MODELLING CONSUMER BEHAVIOUR: A COMPARISON BETWEEN AIDS AND THE STONE-GEARY MODELS

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This work responds to the question: Why are the Almost Ideal Demand System (AIDS) and its quadratic extension, the QUAIDS model, better ways of modelling consumer behaviour than the Stone-Geary Models? . Thus, the work shall deal with proposals of single-equation (or separated equations) and with systems of equations where the theory is more striking.

The work starts by defining what is known as consumer behaviour. Secondly, it makes a presentation of the restrictions or properties that demand functions have to possess. Thirdly, it makes a brief presentation of each model and its main characteristics, limitations and empirical evidence of using each. In this paragraph, the essay illustrates how can changes in some exogenous variables affect on the consumption of particular commodities with especial reference to the way in which different models can allow for the impact of changes in income. Finally some concluding remarks are made.

1. CONSUMERS BEHAVIOUR

Rationality is a good place to start when analysing the consumer's behaviour. It is assumed that the consumer chooses, among all possible consumption alternatives, that bundle that gives her or him

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the highest utility (satisfaction). All the information related to her or his satisfaction is included into her or his utility function[1].

This utility function is assumed well-behaved in the sense it satisfies the axioms usually imposed to the ordering of goods or bundles built by the consumer according to her or his preferences.

Consumers, in general, are assumed to face the primal problem of maximizing their utility subject to a limited income with given prices. The Marshallian demand functions are obtained from of this process of maximization.

The dual problem is read as the minimization of the expenditure to reach (or to maintain) a given level of satisfaction. From here we obtain the Hicksian demand functions.

2. *PROPERTIES OF DEMAND FUNCTIONS*

Those theoretical assumptions allow to characterize demand functions in terms of the following properties:

i) Adding-up:

The expenditure in the n goods has to be equal to the income.

$$M = \sum_i^n P_i q_i$$

That is, the value of demands is equal to the total expenditure.

ii) Homogeneity

The Marshallian demands are homogeneous of degree zero in money income and prices, which implies that (by using the Euler's Theorem) the following expression has to be satisfied:

$$0 = \frac{\partial q_i}{\partial P_i}\frac{P_i}{q_i} + \sum_j^n \frac{\partial q_i}{\partial P_j}\frac{P_j}{q_i} + \frac{\partial q_i}{\partial M}\frac{M}{q_i}$$

This means that consumer has no money illusion. For the Hicksian demands case, the functions (qi') have to be homogeneous of degree zero in prices.

1. It refers to the direct utility fuction.

$$0=\frac{\partial q'_i}{\partial P_i}\frac{P_i}{q'_i}+\sum_j^n\frac{\partial q'_i}{\partial P_j}\frac{P_j}{q'_i}$$

These first two restrictions are consequence of a linear budget constraint.

iii) Negativity of the substitution effect:

Intuitively this restriction implies that demanded quantity of the ith good reduces when Pi increases but not because of the reduction in the consumer's purchasing power (as it in fact occurs unless any compensation would take place) but because the increasing in the relative price of good ith itself; i.e. the demand curve has to be downwards slope in qi,pi space:

$$\frac{\partial q'_i}{\partial P_i}\leq 0$$

Hence, an increase in price (Pi) with utility held constant will lead to a fall in the demand for that good (qi). This is not other thing that the «law of demand»[2].

iv) Sýmmetry (of the cross-substitution matrix)[3]: This restriction states:

$$\frac{\partial q'_i}{\partial P_j}=\frac{\partial q'_j}{\partial P_i}$$

for each pair $_{i,j}$[4].

The fulfilment of negativity and symmetry is evidence of consistency of preferences of consumer. In practice, once demand functions are estimated, it is required to check out the consistency of the results with the theory together properties i) and ii). The restrictions are satisfied when demand functions are derived by utility maximization.

Equivalent to that, it would be to impose the restrictions a priori and test its validity using standard statistical methods.

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2. Through the Slutsky equation it can be shown that the law of demand has to be a property of Marshallian demand as well.

3. Mathematically this property is based on that proposition known as Young's theorem (Sea Chiang, 1984, page 313).

4. To Show the property it is useful to refer to the expenditure function that is obtained by solving for income from the indirect utility function.

This procedure is also equivalent to test the approval of axioms for the existence of a utility function (or set of preferences) for each consumer or group of them and that demand functions come from optimization behaviour.

When demand functions have the properties the estimation process is easier since the degrees of freedom increase and it is also possible to solve the integrability problem. This information can be exploited in multiple ways.

3. *THE MODELS AND THEIR CHARACTERISTICS*

In this paragraph we show the main characteristics of the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS) and its further quadratic development (QUAIDS).

3.1 STONE-GEARY PREFERENCES AND THE LINEAR EXPENDITURE SYSTEM (LES).

The LES is originated in the Stone-Geary direct utility function, which takes the form:

$$U_{(q)} = A\prod_i^n (q_i - V_i)^{\beta_i} \qquad \text{Where } \sum_i^n \beta_i = 1 \qquad (1)$$

This expression makes economic sense for $q_i > V_i$, where V_i's are minimum possible of required quantities (subsistence) of each good. In (1) A and β_i's are positive parameters[5]. In (1) the sum of β_i is restricted to unity[6].

The income expansion paths here are straight lines through the intersection between V_1, V_2 in quantities space. See figure 1. The Stone-Geary direct utility function belongs to that kind of functions called quasi-homothetics. Homothetic functions are those in which

5 When the subsistence levels are dropped, this utility function becomes in the Cobb-Douglas utility function.

6 These arguments are taken from Deaton and Muellbauer (1980b).

the marginal rates of substitution are unaffected by same proportional changes in all quantities: In other words, income elasticity is one. The income expansion paths are straight lines through the origin.

As we shall see that quasi-homotheticity is one of the main features of LES.

The Langrangean procedure to solve theoretically the problem consumer faces yields demand functions of the form[7]:

$$q_i = V_i + \frac{\beta_i}{P_i}(M - \Sigma_j^n P_j V_j) \quad (2)$$

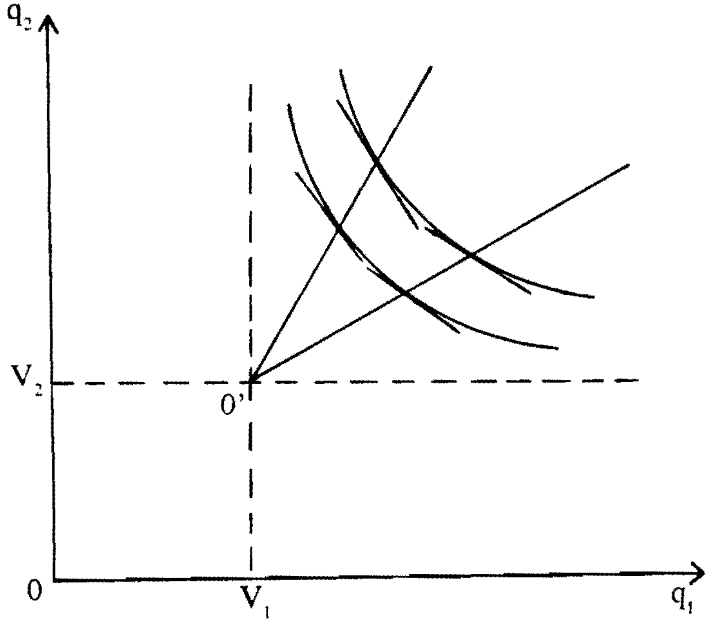

Figure 1. Income expansion paths of Stone-Geary preferences.

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7 Richard Stone was who originally imposed the algebraic restrictions of adding-up, homogeneity and symmetry to a simple linear demand:

$$P_i q_i = \beta_i M + \Sigma_{j=1}^n \beta_{ij} P_j$$

Those restrictions can only be satisfied by an eguation as (2).

The group of n equations (2) is called the Linear Expenditure System (LES)[8]. The summation of P_jV_j discounted of the money income in called the supernumerary expenditure that is interpreted as a residual income to purchase the additional quantities of good ith over V_i. This assignment is made in fixed proportions β_i[9].

In (2) we can see the first limitation of LES: prices different of the own-price are present into the demand of good i, but those prices (P_j's) only affect qi through the subsistence expenditure. The substituibility between goods is because the competition for the supernumerary. In addition, P_j and V_j play the same role in determining q_i.

For the two goods case, it can be shown that the own-price, cross-price and income elasticities are, respectively:

$$e_{11} = [\frac{\beta_1 P_2 V_2 - \beta_1 M}{P_1 q_1}] \qquad (3)$$

$$e_{12} = -[\frac{\beta_1 P_2 V_2}{P_1 q_1}] \qquad (4)$$

$$e_{1M} = [\frac{\beta_1 M}{P_1 q_1}] \qquad (5)$$

In equation (3) it can be seen that $-1<e_{11}<0$ for all the cases. That is, algebraic values less than minus one are not possible. In equation (4) the cross-price elasticities (say e_{ij} and e_{ik}) are proporcional in the term P_jV_j/P_kV_k.

Inferior goods are excluded in (5), since a negative value of β_i violates the concavity requirement of (1); if it were allowed, the own-price elasticity would show positive responses. However, it would allow for the possibility of substitutes goods in (4)[10].

On the other hand, according to the LES the elasticities of substitution between goods are equal to one and in log-lin form this function is strong additive so that the values of e_{ii} and e_{iM} are almost proportional: the knowledge of e_{1M} and V_2 are enough to know e_{11}. It could be though such as an advantage in the sense that to estimate e_{ii}

8 Note that this model is linear in the variables but not in the parameters Vi and β_i

9 With this reading of (2) it should be clear that we are dealing with a equation-by-equation system.

10. That is, all goods complement each other.

in a cross-section data it would be enough to know e_{IM}, but it actually is a restriction.

With respect to the latter limitation of LES Deaton and Muellbauer (1980b) point at:

«.. Unless we have grounds for believing that elasticities should be proportional, and there is a good deal for evidence, both a priori and empirical, against such a position, even for broad group of goods, then we have good reason for considering the LES too restrictive and for passing on to more general models.»

For those reasons sometimes researchers refer to LES as overly restrictive[(11)].

However, knowing its restrictions, which come from the functional form itself, this model could carefully be used in practice when such limitations are not thoughc to be serious.

It can be shown that (at least theoretically) LES obeys adding up and homogeneity, but symmetry has to be tested by standard methods once regressions have been fitted.

The performance of LES in practice -taken from Stone's results in 1954 using British data from 1920 to 1938-[(12)] is an evidence of the theoretical set up since the income elasticity is always positive (no inferior baskets) for the six groups of expenditures he considered. However, the main feature is that income elasticities are (approximately) twice that own-price elasticities (in absolute value) and theory does not give reasons to expect such situation.

Therefore, the overrestrictiveness of LES is clear and affects seriously the measures of consumer behaviour due to its narrowness to treat the theory.

3.2 THE ALMOST IDEAL DEMAND SYSTEM (AIDS)

This system is a more flexible functional form derived from an expenditure function instead from a direct utility function as LES is.

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11 See, for example, Deaton and Muellbauer (1980b) and Chesher and Rees (1987).

12 See Deaton and Muellbauer (1980b), paga 67.

The expenditure function is defined as the minimum expenditure necessary to attain a specific utility level at given prices (Deaton and Muellbauer, 1980a)[(13)].

The expression they used relates the value of shares to the logarithm of total expenditure[(14)]:

$$W_i = \alpha_i + \beta_i \log M \quad (6)$$

where α_i and β_i are parameters to be estimated, which has to be function of prices to include them into the model, M is the money income and wi are the budget shares.

The general form of the expenditure function is:

$$\log E(P,u) = a(P) + u\, b(P) \quad (7)$$

where a (P) and b (P) are functions of prices, defined respeccively, as:

$$a(P) = \alpha_0 + \sum_k \alpha_k \log P_k + \frac{1}{2}\sum_k \sum_1 \mu^*_{k1} \log P_k \log P_1 \quad (8)$$

and,

$$b(P) = \beta_0 \prod_k P_k^{\beta_k} \quad (9)$$

where α, β and μ^* are parameters. For homogeneity of degree 1 in prices, log E(P,u) requires:

$$\sum_1^n \alpha_k = 1\ ; \quad \sum_k \mu^*_{k1} = \sum_1 \mu^*_{k1} = \sum_1^n \beta_k = 0 \quad (10)$$

Substituting (9) and (8) in (7) and using the Shephard's lemma it gives the budget shares as a function of prices and income:

13 AIDS by difference with LES suggests a proper way of aggregation. Deaton and Muellbauer start from a particular set of preferences that permits exact aggregation over consumers. Previously, Muellbauer (1976) had suggested an alternative to work exact non- linear aggregation over consumers, by means of a generalization of Gorman's polar form. According to the proposition of «arithmetic mean consumer», Gorman assumed that m consumers, each with arithmetic mean income, behave exactly as a community of m consumers each with different income (See, Cornes 1992, p 196).

14 This expression is due to Working and Leser.

$$W_i = \alpha_i + \Sigma_j \mu_{ij} \log P_j + \beta_i \log(\frac{M}{P}) \quad \text{P=PriceIndex} \quad (11)$$

Note that equation (11) is the expanded form of (6) which is the Engel curve used by Working and Leser[15].

The price index is defined by:

$$\text{Log P} = \alpha_0 + \Sigma \alpha_k \log P_k + \frac{1}{2} \Sigma_k \Sigma_l \mu_{kl} \log P_k \log P_l \quad (12)$$

where,

$$\mu_{i\,j} = \frac{1}{2}(\mu^{*}_{i\,j} + \mu^{*}_{j\,i}) = \mu_{j\,i} \quad (13)$$

Equations (11) to (13) are called the Almost Ideal Demand System of Deaton and Muellbauer (1980a), where (11) one is a first-order approximation to the general unknown relation between wi, log M and log P's. Given this, the AIDS shows that in the absence of changes in relative prices and «real» expenditure (M/P) the budget shares are constant and this is the natural starting point for predictions using the model. Changes in relative prices are in the terms μ_{ij} under (M/P) constant.

Wich respect to the restrictions, adding up requires, for all j:

$$\Sigma \alpha_k = 1; \quad \Sigma_k \mu_{k\,j} = 0; \quad \Sigma_k \beta_k = 0 \quad (14)$$

Homogeneity, requires for all j,

$$\Sigma_k \mu_{j\,k} = 0 \quad (15)$$

and symmetry is sacisfied if:

$$\mu_{i\,j} = \mu_{j\,i} \quad (16)$$

Conditions (15) and (16) are linear restrictions which may be tested by standard techniques, while condition (14) is imposed by the

15 See Chesher and Rees (1987).

model and so is not testable(16). The latter is because the model is defined for shares.

It can be shown that income elasticities of expenditures, given by:

$$e_i = \frac{\partial \log W_i}{\partial \log M} = 1 + \frac{\beta_i}{W_i} \quad (17)$$

depend on both prices and total expenditure through the budget shares wi.

Goods for which $\beta_i<0$ (and $|\beta_i|<w_i$) are identified as necessities; those for which $\beta_i<0$ (and $|\beta_i|>w_i$) are not only necessities but also inferior goods. Finally, goods for which $\beta_i>0$, w_i increases with M so that good i is a luxury.

The μ_{ij}'s measure the change in the budget share w_i because of a unit proportional change in P_j with (M/P) held constant.

It is important to point out that in empirical works it is used:

$$P = \sum w_k \log P_k \quad (18)$$

instead of (12) because of the relative collinearity between prices. Nonetheless, there is some evidence of bias brought by using this approximation of index prices when AIDS is estimated using micro data(17).

The quadratic extension of AIDS, known as QUAIDS, is found in Blundell et.al (1993). It keeps the main characteristics of its predecessor but incorporates a second term with the log of real but in squares. That is,

$$W_i = \alpha_i + \sum_j \mu_{i\,j} \log P_j + \beta_{i1} \log(\frac{M}{P}) + \beta_{i2} \log(\frac{M}{P})^2 \quad (19)$$

Blundell et. al., justifies the introduction of QUAIDS in their wishes by capturing the variability of demand patterns when micro-data are used in practice. They model such variability (across households with different household characteristics and with different income levels) by making intercept and slope parameters in the budget-

16 Molina (1994).

17. See Deaton and Muellbauer (1980a), Pag 254.

share equations to depend on household characteristics and by allowing for nonlinear total log- expenditure terms.

Accordingly, at this stage should be clear that flexibility which entails AIDS makes it more attractive that LES which is forced and overrestricted . Now we turn to the use of this models in practice.

In Deaton and Muellbauer's work the estimation of β_i's allow for luxury goods (clothing, fuel, drink and tobacco etc.) and necessities ones (food and housing). The estimates μ_i's allow for different relationships between goods since the signs and numbers vary across.

However, it is important to say that in the empirical use of AIDS (Annual British data from 1954 to 1974 using PIGLOG for aggregation) it showed a failure of homogeneity[(18)]. Specifically, the outcome of a proportional increase in prices and expenditure is a decrease in expenditure on food and on clothing whereas it is an increase expenditure on housing and transport and communication. Possible explanations of that are in the inflexibility of expenditures in the short-run, omission of lagged variables and price expectations (dynamic conditions of the model) and in the very method of aggregation data.

Symmetry was rejected as well. The explanation for that is in the lack to allow for habits in consumption.

In an application of AIDS for food demand in Spain, Molina (1994)[(19)] found that bread and cereals and meat and fish are necessities while milk and eggs, vegetables and fruit and other food are luxuries, in the second-stage. The estimation of elasticities with respect to the total expenditure (first-stage) shows that bread and cereals, meat, fish and milk and eggs are necessities, while vegetables and fruit and other food are luxuries.

Molina also estimated (statiscally significant) own-price elasticities -for Marshallian demands- between minus one and zero for meat, milk and eggs, vegetables and fruits and other food[(20)]. The own-price elasticities are higher in absolute than those estimated for Hicksians demands. However, the cross-price elasticities are not the same

18 See Deaton and Muellbauer (1980a), page 320.

19 Molina assumed weak separability of preferences as a necessaryand sufficient condition for a two-stage budgeting process.

20 This a point in favour of LES since all the statiscally significant estimated elasticities are between zero and minus one such as LES would respond.

(even for statistically significant estimators) neither for Marshallian nor Hicksian demands.

Chesher and Rees (1987) make -as Deaton and Muellbauer (1980a) and Blundell et.al. (1993)- explicit the issue concerning with periods of non-zero expenditure in some set of goods during the sample period. However, the AIDS expenditure-income relationship can be estimates without removing zero expeditures.

4. *CONCLUSIONS*

According to the theoretical interpretations and empirical results the LES model is very restrictive. The problem arises due to the underlying direct utility function. Undesirable proportionalities among elasticities are predictable from the theory and proved in practice. AIDS (and QUAIDS) is more flexible to represent the behaviour of consumers in market of goods and services. The latter has been used with more, but not complete, succesful since the set of properties has not been carried out in all cases. AIDS is said to be a better way of representing the consumer behaviour than Stone-Geary models.

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