A Model of Oligopoly*

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Abstract
This article develops a simple linear model of oligopoly and uses it to provide a detailed characterization of equilibrium prices, quantities, mark-ups, price elasticities of market demand; and welfare, all in terms of the parameters of the model. This is done under five different conjectures: Collusion, Threat, Cournot, Stackelberg, and Bertrand. The results of the model are used for comparative statics.

JEL: C70; C71; D43; L13.

Keywords: Oligopoly; collusion; threat; Cournot; Stackelberg; Bertrand, mark-up.

Un modelo de oligopolio

Resumen
Este artículo construye un modelo simple y lineal de oligopolio y lo utiliza para hacer una caracterización detallada del equilibrio en términos de precios, cantidades, mark-ups, elasticidades precio de la demanda del mercado y el bienestar, en función de los parámetros del modelo. Esto se hace bajo cinco conjeturas diferentes -Colusión, Amenaza, Cournot, Stackelberg y Bertrand-. Los resultados del modelo se utilizan para hacer ejercicios de estática comparativa.

Palabras clave: oligopolio; colusión; amenaza; cournot; Stackelberg; Bertrand; mark-up.

Um modelo de oligopólio

Resumo
Este artigo constrói um modelo simples e linear de oligopólio e o utiliza para fazer uma caracterização detalhada do equilíbrio em termos de preços, quantidades, margens de lucro, elasticidades-preço da demanda do mercado e bem-estar, em função dos parâmetros do modelo. Isso é feito sob cinco conjecturas diferentes: conluio, ameaça, Cournot, Stackelberg e Bertrand. Os resultados do modelo são utilizados para fazer exercícios de estética comparativa.

Palavras-chave: oligopólio; conluio; ameaça; cournot; Stackelberg; Bertrand; margem de lucro.

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Introduction

This article presents a simple linear model of oligopoly. To do so, the same market Vdemand and the same cost structure per firm are used, to characterize the market equilibrium in terms of the parameters of the model, under five different conjectures: Collusion, Threat, Cournot, Stackelberg, and Bertrand. The results are used for comparative statics.

Previous literature

The term oligopoly has been traced back by Chamberlain (1957) to St. Thomas More's "Utopia" (1516), where he used it to refer to "the sale by the few". Schlesinger (1914) used the German word "oligopolische" without a theory and without influence. Oligopoly as a term was later used and massified by Chamberlin (1933) in his book "The Theory of Monopolistic Competition".

Regarding the theory itself, Cournot (1838) presented the first formal model of oligopoly, which remains at the core of this theory and is regarded by authors such as Shapiro (1989), as one of the most important —if not the most important— contributions to the topic.

However, Bertrand (1883) criticized Cournot and proposed that if players could choose between quantity competition and price competition, price competition would prevail. With constant returns to scale, the Bertrand price equals the competitive price, which equals the marginal cost. Edgeworth (1897) also criticized Cournot, pointing out that his results had been shown to be incorrect, regardless of the cost structures.

Furthermore, Stackelberg (1934) analyzed different types of market structures and proposed a sequential oligopoly equilibrium in which one firm was a market leader in quantities. Another firm was a follower, taking the quantity produced by the leader firm as given.

Kleinwächter (1883) introduced and formalized the concept of cartels, which refers to cooperation between firms. In addition, Bain (1956) explained that effective impedance —blocking the entry of a new firm into a market— can occur when the price and quantities set by an incumbent firm make it unprofitable for a potential competitor to enter such a market.

Decades later, Shapiro (1989) conducted a thorough review of oligopoly theory for static and dynamic games. It has long been acknowledged that oligopolies have an equilibrium that can change depending on the assumptions made and the way firms interact among themselves. The way firms interact with one another may also change over time.

In a specific comparison, Levin (1988) analyzed Collusion, Cournot, and Stackelberg, and concluded that at equilibrium, collusion yields higher prices than Cournot, and Cournot yields higher prices than Stackelberg.

As for measuring market power, Lerner (1934) proposed a measure of monopoly power as $\frac{p-MgC}{p}$, where p is the unit price and MgC is the marginal cost of production of the good or service. This measure is known as Lerner's Index and is probably the most widely used formula to measure market power.

From the consumer's perspective, Dupuis (1844) proposed the difference between maximum willingness to pay and actual payments as a measure of the utility generated by public policies. Marshall (1980) formalized the concept of consumer surplus as a graph under a demand curve and above the market price.

This article develops a simple and linear model of oligopoly, reflecting some of the basic results obtained in the literature, and expresses them in terms of the model's parameters. Vallejo (2024) adopts a similar approach for oligopsony in a factor (labor) market, which has mirror-opposite features to this article, in order to highlight the explicit mirror symmetry between oligopoly and oligopsony.

The literature on oligopoly is broad. For less stringent and more generalized approaches, readers may want to refer to authors such as Friedman (1983). For non-linear Cournot oligopolies, readers may wish to refer to authors such as Naimzada et al. (2006). For oligopolies with non-linear pricing, readers may wish to refer to authors such as Ireland (1981). For dynamic games, readers may refer to authors such as Shapiro (1989).

A Model of the Oligopoly

The model constructed in this article aims to characterize the oligopoly equilibrium in goods and services markets in detail, under the conjectures of Collusion, Threat, Cournot, Stackelberg, and Bertrand.

General Assumptions

To achieve the objectives of this article, the following assumptions are made.

There are many consumers of a good or service, who are price takers. The market demand curve is linear, or is linearized around the relevant equilibrium, as:

$$p = a - bQ \qquad (1),$$

where p is the unit price and Q are the total quantities produced and sold in the market, and a and b are positive parameters (a>0 and b>0).

Consider two identical firms, A and B, that produce a homogeneous product. Firms have identical total cost functions (TC), with a non-negative and constant marginal cost $d \geq 0$, and no fixed costs. Fixed costs are zero, or are sunk, to focus on the role of conjectural variations, and not on the role of economies of scale.

Thus, the total cost of any firm, as a function of a firm's output *q*, can be written as:

$$TC = dq \qquad (2)$$

The marginal cost of any firm is thus constant, and non-negative, as pointed out before:

$$MgC_i = d \qquad (3)$$

Just as in monopoly or monopolistic competition, there is no supply curve in oligopoly, since firms maximize profits by selling at the highest possible price, which is determined by the demand curve. This implies that all combinations of prices and quantities are on the demand curve, and thus, it is impossible to find a combination of prices and quantities sold that is independent of such demand curve.

Given the market demand, the market price is a function of the quantities produced and sold by firms *A* and *B*:

$$p = a - b(q_A + q_B) \qquad (4)$$

The market existence condition implies that the demand intercept on the vertical axis is greater than the marginal cost ($a>d$). If this condition does not hold, the market collapses.

Perfect Competition

Perfect competition is not an oligopoly equilibrium in its own right, since there is no scope for strategic interactions in this market structure. However, its equilibrium is relevant as a benchmark for the welfare impacts of the different conjectures considered here, under oligopoly.

In perfect competition, the profit maximization condition is that price equals marginal cost, so the equilibrium prices, total quantities, and the profits of any firm Π_i, can be written as:

$$p = d \quad (5)$$

$$a - bQ = d \qquad (6)$$

$$Q = \frac{a-d}{b} (7)$$

$$\Pi = \Pi_i = 0 \qquad (8)$$

Since price equals marginal cost, there is no mark-up (*MU*) in perfect competition. Measuring such mark-up with the Lerner Index:

$$MU = \frac{p - MgC}{p} = \frac{d-d}{d} = 0 \quad (9)$$

At the equilibrium price, the price elasticity of the market demand (∈) is:

$$\in = -\frac{\partial Q}{\partial p}\frac{p}{Q} = \frac{1}{b}\frac{d}{\left(\frac{a-d}{b}\right)} = \frac{d}{a-d} > 0 \qquad (10)$$

Firms have no surplus and no profits, so social welfare (*W*) is equal to consumer surplus (*CS*):

$$W = CS = \frac{1}{2}(a-d)\frac{a-d}{b} = \frac{(a-d)^2}{2b} \; (11)$$

Collusion

Collusion or cartel –originally proposed by Kleinwächter (1883)– is the only cooperative conjecture considered in this article. It occurs here when firms cooperate to maximize total profits. As such, they replicate a monopoly.

The payoff function represented by the total profits of the cartel (Π) can be written as:

$$\Pi = [a - bQ]Q - dQ \qquad (12)$$

The first order condition for profit maximization is:

$$\frac{\partial \Pi}{\partial Q} = a - 2bQ - d = 0 \qquad (13)$$

$$Q = \frac{a-d}{2b} > 0 \qquad (14)$$

Given that the marginal cost is constant, there is no *a priori* mechanism to allocate output between firms *A* and *B*. However, in a symmetric equilibrium:

$$q_A = q_B = \frac{a-d}{4b} > 0 \qquad (15)$$

Substituting *Q* in the demand equation:

$$p = \frac{a+d}{2} > 0 \qquad (16)$$

The mark-up under Collusion, measured with the Lerner Index, can be expressed as:

$$MU = \frac{\frac{a+d}{2} - d}{\frac{a+d}{2}} = \frac{2(a+d-2d)}{2(a+d)} = \frac{a-d}{a+d} > 0 \qquad (17)$$

The price elasticity of the market demand at equilibrium is:

$$\in = \frac{1}{b}\frac{\frac{a+d}{2}}{\left(\frac{a-d}{2b}\right)} = \frac{a+d}{a-d} > 0 \qquad (18)$$

The profits of each firm are:

$$\Pi_A = \Pi_B = \left[\frac{a+d}{2} - d\right]\frac{a-d}{4b} = \frac{(a-d)^2}{8b} > 0 \qquad (19)$$

The total profits Π are:

$$\Pi = \Pi_A + \Pi_B = \frac{(a-d)^2}{4b} \qquad (20)$$

The social welfare loss (*WL*) with the Collusion outcome, with respect to the welfare under the perfectly competitive equilibrium, is:

$$WL = \frac{1}{2}\left[\frac{a+d}{2} - d\right]\left[\frac{(a-d)}{b} - \frac{(a-d)}{2b}\right] = \frac{(a-d)^2}{8b} \quad (21)$$

Threat

Threat is a conjecture where there is an established firm *A* that acts as a monopolist, and another firm *B* considers entering the market, along the lines proposed by Bain (1956). *A* decreases the price or expands production to discourage the entry of *B*, and once *B* desists from entering the market, *A* moves back to being a monopolist. Thus, the Threat equilibrium is the Monopoly equilibrium, which is the Collusion equilibrium, but all the production is done by firm *A*, and all the profits are for firm *A*. Thus, the Threat equilibrium can be characterized in terms of the parameters of the model, as follows:

$$Q = q_A = \frac{a-d}{2b} > 0 \qquad (22)$$

$$p = \frac{a+d}{2} > 0 \qquad (23)$$

$$MU = \frac{a-d}{a+d} > 0 \quad (24)$$

$$\epsilon = \frac{a+d}{a-d} > 0 \qquad (25)$$

$$\Pi = \Pi_A = \frac{(a-d)^2}{4b} \quad (25)$$

$$WL = \frac{(a-d)^2}{8b} \qquad (26)$$

Cournot

According to the Cournot (1838) conjecture, firms choose the level of production that maximizes their profits, given the production of the other firm. Output decisions affect market prices.

Given the Cournot conjecture, the profits of firm *A* can be written as:

$$\Pi_A = [a - b(q_A + q_B)]q_A - dq_A \quad (27)$$

The first order condition for profit maximization is:

$$\frac{\partial \Pi_A}{\partial q_A} = a - bq_B - 2bq_A - d = 0 \quad (28)$$

The reaction function (optimal strategy) for firm *A* is:

$$q_A = \frac{a-d-bq_B}{2b} \quad (29)$$

By symmetry it can be verified that the reaction function for firm *B* is:

$$q_B = \frac{a-d-bq_A}{2b} \quad (30)$$

The profit maximizing output of firm *A*, in terms of the parameters of the model, can be found replacing the reaction function of *B* into the reaction function of *A*:

$$q_A = \frac{a-d-b\left[\frac{a-d-bq_A}{2b}\right]}{2b} \quad (31)$$

$$2bq_A = 2a - 2d - \left[\frac{a-d-bq_A}{2}\right] \quad (32)$$

$$4bq_A = 2a - 2d - a + d + bq_A \quad (33)$$

$$3bq_A = a - d \quad (34)$$

$$q_A = \frac{a-d}{3b} > 0 \quad (34)$$

By symmetry, it can be verified that:

$$q_B = \frac{a-d}{3b} > 0 \quad (35)$$

Total output in the market is:

$$Q = \frac{2(a-d)}{3b} > 0 \quad (36)$$

The market price under Cournot is:

$$p = a - b\left[\frac{2(a-d)}{3b}\right] = \frac{a+2d}{3} > 0 \quad (37)$$

The mark-up estimated as before, is:

$$MU = \frac{\left[\frac{a+2d}{3}\right]-d}{\frac{a+2d}{3}} = \frac{a-d}{a+2d} > 0 \qquad (38)$$

The price elasticity of the market demand at equilibrium is calculated as:

$$\in = \frac{1}{b}\frac{\frac{a+2d}{3}}{\frac{2(a-d)}{3b}} = \frac{a+2d}{2(a-d)} > 0 \quad (39)$$

The profits of each firm and the total profits are:

$$\Pi_A = \Pi_B = \left[\frac{a+2d}{3} - d\right]\frac{(a-d)^2}{3b} = \frac{(a-d)^2}{9b} > 0 \ (40)$$

$$\Pi = \frac{2(a-d)^2}{9b} > 0 \quad (41)$$

The welfare loss with the Cournot outcome, with respect to the welfare under the perfectly competitive equilibrium is:

$$WL = \frac{1}{2}\left[\frac{a+2d}{3} - d\right]\left[\frac{(a-d)}{b} - \frac{2(a-d)}{3b}\right] = \frac{(a-d)^2}{18b} \qquad (42)$$

Stackelberg

Under the Stackelberg (1934) conjecture, firms do not cooperate; rather they compete, with one firm acting as a leader in the output produced and sold in the market, and the other firm acting as a follower. Assume firm *B* is the leader and firm *A* is the follower, and thus, firm *A* takes the output of firm *B* as given. The production decisions of *A* and *B* affect the output market prices.

The reaction function for firm *A* is the same as in Cournot:

$$q_A = \frac{a-d-bq_B}{2b} \qquad (43)$$

Firm *B* knows it is the market leader, and it is aware that the output decisions of *A* depend on its own output decision. In fact, *B* knows the reaction function of *A*. Thus, profit maximization for firm B can be expressed as:

$$\Pi_B = [a - b(q_A + q_B)]q_B - dq_B = \left[a - b\left(\frac{a-d-bq_B}{2b} + q_B\right)\right] q_B - dq_B \qquad (44)$$

$$\Pi_B = \left[\frac{aq_B + dq_B - bq_B{}^2}{2}\right] - d \, (45)$$

The first order condition for profit maximization is:

$$\frac{\partial \Pi_B}{\partial q_B} = \frac{a+d-2bq_B-2d}{2} = 0 \quad (46)$$

$$q_B = \frac{a-d}{2b} \qquad (47)$$

Replacing the optimal output of the leading firm *B*, in the reaction function of firm *A*:

$$q_A = \frac{a-d}{4b} \qquad (48)$$

Total output in the market under Stackelberg is:

$$Q = \frac{3(a-d)}{4b} \qquad (49)$$

Replacing in the demand curve, the price is:

$$p = \frac{a+3d}{4} \qquad (50)$$

And the mark-up is:

$$MU = \frac{\left[\frac{a+3d}{4}\right]-d}{\frac{a+3d}{4}} = \frac{a-d}{a+3d} > 0 \qquad (51)$$

The price elasticity of the market demand is:

$$\epsilon = \frac{1}{b}\frac{\left[\frac{a+3d}{4}\right]}{\left[\frac{3(a-d)}{4b}\right]} = \frac{a+3d}{3(a-d)} > 0 \ (52)$$

The profits for *A*, *B* and the total profits, are:

$$\Pi_A = \left[\frac{a+3d}{4b} - d\right]\frac{(a-d)^2}{4b} = \frac{(a-d)}{4}\frac{(a-d)}{4b} = \frac{(a-d)^2}{16b} > 0 \quad (53)$$

$$\Pi_B = \frac{(a-d)}{4}\frac{(a-d)}{2b} = \frac{(a-d)^2}{8b} > 0 \qquad (54)$$

$$\Pi = \frac{3(a-d)^2}{16b} > 0 \ \ (55)$$

The welfare loss with the Stackelberg outcome, with respect to the welfare under the perfectly competitive equilibrium is:

$$WL = \frac{1}{2}\left[\frac{a+3d}{4b} - d\right]\left[\frac{(a-d)}{b} - \frac{3(a-d)}{4b}\right] = \frac{(a-d)^2}{32b} \qquad (56)$$

Bertrand

In Bertrand (1883), firms compete in prices, meaning they set their prices to maximize their profits, assuming the price of their competitor is given. Starting with any price, for example the Cournot price, which as shown before has a mark-up over the marginal cost, one firm will have the incentive to decrease the price and capture the whole market. However, the other firm will do the same and capture the whole market. At equilibrium, one or both firms will set the price equal to the marginal cost of production and sell all the quantities demanded in the market.

Thus, under the setting of this article, with the market price depending on the total quantities available in the market, the Bertrand equilibrium replicates the perfect competition equilibrium, since the equilibrium price equals the marginal cost of production.

Profit maximization in Bertrand can be expressed as:

$$p = d \quad (57)$$

$$a - bQ = d \quad (58)$$

$$Q = \frac{a-d}{b} \quad (59)$$

Since the marginal cost is assumed constant in this model, there is no *a priori* mechanism to allocate production between firms. However, in a symmetric equilibrium:

$$q_A = q_B = \frac{a-d}{2b} \quad (60)$$

$$\Pi_A = \Pi_B = 0 \quad (61)$$

$$MU = 0 \quad (62)$$

At the equilibrium price, the price elasticity of the market demand is:

$$\in = \frac{d}{a-d} > 0 \quad (63)$$

The consumers' surplus is:

$$CS = \frac{(a-d)^2}{2b} \quad (64)$$

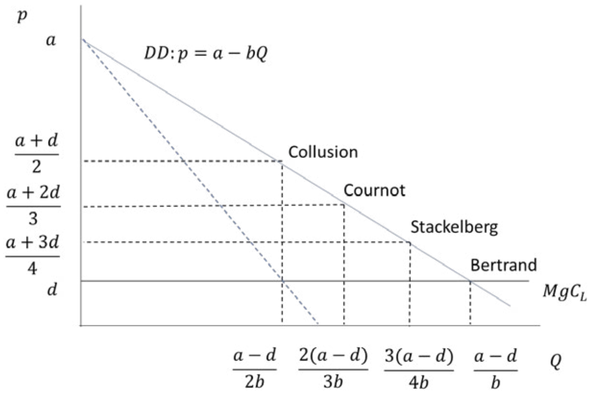
And the welfare loss with respect to perfect competition is:

$$WL = 0 \quad (65)$$

Summary

Graphically the results obtained in the model can be summarized as shown in Figure 1:

Figure 1. Summary of the Results of the Model of Oligopoly



Source: Own elaboration.

Table 1. Summary of the Results of the Oligopoly Model[1]

Market	Collusion	Threat	Cournot	Stackelberg	Bertrand
q_A	$\frac{a-d}{4b}$	$\frac{a-d}{2b}$	$\frac{a-d}{3b}$	$\frac{a-d}{4b}$	$\frac{a-d}{2b}$
q_B	$\frac{a-d}{4b}$	0	$\frac{a-d}{3b}$	$\frac{a-d}{2b}$	$\frac{a-d}{2b}$
Q	$\frac{a-d}{2b}$	$\frac{a-d}{2b}$	$\frac{2(a-d)}{3b}$	$\frac{3(a-d)}{4b}$	$\frac{a-d}{b}$
p	$\frac{a+d}{2}$	$\frac{a+d}{2}$	$\frac{a+2d}{3}$	$\frac{a+3d}{4}$	d
Π_A	$\frac{(a-d)^2}{8b}$	$\frac{(a-d)^2}{4b}$	$\frac{(a-d)^2}{9b}$	$\frac{(a-d)^2}{16b}$	0
Π_B	$\frac{(a-d)^2}{8b}$	0	$\frac{(a-d)^2}{9b}$	$\frac{(a-d)^2}{8b}$	0
Π	$\frac{(a-d)^2}{4b}$	$\frac{(a-d)^2}{4b}$	$\frac{2(a-d)^2}{9b}$	$\frac{3(a-d)^2}{16b}$	0
MU	$\frac{a-d}{a+d}$	$\frac{a-d}{a+d}$	$\frac{a-d}{a+2d}$	$\frac{a-d}{a+3d}$	0
$\in$	$\frac{a+d}{a-d}$	$\frac{a+d}{a-d}$	$\frac{a+2d}{2(a-d)}$	$\frac{a+3d}{3(a-d)}$	$\frac{d}{a-d}$
WL	$\frac{(a-d)^2}{8b}$	$\frac{(a-d)^2}{8b}$	$\frac{(a-d)^2}{18b}$	$\frac{(a-d)^2}{32b}$	0

Source: Own elaboration.

1. The equilibria of Collusion and Bertrand are presented assuming the symmetric equilibrium. Given that there are no fixed costs, the equilibrium of Bertrand is identical to the equilibrium of Perfect Competition, although in Perfect Competition, there would be many firms. Note that given the assumptions of the model and in particular since $a > 0$, $b > 0$, $d \geq 0$, and $a > d$, all the results obtained are well defined.

Within the simple model structure presented in this article, the algebraic results allow us to conclude unambiguously that, under the assumptions used:

Collusion and Threat replicate Monopoly, have the highest prices, the lowest quantities produced, the highest mark-up, the highest price elasticity of market demand, the highest profits, and the lowest welfare. When compared to Collusion, Cournot has a lower price, higher total quantities produced, lower price elasticity of market demand, lower mark-up, lower profits and higher welfare. When compared to Cournot, Stackelberg has a lower price, higher total equilibrium quantities, lower price elasticity of market demand, lower mark-up, lower profits, and higher welfare. Bertrand has the lowest price, the highest total equilibrium quantities, the lowest price elasticity of market demand, the lowest mark-up, the lowest profits, and the highest welfare, when compared to any of the other conjectural variations considered in this model, and replicates in this framework, the equilibrium that would prevail under perfect competition.

Thus, in general, the conjectures with lower prices have higher equilibrium quantities and *vice versa*. These results are in line with —and extend— those derived by Levin (1988). The conjectures with lower prices also have higher welfare, lower price elasticity of market demand, lower mark-ups, lower profits, and *vice versa*.

Note the counter-intuitive results regarding equilibria with lower mark-ups and lower price elasticities of market demand. This is because lower mark-ups imply lower prices closer to the marginal cost. Given linear market demands, lower equilibrium prices imply lower equilibrium price elasticities of market demand.

Comparative Statics

The results presented in the previous sections allow to perform comparative statics that yield the following results:

All conjectures, except for Bertrand

In all conjectures except Bertrand, an increase in the demand parameter a (i.e. an increase in demand) leads to increases in equilibrium prices, output, profits, and mark-up. Increases in a also lead to decreases in the price elasticity of market demand, and welfare as compared to the welfare that would prevail under perfect competition. And *vice versa*.

In these same conjectures, decreases in demand parameter b (an increase in demand) lead to increases in equilibrium output and profits. The unit price, the mark-up, and the price elasticity of market demand remain unchanged. Welfare would fall, as compared to the welfare that would prevail under perfect competition. And *vice versa*.

In this scenario, decreases in the marginal cost d, at equilibrium, would lead to increases in output, profits, and mark-up. Decreases in d would also lead to decreases in prices, the price elasticity of market demand, and welfare, when compared to the welfare under perfect competition. And *vice versa*.

Bertrand

In Bertrand, increases in demand (increases in *a* and/or decreases in *b*) or decreases in the marginal cost *d*, lead to increases in output, with no changes in profits, mark-up, or welfare —when compared to the welfare that would prevail under perfect competition—. Decreases in the marginal cost would decrease the unit price and the price elasticity of market demand. And *vice versa*.

Conclusions

This article has shown that, given the model's assumptions, when firms interact as in Bertrand, they charge lower prices and have higher total equilibrium quantities than when they interact as in Stackelberg. When firms interact as in Stackelberg, they charge lower prices and have higher total equilibrium quantities than when they interact as in Cournot. And when firms interact as in Cournot, they have lower prices and have higher total equilibrium quantities than when they collude.

In general, the conjectures with lower prices have higher total equilibrium quantities, higher welfare, lower price elasticity of market demand, lower mark-ups, and lower profits. In all conjectures except in Bertrand, increases in demand (as increases in *a*) and decreases in the marginal cost *d* increase quantities, prices and mark-ups, and reduce welfare when compared to perfect competition. Increases in demand (as decreases in *b*) also increase quantities, mark-up and reduce welfare when compared to perfect competition.

In Bertrand, increases in demand (as increases in *a*) and decreases in marginal costs, increase equilibrium quantities and reduce the price elasticity of market demand. Increases in demand (as decreases in *b*) increase output. Decreases in marginal cost decrease prices and decrease the price elasticity of market demand. In Bertrand, none of the changes in the parameters considered here alter the mark-up, the profits, and welfare —when compared to the welfare under perfect competition—.

Although the results suggest lower mark-ups and lower price elasticities of market demand, this counter-intuitive result arises since lower mark-ups imply lower prices closer to the marginal cost. Given linear market demands, lower equilibrium prices imply lower equilibrium price elasticities of market demand.

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