck} was calculated by applying the Eurocode equation: $f_{ck} = f_m - 8$ MPa.
Source: Adapted from Ke[12]

of the aggregate (V_a); and the Young modulus of the mortar (E_M). The main advantage of the proposed methodology, when compared with Eqs. 1 and 2, is the fact that it does not demand previous knowledge of the concrete's compressive strength. Once the mortar elastic modulus is obtained, even for a different kind and/or amount of aggregate in the concrete, the proposed methodology is able to fairly predict E_c.

2. Proposed methodology

In order to predict Ec, the basic function presented in Eq. 5 was taken as a starting point, based on the parameters to be adjusted according to the experimental database:

$$E_c = E_M \, \Gamma(\lambda) \tag{4}$$

Where E_M is the elastic modulus of the mortar, standing for its influence on E_c, and $\Gamma(\lambda)$ represents the contribution of LWA for the E_c, where $\lambda = [(\rho_a/1000)/V_a]$.

Admitting that the mortar has an elastic modulus equal or superior to the LWA's, the maximum value of E_c should be E_M and $\Gamma(\lambda) \leq 1$. The next step is to identify the function $\Gamma(\lambda)$. To this end, a set of experimental results, presented by Ke[M1] [12] in his PhD thesis, was used. Three kinds of mortar for five different types of LWA and five levels for the amount of concrete, resulting in 75 different mixes were tested. For each mix, three samples were tested, leading to 225 samples, and the mean values were named as E_c. Tab. 1 summarizes Ke's [12] results. In this table only the mean values for E_c (column $E_{c,exp}$ in Table 1) are presented and f_{ck} is omitted. The concrete number (column # in Table 1) is followed by a letter (a, b or c) indicating the respective mortar. Two kinds of LWA were tested: expanded clay and shale. The oven-dry density (column ρ_s in Table 1) is followed by the aggregate type: "*A*" for expanded clay and "*B*" for expanded shale. The shape factor (I_s) for clay and shale are, respectively, 1.240 and 1.873.

Concretes from #1a to #45c were used to investigate function $\Gamma(\lambda)$. Figs 1 to 3 show a comparison between experimental results and E_c obtained with Eq. 5, considering $\Gamma(\lambda)$ as described in Eq. 6:

$$\Gamma(\lambda) = [1+\exp(-\alpha\,\lambda)]^{-1} \tag{5}$$

Where $\alpha = 30.82$ m^3/kg.

The parameter α is achieved by curve fitting Eq. 5, with $\Gamma(\lambda)$ showed in Eq. 6, for each analyzed mortar, resulting in three α parameters. The adopted value for α is the mean of them. The applied methodology for curve fitting was the mean square method.

3. Validation of the proposed methodology

By applying the achieved expression for E_c to the concretes from #46a to #75c the validation of the proposed methodology is carried out. Fig. 4 presents a comparison between the experimental results for E_c and the predicted counterparts.

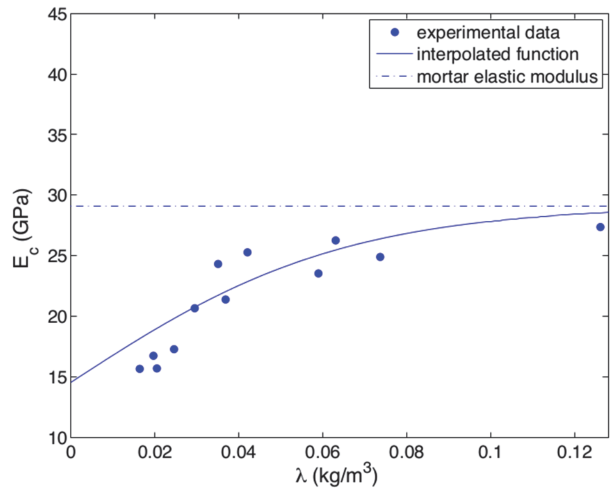

Figure 1. Analysis of Ec for mortar a.
Source: The authors

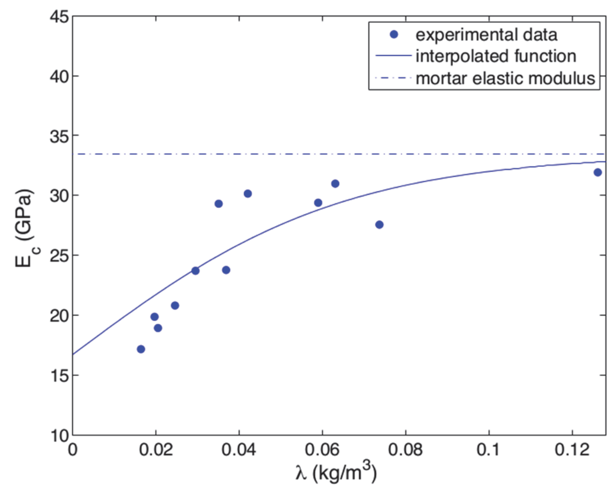

Figure 2. Analysis of Ec for mortar b.
Source: The authors

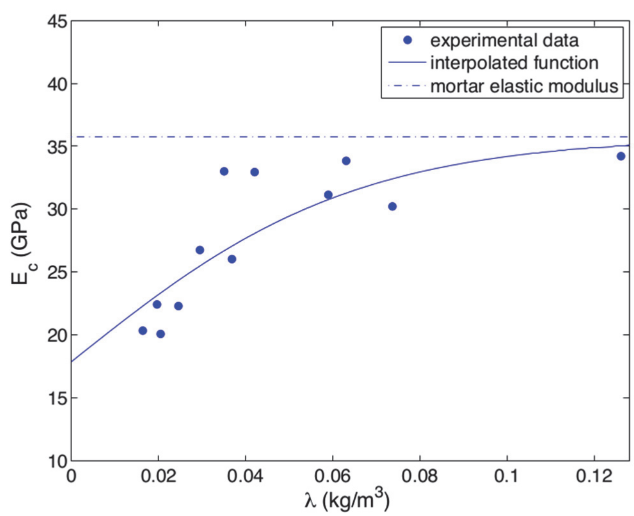

Figure 3. Analysis of Ec for mortar c.
Source: The authors

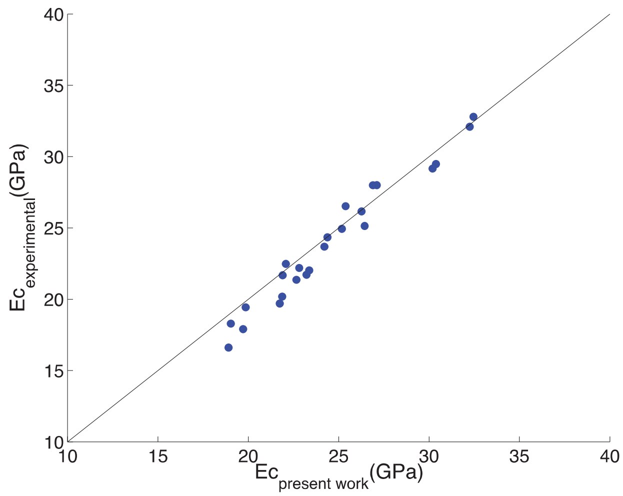

Figure 4. Comparison between experimental results and predictions for E_c
Source: The authors

It is possible to observe in Fig. 4 that the proposed methodology allows a good prediction for E_c.

4. Comparisons with available expressions

The performance of the proposed formula was assessed by comparing its results to those obtained from expressions available in the literature (Eqs. 1 and 2).

For comparison purposes, a multilayer perceptron artificial neural network was adopted, which is a technique applied to several kind of problems [13]. The network adopted herein has one hidden layer and eight neurons in the hidden layer. Concretes from #1 to #45 were used for the network training.

The performance of each predictor can be better observed in Figs. 5-9. In order to avoid distorted results in favor of the presented methodology, only the concretes used in the validation process were considered in these figures. For the proposed methodology, the neural network, and Cui *et al* [5] results, the predictions for E_c were multiplied by 0.85 aiming to consider a safety design parameter. This value was arbitrarily chosen and it tries to assure that practically all predictions for E_c are inferior to the experimental counterparts. For ACI and Eurocode, safety design coefficients are implicitly included in the respective Eqs. 1 and 2.

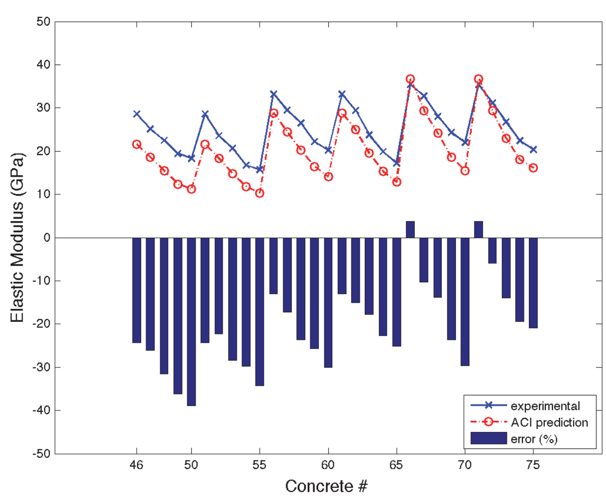

Figure 5. Evaluation of ACI [8] predictions for Ec.
Source: The authors

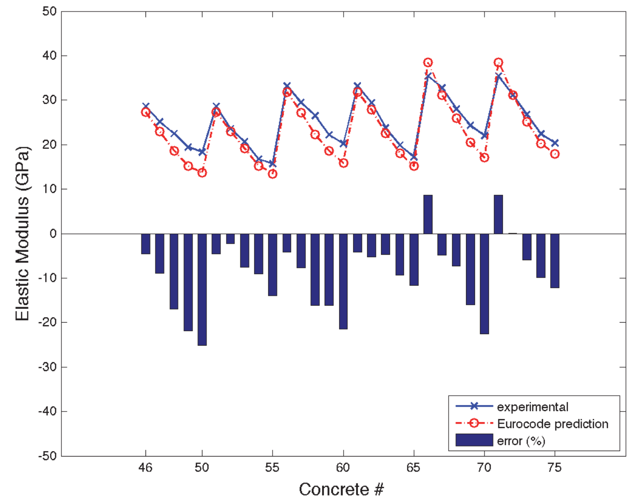

Figure 6. Evaluation of Eurocode [9] predictions for Ec.
Source: The authors

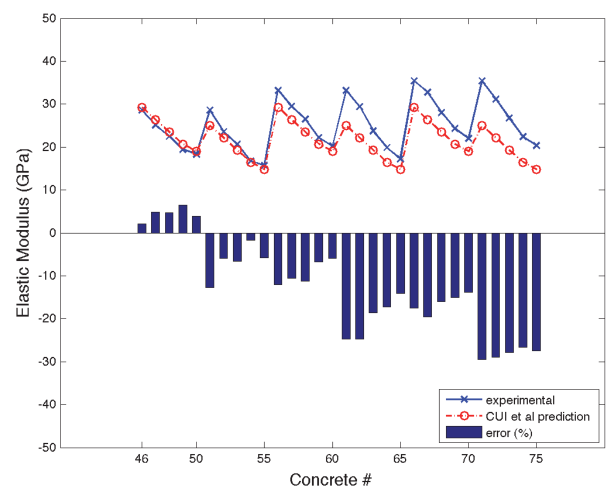

Figure 7. Evaluation of Cui Cui et al [5] predictions for Ec.
Source: The authors

It is possible to observe from Figs. 5-9 that all formulations give conservative predictions for E_c for practically all concretes. Only a limited number of concretes had estimations for E_c slightly superior than the experimental counterparts.

THE results of the overall comparison are calculated in Table 2.

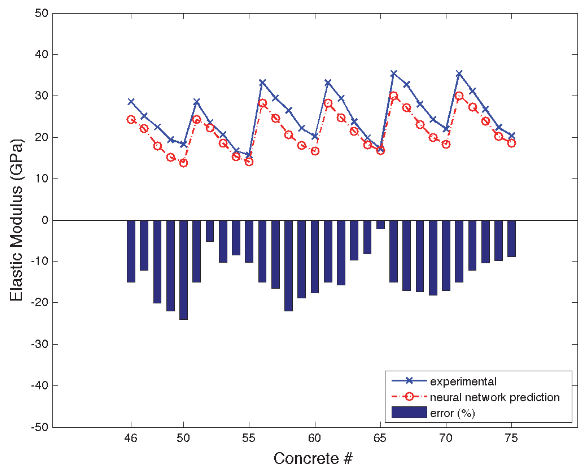

Figure 8. Evaluation of Neural Network predictions for Ec.
Source: The authors

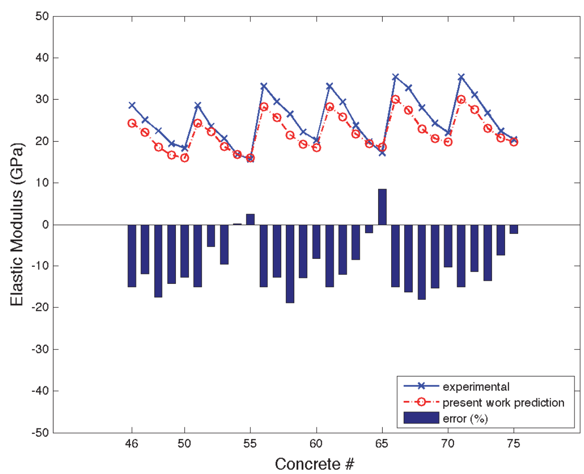

Figure 9. Evaluation of the present work predictions for Ec.
Source: The authors

Table 2:
Comparisons with available expressions

Reference	maximum absolute error (%)	mean absolute error (%)	standard deviation of the error (%)
ACI [8]	38.88	21.48	10.32
Eurocode [9]	25.19	10.40	8.15
Cui et al [5]	29.47	14.11	10.88
Neural network	24.03	14.11	5.10
Present work	18.90	11.42	6.52

Source: The authors

From Table 2 and Figs. 5-9 one can observe that:

- ACI [8] results are the most conservative. Moreover, one verifies that the ACI method allows the greatest maximum and mean absolute errors;
- CUI *et al* [5] results allow the second biggest maximum absolute error and the greatest standard deviation;

Table 3:
Performance of the three best predictors

Rank	Maximum absolute error (%)	Mean absolute error (%)	Standard deviation of the error (%)
#1	Present work	Eurocode	Neural Network
#2	Neural Network	Present work	Present work
#3	Eurocode	Neural Network	Eurocode

Source: The authors

- Eurocode, neural network and the present work have good performances amongst the studied criteria. These methods were considered as the best predictors for Ec. Table 3 aims to rank the three best predictors:

Considering Tables 2 and 3, it is possible to conclude that, for the set of studied concretes, Eurocode, neural network and the present work present fair results for the prediction of E_c. Moreover, in view of the fact that the present work has the best performance in terms of maximum absolute error and the second best performance for the other two criteria, it is possible to consider that the fair results achieved by applying the proposed methodology are slightly better than the other methods.

5. Conclusions

The present work deals with an analytical expression to evaluate the elastic modulus of Lightweight Aggregate concretes,aimed towards practical applications by design engineers. The main feature of the proposed formula is the fact that the input parameters are: mortar Young's modulus, instead of concrete compressive strength; aggregate's density and amount of aggregate. The principal advantage of the proposed methodology is to avoid laboratory tests to determine concrete compressive strength for any prediction of E_c. Once the Young's modulus of the mortar is obtained, the estimation of E_c may be fairly achieved without further laboratory tests, even for different kinds or/and amounts of aggregates. The results for the set of analyzed concretes are considered as fair and the performance, when compared to other formulas, was slightly superior to Eurocode and neural network, and clearly superior to the other evaluated formulations.

Finally, It is important to observe that this article proposes a methodology and not an expression for the estimation of E_c. The results were achieved by analyzing two kinds of LWAs. A general formula demands more laboratory tests considering a large number of LWA types. Despite this, the proposed methodology could be applied for other kinds of aggregates, by adjusting α parameter for each LWA type. In the present work, the results were considered as fair for LWACs made by expanded clay and expanded shale, using the same adjusted α parameter. A separate analysis for each LWA would be less generic but more accurate.

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