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Multivariable Regression 3D Failure Criteria for In-Situ Rock

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ABSTRACT

Wellbore stability problems increase with the exploration and development of oil and gas reservoirs. A new 3D non-linear failure criterion is proposed as a trigonometric function considering the intermediate principal stress (σ_2) on the triaxial compression test data. Mohr-Coulomb and Hoek-Brown are well-known failure criteria, but they do not consider the influence of (σ_2) on rock strength. This new criterion produces a concave surface on the principal stress space (σ_1,σ_2,σ_3) with the influence of intermediate principal stress. In this study, sensitivity analysis for the variable is also done to understand the significant influence of parameters on the accuracy of the proposed criterion. Further validation of this non-linear criterion on three principal stresses (σ_1,σ_2,σ_3) was done compared with linear regression and second-degree polynomial regression results. It has been observed that the new non-linear 3D criterion with five material parameters reveals a good fit compared to linear regression and second-degree polynomial regression, which have four and six material parameters, respectively. The new non-linear criterion was further validated by comparison with existing criteria like the Priest, Drucker-Prager, and Mogi-Coulomb. It has been observed that the new 3D non-linear criterion shows a more accurate result than these existing criteria as certain rock types exhibit coefficient of determination (DC) values near one, precisely 0.95 for inada granite, 0.94 for orikabe monzonite, and 0.91 for KTB amphibolite. In contrast, other rock types have DC values ranging from 0.7 to 0.9. The new 3D non-linear criterion also yields lower root means square error (RMSE) values than the Mogi-Coulomb criterion for seven rock types. Specifically, the RMSE values by the new criterion are as follows: KTB amphibolite - 40.03 MPa, Dunham dolomite - 15.16 MPa, Shirahama sandstone - 9.08 MPa, Manazuru andesite - 22.14 MPa, Inada granite - 35.47 MPa, and Coconino sandstone - 19.047 MPa. This new 3D criterion gave precise predictions of the failure of the formation under in-situ stresses and was further helpful for the simulation of the wellbore in the petroleum industry. The variable in the new 3D criterion should be calculated from triaxial compression test data for each formation rock before applying this criterion to the wellbore stability problem and the sand production problem.

Keywords: 3D failure criterion; Non-linear failure criterion; Linear regression; Polynomial multivariable regression; Geomechanics; In-situ stresses; Principal stresses.

Criterio de falla tridimensional por regresión multivariable en rocas *in-situ*

RESUMEN

Los problemas de estabilidad de un pozo se incrementan con la exploración y el desarrollo de los reservorios de petróleo y gas. En este trabajo se propone un nuevo criterio de falla tridimensional y no lineal como una función trigonometrica que considera la tensión principal intermedia (σ_2) en la información para la evaluación de compresión triaxial. Los criterios de falla de Mohr-Coulomb y Hoek-Brown son bien conocidos, pero estos no consideran la influencia de la tensión principal intermedia en la dureza de la roca. Este nuevo criterio produce una superficie cóncava en el espacio de las tensiones principales máxima, intermedia y mínima (σ_1,σ_2,σ_3) con la influencia de la tensión principal intermedia. Para este trabajo también se hizo el análisis de sensibilidad con el fin de entender la influencia de los parámetros en la exactitud del criterio propuesto. Se realizó una validación adicional de este criterio en las tres tensiones principales (máxima, intermedia y mínima) y se comparó con los resultados de la regresión lineal y la regresión polinómica de segundo grado. En este proceso se observó que el criterio tridimensional no lineal con cinco parámetros materiales revela un buen ajuste en comparación con las regresiones lineal y polinómica de segundo grado, que se trabajaron con cuatro y seis parámetros materiales, cada una. El nuevo criterio no lineal se validó adicionalmente con criterios existentes como el de Priest, Drucker-Prager y Mogi-Coulomb. Los autores observaron que el nuevo criterio tridimensional no lineal muestra un resultado más exacto que aquellos de los criterios existentes ya que ciertos tipos de roca exhiben valores en los coeficientes de determinación cerca de uno. Precisamente, 0.95 para granito inada, 0.94 para monzonita orikabe, y 0.91 para anfibolitas KTB. En contraste, otros tipos de rocas tienen valores en los coeficientes de determinación que van de 0.7 a 0.9. El nuevo criterio tridimensional no lineal también produce valores de error cuadrático medio (RMSE) más bajos que el criterio Mogi-Coulomb en siete tipos de rocas. Específicamente, los valores RMSE para el nuevo criterio son los siguientes: anfibolitas KTB = 40.03 MPa; dolomitas Dunham = 15.16 MPa; arenisca Shirahama = 9.08 MPa; andesita Manazuru = 22.14 MPa; granito inada = 35.47 MPa, y arenisca Coconino = 19.047 MPa. Este nuevo criterio tridimensional ofreció predicciones precisas de falla de la formación bajo tensiones in-situ y luego fue útil en la simulación de pozos en la industria petrolera. La variable en el nuevo criterio tridimensional debe de ser calculada con información de la evaluación de compresión triaxial en cada formación rocosa antes de aplicar este criterio en los problemas de estabilidad de pozos y en los problemas de producción de arena.

Palabras Clave: criterio de falla tridimensional; criterio de falla no lineal; regresión lineal; regresión polinomial multivariable; geomecánica; tensiones in-situ; tensiones principales.

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List of symbols

σ_1, σ_2, σ_3	Principal stresses the failure in MPa
ξ, ρ, θ	Haigh-Westergaard coordinates in MPa
I_1	First invariant of stress tensor and the mean stress
J_2,J_3	Second invariants of the deviatoric stress tensor.
m_b,s,a	Empirical constants of the generalized Hoek–Brown criterion
τ_{oct}	The octahedral shear stress in MPa
σ_{oct}	The normal octahedral stress in MPa
σ_{1f}	Three-dimensional effective failure stress MPa
w	The weighting factor for Hoek–Brown criterion
σ_{3hb}	Minimum 2D Hoek–Brown effective stress at failure in MPa
σ_{1hb}	Maximum 2D Hoek–Brown effective stress at failure in MPa
P	The weighting of σ_2
RMSE	The root-mean-square-error in MPa
q_1,q_2	The material constant of intact rock
r_i	Prediction error or residuals i-th test
DC	Coefficient of determination.
ε_i	Error percentage

1. Introduction

Better prediction of the failure of rock near the wellbore has been essential over the last three to four decades. New oil and gas fields develop on the uncertainty of subsurface reservoir pressure, tectonic effect, and temperature gradient. More recent technology is required in drilling and production operations for deeper reservoirs. The problem of wellbore stability increased at high-pressure, high-temperature reservoirs with an inclination of the well in directional drilling. The prominent wellbore instability is brack-out of rock by a combination of tensile and compressive force on the geomechanical stress near the wellbore results from formation, overburden, and hydrostatic pressure (Mahetaji et al., 2020). Lots of wells are missed due to the instability of rock fractures. Geomechanical modeling with maximum wellbore stability solves uncertainty about the drilling well. Geomechanical stress is vital in numerical modeling (Zhang, 2019).

To recognize a failure phenomenon, a compatible and specific criterion must be used (Bou-Hamdan, 2022). While some materials fail in shear, some may fail due to plastic deformation. The reasons behind the near-wellbore instability problems are tensile failure of formation, shear failure without plastic deformation, formation collapse or compressive failure, erosive-cohesive failure, and creep failure during drilling. (Aadnoy and Looyeh, 2019).

Various failure criteria are introduced in the literature for the failure of different-different materials. Mohr-Coulomb criterion and Hoek-Brown failure criterion are well-known failure criteria that consider maximum and minimum principal stress (σ_1, σ_3) and neglect the intermediate principal stress (σ_2) (Culshaw and Ulusay, ISRM Suggested Method 2007-2014). Mohr-Coulomb criterion is driven by the linear relation of mean and shear stress. Mohr-Coulomb failure criterion is mainly recommended when principal stresses σ_1, σ_2, and σ_3 are compressive and in a limited range of mean stress. The Hoek-Brown failure criterion is the empirically derived relation between two principal stress σ_1 and σ_3 (Hoek and Brown, 1997; Hoek et al., 2002). Hoek-Brown criterion is a non-linear, parabolic curve with the independence of the effect of intermediate principal stress. Experimental evidence shows that the intermediate principal stress influences the failure of rock material (Takahashi and Koide, 1989). Mohr-Coulomb and Hoek-Brown's failure criterion does not consider the effect of intermediate principal stress, so the rock failure prediction under the triaxial stress condition is unreliable. These lead to developing several three-dimensional Hoek-Brown criteria given by Pan and Hudson (1988); Priest (2005); Zhang and Zhu (2007); Zhang (2008); Jiang and Yang (2020). Singh et al. (2011) and Mahetaji et al., (2023) developed a modified non-linear Mohr-Coulomb criterion considering the effect of intermediate principal stress on strength behavior.

Furthermore, failure criteria are introduced after considering the effect of intermediate principal stress, such as the Drucker and Prager criterion (Drucker and Prager, 1952) and the Lade criterion (Kim and Lade, 1984). Drucker-Prager criterion was a curvilinear 3D generalized form of the Mohr-Coulomb criterion for soils. Drucker-Prager criterion is the relation between normal and octahedral shear stress through material constant. The Lade criterion (Kim and Lade, 1984) and Modified Lade criterion (Ewy, 1999) relate to the respective first and third invariants of the effective stress tensor at the failure. Modified lade criterion is further used for sand production to produce well and wellbore stability problems with the effect of intermediate stress (Yi et al., 2005; Yi et al., 2006).

The failure criterion from the triaxial experimental testing data in terms of the power law, considering three principal stresses, is given by Mogi (1971). Other rock triaxial testing data also validate failure criterion based on the experiment data by Haimson and Chang (2000), Chang and Haimson (2000), Takahashi and Koide (1989), and Xiaodong and Haimson (2016). Priest (2005) proposed the 3D failure criterion based on the extended Hoek-Brown yield criterion combined with the Drucker-Prager criterion. Melkoumian et al., 2008 further developed the 3D Hoek-Brown yield criterion by considering the weighting of intermediate principal stress. Further research and triaxial rock testing are necessary before applying any of the 3D failure criteria.

The study aims to propose a 3D non-linear failure criterion from the actual triaxial compression data. The proposed criterion may apply to a subsurface rock formation using the triaxial compression test of that rock formation. After that, this failure criterion is helpful for the wellbore stability problem and the prediction of sand production in producing well. A new failure criterion is proposed by considering the effect on the strength of the rock. Proposed failure criteria produce a curve surface as a failure envelope on a 3D space of (σ_1, σ_2, σ_3) that is easy to derive and interpret. Available triaxial data in the literature from various researchers (Mogi, 1971; 1974; 2007; Takahashi and Koide, 1989; Chang and Haimson, 2000; and Xiaodong and Haimson, 2016) as shown in Table 1 are used for the non-linear regression analysis of the proposed 3D criterion on principal stress space (σ_1, σ_2, σ_3).

Table 1. Sources of Laboratory Test Data for Ten Rock Samples.

Rock Type	Source of the Tabular Data.	Original Source of Data.	Specimen Size
KTB amphibolite (German Continental Deep Drilling Program)	Colmenares and Zoback (2002)	Haimson and Chang (2000)	-
Shirahama sandstone	Colmenares and Zoback (2002)	Takahashi and Koide (1989)	Rectangular prism 3.5 cm × 3.5 cm × 7.0 cm
Dunham dolomite	Mogi (2006)		Circular cylinders 1.6 cm × 5.0 cm
Solnhofen limestone	Mogi (2006)		Circular cylinders 1.6 cm × 5.0 cm
Yamaguchi marble	Mogi (2006)		Rectangular prism 1.5 cm × 1.5 cm × 3.0 cm
Mizuho trachyte	Mogi (2006)		Rectangular prism 1.5 cm × 1.5 cm × 3.0 cm
Manazuru andesite	Mogi (2006)		Rectangular prism 1.5 cm × 1.5 cm × 3.0 cm
Inada granite	Mogi (2006)		Rectangular prism 1.5 cm × 1.5 cm × 3.0 cm
Orikabe monzonite	Mogi (2006)		Rectangular prism 1.5 cm × 1.5 cm × 3.0 cm
Coconino sandstone	Xiaodong and Haimson(2016)		Rectangular prism 1.9 cm × 1.9 cm × 3.8 cm

Further validation of the proposed 3D criterion is done by comparison of predicted results with actual triaxial data on ten rock types on 3D Haigh-Westergaard coordinates stress space (ξ, ρ, θ). In addition, the sensitivity of the parameter on the prediction error and comparison of the proposed criterion with the Priest criterion are discussed on the ten rock types. The priest criterion combines the three-dimensional Drucker-Prager criterion (Drucker and Prager, 1952) and the two-dimensional Hoek–Brown criterion. The proposed 3D non-linear failure criterion considers the trigonometric relation between on principal stress space (σ_1, σ_2, σ_3). Based on that, predicted strength by the new 3D criterion might indicate a more accurate result compared to the existing failure criteria.

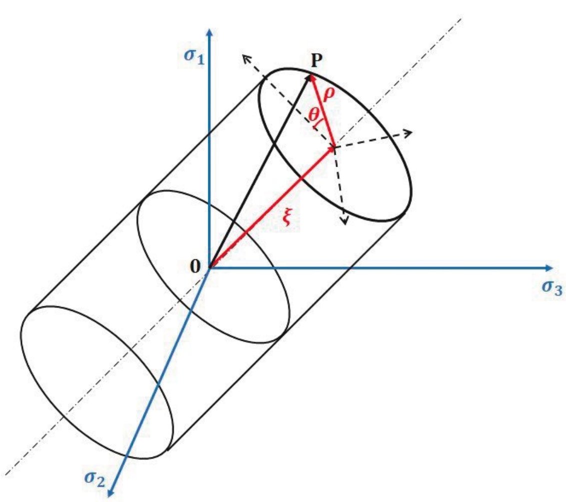


Figure 1. Geometric Representation of a Stress State in the Haigh–Westergaard and Principal Stresses Space (Recreated after Lian et al., 2013).

2. Theory

The failure criteria are visualized and expressed with principal stress space $f(\sigma_1,\sigma_2,\sigma_3)$, stress invariant $f(I_1,J_2,\theta)$, $f(J_1,J_2,J_3)$ or $f(I_1,I_2,I_3)$, or a version of 3D Haigh- Westergaard coordinates stress space (ξ, ρ, θ) which is a direct physical interpretation geometrically (Ottosen, 1977). The Geometric representation of a stress state in Haigh–Westergaard and principal stresses space is given in Figure 1. Haigh- Westergaard coordinates are widely used invariant in (ξ, ρ, θ) where ξ is the projection on the unit of the hydrostatic axis, ρ is the radial distance from a failure point P and θ is the similarity angle range of 0 to π/3 from the rotation of the axis.

$$\xi = \sqrt{3}\,\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}} \tag{1}$$

$$\rho = \sqrt{3}\tau_{oct} = \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}}{\sqrt{3}} \tag{2}$$

$$\theta = \frac{1}{3}\cos^{-1}\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{\frac{3}{2}}}\right) = \tan^{-1}\left[\sqrt{3}\frac{\sigma_2 - \sigma_3}{2\sigma 1 - \sigma_2 - \sigma_3}\right] \tag{3}$$

$$|OP| = \sqrt{\xi^2 + \rho^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \tag{4}$$

The ξ-ρ plane is also called the rendulic plane. Principal stress σ_1,σ_2, and σ_3 in the terms of the (ξ, ρ, θ) are given by Brannon et al., 2009.

$$\sigma_1 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(\cos\theta - \frac{\sin\theta}{\sqrt{3}}\right) \tag{5}$$

$$\sigma_2 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(\frac{2\ \sin\theta}{\sqrt{3}}\right) \tag{6}$$

$$\sigma_3 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(-\frac{\sin\theta}{\sqrt{3}} - \cos\theta\right) \tag{7}$$

Angle θ is the stress angle, and cos3θ is the Load parameter. When θ equals zero in compressive meridian and for θ is equal to 60° in tensile meridian

New Failure Criterion

In the conventional Mohr-Coulomb and Hoek-Brown criterion, the effect of the intermediate stress is neglected on the rock failure prediction. The experimental result of the triaxial compression failure test indicates the impact of the intermediate principal (σ_2) on the rock strength. Rock strength increases with the increment in the value. Mogi's (1971) failure criterion is based on the true triaxial testing data regarding the power law. That failure criterion best fits the actual triaxial experimental data by obeying the power law between normal octahedral stress and octahedral shear stress τ_{oct}.

The sine function, often denoted as sin(x), is a fundamental trigonometric function that relates an angle in a right triangle to the ratio of the length of the side opposite the angle to the length of the hypotenuse. However, the sine function can also be extended to real numbers beyond the scope of triangles. The plot of the sine function exhibits a smooth wave that oscillates for the value of the θ in the range of 0 to 2π. It has been observed from the experimental data that the non-linear relation between principal stresses is matched with the upper crest side oscillating sine function wave that is constructed in a curvilinear plotting range of 0 to π (Shvarts and van Helden, 2022). So, choosing this sine function for the regression analyses of principal stress ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) for actual triaxial compression test data with five material parameters gives the best fitting result compared to linear and second-degree polynomial regression. The new 3D non-linear criterion as trigonometric sine function between the principal stresses as proposed,

$$\sigma_1 = a \sin(b_1\sigma_2 + d_1) \sin(b_2\sigma_3 + d_2) \tag{8}$$

where σ_1,σ_2, and σ_3 are the principal stresses and a,b_1,b_2,d_1,and d_2 are material constants for the best fitting with the actual triaxial data point.

Relation of the a and UCS with adopting $\sigma_2 = 0$ and $\sigma_3 = 0$ as,

$$UCS = a \sin d_1 \sin d_2$$

The value of σ_1 must be positive for $\sigma_1 \geq \sigma_2 \geq \sigma_3$ compression failure in equation (8). For a positive value of σ_1, the value of $\sin(b_1\sigma_2 + d_1)$ and the value of $\sin(b_2\sigma_3 + d_3)$ must be positive, so the range of $(b_1\sigma_2 + d_1)$ and $(b_2\sigma_3 + d_3)$ must be between 0 to π. All the strength parameters are easy to derive by the triaxial compression test data for a minimum six-core sample.

Equation 8 is a proposed non-linear failure criterion directly derived from non-linear 3D regression analysis considering the effect of σ_2 on the rock strength. Equation 8 produces a concave surface on the principal stress space (σ_1,σ_2,σ_3).

Proposed non-linear failure equation in the Haigh- Westergaard coordinate is given by the function of (ξ,ρ,θ) as

$$\begin{aligned} f(\xi,\rho,\theta) = {} & \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(\cos\theta - \frac{\sin\theta}{\sqrt{3}}\right) \\ & - a \sin\left[b_1\frac{1}{\sqrt{3}}\xi + \frac{\rho b_1}{\sqrt{2}}\left(\frac{2\ \sin\theta}{\sqrt{3}}\right) + d_1\right] \sin\left[b_2\frac{1}{\sqrt{3}}\xi \right. \\ & \left. + \frac{\rho b_2}{\sqrt{2}}\left(-\frac{\sin\theta}{\sqrt{3}} - \cos\theta\right) + d_2\right] \end{aligned} \quad (9$$

This equation is complex to solve with the arithmetic method but easily solved by the numerical method. Newton Raphson's method is recommended to solve this equation with a quick convergent rate. Newton Raphson's method has a limitation in that f'(ξ,ρ,θ) is not zero. If that f'(ξ,ρ,θ), then it must be required to modify the Newton-Rapson method. Modifying code for the Newton-Rapson method is done by making a bridge for that range at f'(ξ,ρ,θ) nearly equal to zero or using another method like the bisection method.

Table 2. Regression Analysis for Ten Rock Types.

ROCK TYPE	Polynomial Degree 1	Polynomial degree 2	Trigonometric function (nonlinear 3D criterion)
KTB amphibolite	σ_1=0.529 σ_2+ 5.315 σ_3+293	σ_1=1.03σ_2+8.56σ_3-0.0013σ_2^2+0.0026σ_2 σ_3-0.0248σ_3^2+187.25	σ_1=1303*sin(0.002σ_2+0.7229)*sin(0.0080σ_3+0.269)
Dunham dolomite	σ_1=0.515 σ_2+ 2.743 σ_3+371	σ_1=1.27σ_2+3.55σ_3-0.0025σ_2^2+0.0059σ_2 σ_3-0.0140σ_3^2+294	σ_1=2193*sin(0.0026σ_2+0.703)*sin(0.0016σ_3+0.2285)
Solnhofen limestone	σ_1=0.294 σ_2+ 2.470 σ_3+346	σ_1=0.509σ_2+2.67σ_3-0.0013σ_2^2+0.0068σ_3-0.0136σ_3^2+331	σ_1=7700*sin(0.0025σ_2+0.8253)*sin(0.0026σ_3+0.059)
Shirahama sandstone	σ_1=0.156 σ_2+ 4.141 σ_3+86.53	σ_1=0.37σ_2+6.10σ_3-0.0052σ_2^2+0.026σ_2 σ_3-0.083σ_3^2+66.35	σ_1=299.3*sin(0.00754σ_2+0.9390)*sin(0.02235σ_3+0.2675)
Yamaguchi marble	σ_1=0.428 σ_2+ 3.582 σ_3+114	σ_1=1.145σ_2+2.35σ_3-0.0055σ_2^2+0.0174σ_2 σ_3-0.0113σ_3^2+106.02	σ_1=11080*sin(0.004σ_2+0.6386)*sin(0.0003σ_3+0.0155)
Mizuho trachyte	σ_1=0.229 σ_2+ 2.789 σ_3+165	σ_1=0.496σ_2+4.20σ_3-0.0014σ_2^2+0.00426σ_2 σ_3-0.018σ_3^2+121.30	σ_1=8884*sin(0.0028σ_2+0.8190)*sin(0.0003σ_3+0.0273)
Manazuru andesite	σ_1=0.551 σ_2+ 5.691 σ_3+276	σ_1=1.29σ_2+11.35σ_3-0.0034σ_2^2+0.0087σ_2 σ_3-0.0818σ_3^2+162.82	σ_1=956.1*sin(0.0039σ_2+0.7607)*sin(0.0088σ_3+0.4191)
Inada granite	σ_1=0.965 σ_2+ 3.796 σ_3+461	σ_1=1.97σ_2+5.44σ_3-0.0057σ_2^2+0.0112σ_2 σ_3-0.0168σ_3^2+358.78	σ_1=1901*sin(0.0023σ_2+0.647)*sin(0.0030σ_3+0.3562)
Orikabe monzonite	σ_1=0.92σ_2+ 2.732 σ_3+458	σ_1=1.89σ_2+4.59σ_3-0.0052σ_2^2+0.0129σ_2 σ_3-0.0207σ_3^2+329.43	σ_1=1572*sin(0.0029σ_2+0.5776)*sin(0.0034σ_3+0.3702)
Coconino sandstone	σ_1=0.073σ_2+3.132 σ_3+161	σ_1=0.201σ_2+0.516σ_3-0.00104σ_2^2+0.0046σ_2 σ_3-0.0202σ_3^2+113.40	σ_1=663.2*sin(0.0028σ_2+0.9803)*sin(0.00675σ_3+0.2436)

Equation 8 and Equation 9 represent failure criterion in the coordinate of (σ_1,σ_2,σ_3) and (ξ, ρ, θ), respectively. Till date, no such equations are available in the literature to predict the rock failure as Equation 8 and Equation 9.

Usually, in-situ condition rock break-out done by the compression and rock fracture is due to the combination of compression and tension (Bui et al., 2023; Ghassemi, 2017; Lee et al., 2012). Experimental data on true triaxial compression tests are available in the literature review. Estimating the strength parameters a,b_1,b_2,d_1,and d_2 is done by two methods: the non-linear regression analysis on the triaxial compression test data result and the analytical solution of the new failure criterion by the triaxial compression test data. Non-linear three-dimensional regression analysis was done to find the material parameters (a,b_1,b_2,d_1,and d_2), as shown in Table 2. The strength parameter is dependent on the rock property. The failure equation for each rock type is shown in Table 2. This equation considers three principal stresses in relation to the function of sine

Linear Multivariable Regression

Linear multivariable regression analysis is conducted on all these ten-rock types to find the linear criterion that considers all three principal stresses in the linear relation. Linear regression analyses give the linear relation for the KBT amphibolite as

$$\sigma_1 = p_1\sigma_2 + p_2\sigma_3 + c_1 \tag{10}$$

Where,
p_1 = co-efferent of the σ_2 =0.529
p_2 = co-efferent of the σ_2 =5.315
c_1 = intersection=293 MPa

The value p_1, p_2,and c_1 is dependent on the formation rock property that is given as the weighting of the intermediate principal stress, minimum principal stress on the strength of the formation, and compressive strength of the formation, respectively. The value of the maximum principal stress is calculated based on the above equation as a combination of the value at the yield point. The yield point is the point where the first crack is initiated. The equation for all ten-rock types based on the linear regression with a variable is given in Table 2.

Second Degree Polynomial Regression

Polynomial regression analysis on these ten-rock types for the failure in true-triaxial compression test is done the finding appropriated relationships between these three principal stresses. Polynomial regression analysis on the data of KBT amphibolite's triaxial compressive test is given as,

$$\sigma_1 = q_1\sigma_2 + q_2\sigma_3 - q_3\sigma_2^2 + q_4\,\sigma_2\sigma_3 - q_5\sigma_3^2 + c_2 \tag{11}$$

Where,
q_1 = co-efferent of the σ_2 = 1.03,
q_2 = co-efferent of the σ_3 = 8.56,
q_3=co-efferent of the σ_2^2 = 0.0013,
q_4 = co-efferent of the $\sigma_2\,\sigma_3$ = 0.0026,
q_5 = co-efferent of the σ_3^2 = 0.0248,
c_2 = intersection =187.25 MPa

The value of q_1, q_2, q_3, q_4, q_5, and c_2 is determined by the formation rock property, which is defined as the weighting of the σ_2, σ_3, σ_2^2, σ_2, σ_3, σ_3^2, and compressive strength of the formation, respectively. The maximum principal stress is calculated using the above equation as a combination of the σ_2 and σ_3 values at the first crack begin. Table 2 shows the equation for all ten rock types based on linear regression with variables.

Priest Criterion

Priest (2005; 2009) developed a three-dimensional failure criterion by combining the three-dimensional Drucker-Prager criterion (Drucker and Prager, 1952) and the two-dimensional Hoek–Brown criterion. Three-dimensional effective failure stress ($\sigma_1 f$) is estimated by

$$\sigma_{1f} = \sigma_{1hb} + 2\sigma_{3hb} - (\sigma_2 + \sigma_3) \tag{12}$$

Where, σ_{3hb} is the minimum two-dimensional Hoek–Brown effective stress at failure, and σ_{1hb} is the maximum two-dimensional Hoek–Brown effective stress at failure given by

$$\sigma_{3hb} = w\sigma_2 + (1 - w)\,\sigma_3 \tag{13}$$

σ_{1hb} is calculated by Hoek–Brown criterion (1997)

$$\sigma_{1hb} = \sigma_{3hb} + \sigma_{ci}\left\{\frac{m_b\sigma_{3hb}}{\sigma_{ci}} + s\right\}^a \tag{14}$$

Where, σ_{1hb} calculated by Hoek-Brown criterion (1997), w is a weighting factor, m_b is the Hoek–Brown constant value, and **s** is a fracture parameter of the rock mass.

Drucker- Prager Criterion

The Drucker-Prager failure criterion is a curvilinear 3D generalized form of the Mohr-Coulomb criterion for soils. Drucker-Prager criterion is the relation between octahedral stress and octahedral shear stress τ_{oct} through material constant.

Drucker-Prager Criterion is given by the equation (Drucker and Prager, 1952)

$$\tau_{oct} = \sqrt{\frac{2}{3}}(3\lambda\sigma_{oct} + k) \tag{15}$$

λ and k are material constants. Drucker-Prager Criterion in Haigh-Westergaard coordinates. (ξ, ρ, θ) is given by equation.

$$\rho = \sqrt{2}(\sqrt{3}\lambda\xi + k) \tag{16}$$

λ and k can be expressed in terms of cohesion intercept and internal friction angle (Yi et al., 2005)

$$\lambda = \frac{2\sin\phi}{\sqrt{3}\,(3 - \sin\phi)};\quad k = \frac{6c\cos\phi}{\sqrt{3}\,(3 - \sin\phi)} \tag{17}$$

Mogi-Coulomb Criterion

Al-Ajami and Zimmerman (2005) proposed a failure criterion by considering the Mogi criterion (1971) and linear Mohr-coulomb failure criterion called the Mogi-Coulomb criterion. The Mogi-Coulomb criterion is the linear relation between octahedral stress (τ_{oct}) and mean effective stress (σ_{m2}).

$$\tau_{oct} = a + b * \sigma_{m2} \tag{18}$$

Here,

$$\tau_{oct} = \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}}{3} \text{ and } \sigma_{m2} = \frac{\sigma_1 + \sigma_3}{2}$$

Material constant a and b is simply related to the angle of internal friction (Ø), and cohesion (c) is given by

$$a = \frac{2\sqrt{2}}{3}\,c\,\cos\emptyset\,;$$

$$b = \frac{2\sqrt{2}}{3}$$

Mogi-coulomb criterion in Haigh-Westergaard coordinates (ξ, ρ, θ) is given as.

$$\rho = \sqrt{3}\,a + \frac{b}{2}(3\,\xi - \sqrt{3}\,b) \tag{19}$$

3. Result and Discussion

Failure Prediction and Validation on Principal Stress Space (σ_1, σ_2, σ_3)

The root means square error (RMSE) function was used to find the misfits between the triaxial test data and predicted strength by the failure criterion. The standard deviation of the residuals defines RMSE.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} r_i^2}, \text{i=1,2,...,n} \tag{20}$$

Where n is the number of the test data point of a specific rock.
Residuals or prediction errors are defined by

$$r_i = \sigma_i^{calc} - \sigma_i^{test} \tag{21}$$

σ_i^{calc} is i^{th} calculated value by the failure criterion, and σ_i^{test} is the i-th tested data. The best fitting RMSE value is the minimum, and if prediction is the best fit of test data, the mean of the residuals is always equal to zero.

Figure 2 indicates the prediction of rock strength by the proposed criterion on ten rock types with a triaxial compression test. Scatter data indicate triaxial compression test data, and solid lines represent the data prediction by a single equation for all rock types shown in Table 2.

In Figure 2, the black dotted line is for $\sigma_1 = \sigma_2 > \sigma_3$ or $\sigma_1 > \sigma_2 = \sigma_3$; color solid is the proposed failure criterion, and scatter data is the experimental triaxial compression data point. The single-color solid line is for the fixed value of the minimum principal stress. For the shirahama sandstone, the RMSE value is minimum among all the rock types,, so we will discuss the shirahama sandstone shown in Figure 2. Color indication for a solid line when the value of σ_3 equal to 5 MPa, 8 MPa, 15 MPa, 20 MPa, 30 MPa, and 40MPa is gradually blue, red, forest green, purple, maroon, and lime. All solid line is plotted by a single equation given in Table 2. The regression analysis estimates parameters a = 299.3, b_1=0.00754, d_1=0.9390, b_2=0.02235, and d_2=0.2695. It has been observed that the parameter is independent of the principal stresses.

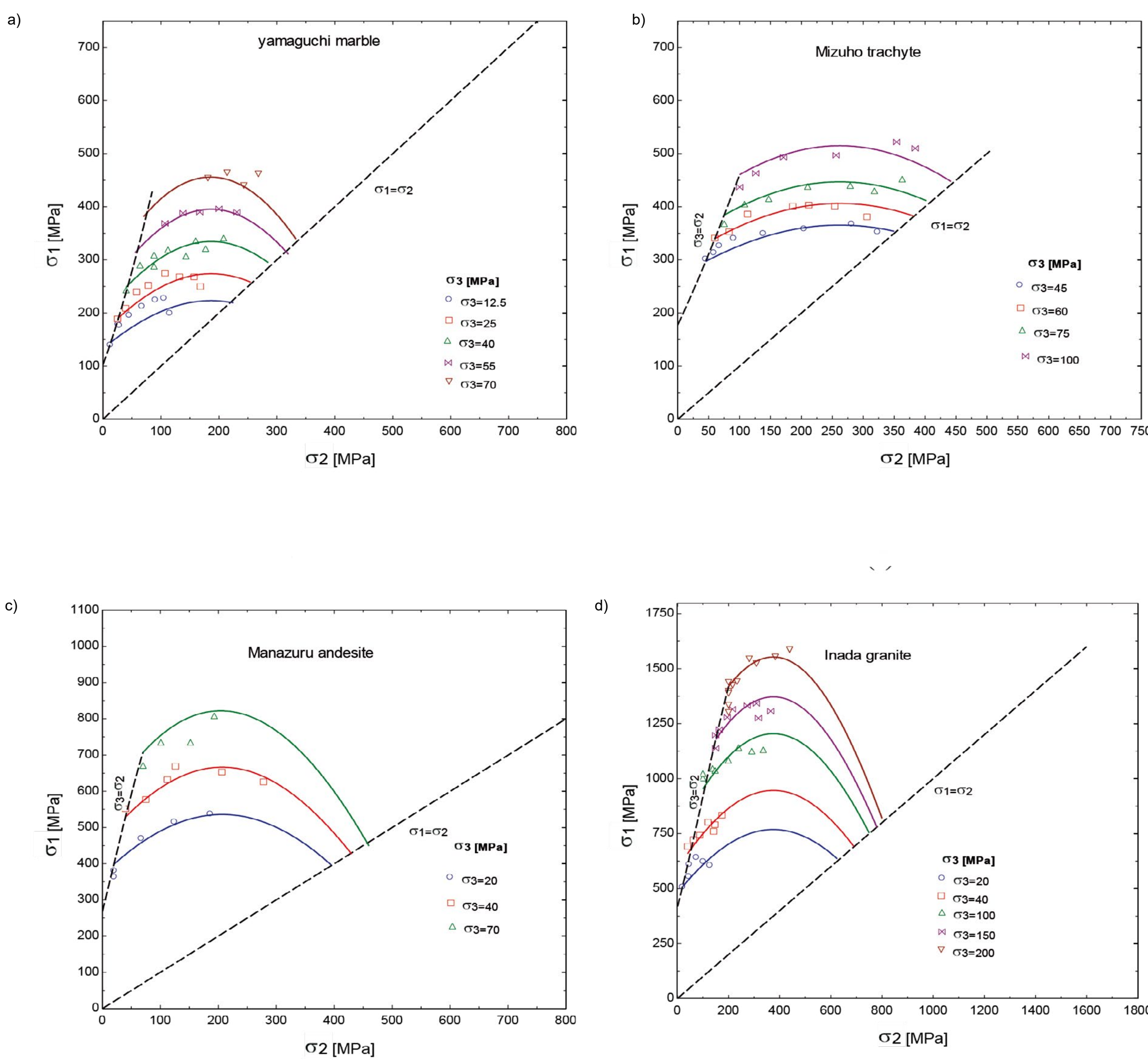


Figure 2. Failure Prediction by Proposed Criterion in Space for Ten Rock Type

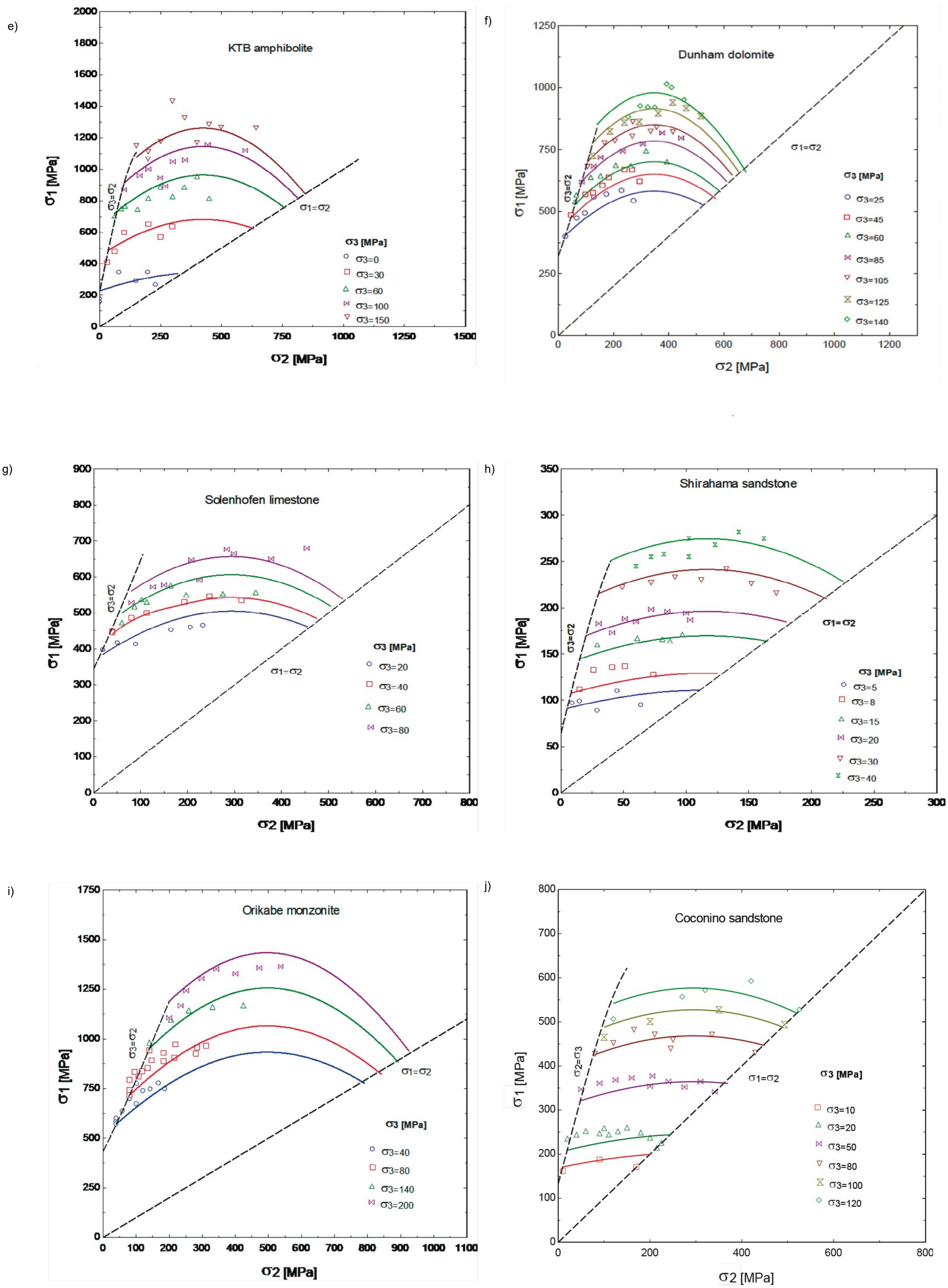

e)
KTB amphibolite
σ1 [MPa]
σ2 [MPa]
σ3=σ2
σ1=σ2
σ3 [MPa]
σ3=0
σ3=30
σ3=60
σ3=100
σ3=150
f)
Dunham dolomite
σ1 [MPa]
σ2 [MPa]
σ3=σ2
σ1=σ2
σ3 [MPa]
σ3=25
σ3=45
σ3=60
σ3=85
σ3=105
σ3=125
σ3=140
g)
Solenhofen limestone
σ1 [MPa]
σ2 [MPa]
σ3=σ2
σ1=σ2
σ3 [MPa]
σ3=20
σ3=40
σ3=60
σ3=80
h)
Shirahama sandstone
σ1 [MPa]
σ2 [MPa]
σ3=σ2
σ1=σ2
σ3 [MPa]
σ3=5
σ3=8
σ3=15
σ3=20
σ3=30
σ3=40
i)
Orikabe monzonite
σ1 [MPa]
σ2 [MPa]
σ3=σ2
σ1=σ2
σ3 [MPa]
σ3=40
σ3=80
σ3=140
σ3=200
j)
Coconino sandstone
σ1 [MPa]
σ2 [MPa]
σ2=σ3
σ1=σ2
σ3 [MPa]
σ3=10
σ3=20
σ3=50
σ3=80
σ3=100
σ3=120

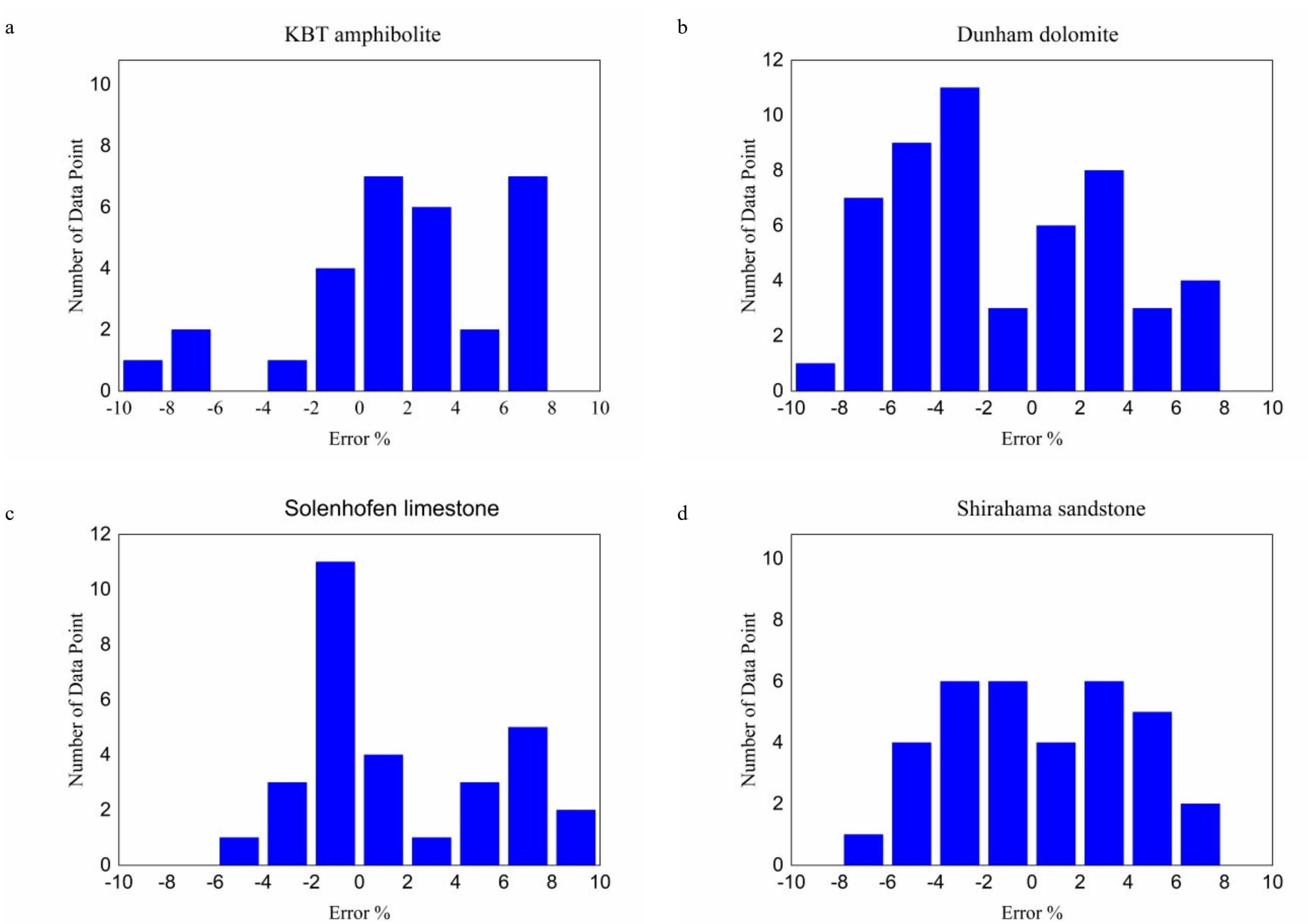
Table 3 shows the RMSE values of σ_1 for shirahama sandstone is a minimum of nearly 9.08 MPa and for the KTB Amphibolite is a maximum of nearly 65.03 MPa for the best fitting triaxial data point. RMSE values of σ_1 are between 9.08 MPa to 65.03 MPa for the rest of all rock types shown in Table 3. Figure 3 shows the distribution of the error percentage for predicted σ_1 for all the rock types. The equation defines the prediction error,

$$\varepsilon_i = \frac{clcuated\ data - tested\ data}{tested\ data} * 100\% \tag{22}$$

Or in terms of the σ_1

$$\varepsilon_i = \frac{\sigma_{1i}^{calc} - \sigma_{1i}^{test}}{\sigma_{i}^{test}} * 100\% \tag{23}$$

σ_{1i}^{calc} is the ith value of predicted σ_1 from the triaxial data point. σ_{1i}^{test} is the tested value of σ_1 on the triaxial data. The predicted error distribution for all the rock types is as a histogram shown in Figure 3. The distribution of data points is excellent and distributed equally in the range of -10 % to 10%, and the maximum number of data points distributed is near zero percentage error. Like Gaussian distribution, this type of indicator with a maximum number of data having an error percentage around zero is preferable. Error distribution for the shirahama sandstone, yamaguchi marble, inada granite, orikabe monzonite, manazuru andesite, and coconino sandstone is distributed assuredly as a Gaussian distribution, so it suggests that the new failure criterion works better for that rock types. Error distribution for the remaining rock types is also good, with a maximum error percentage near zero, but the distribution shape is not like a bell curve, shown in Figure 3.



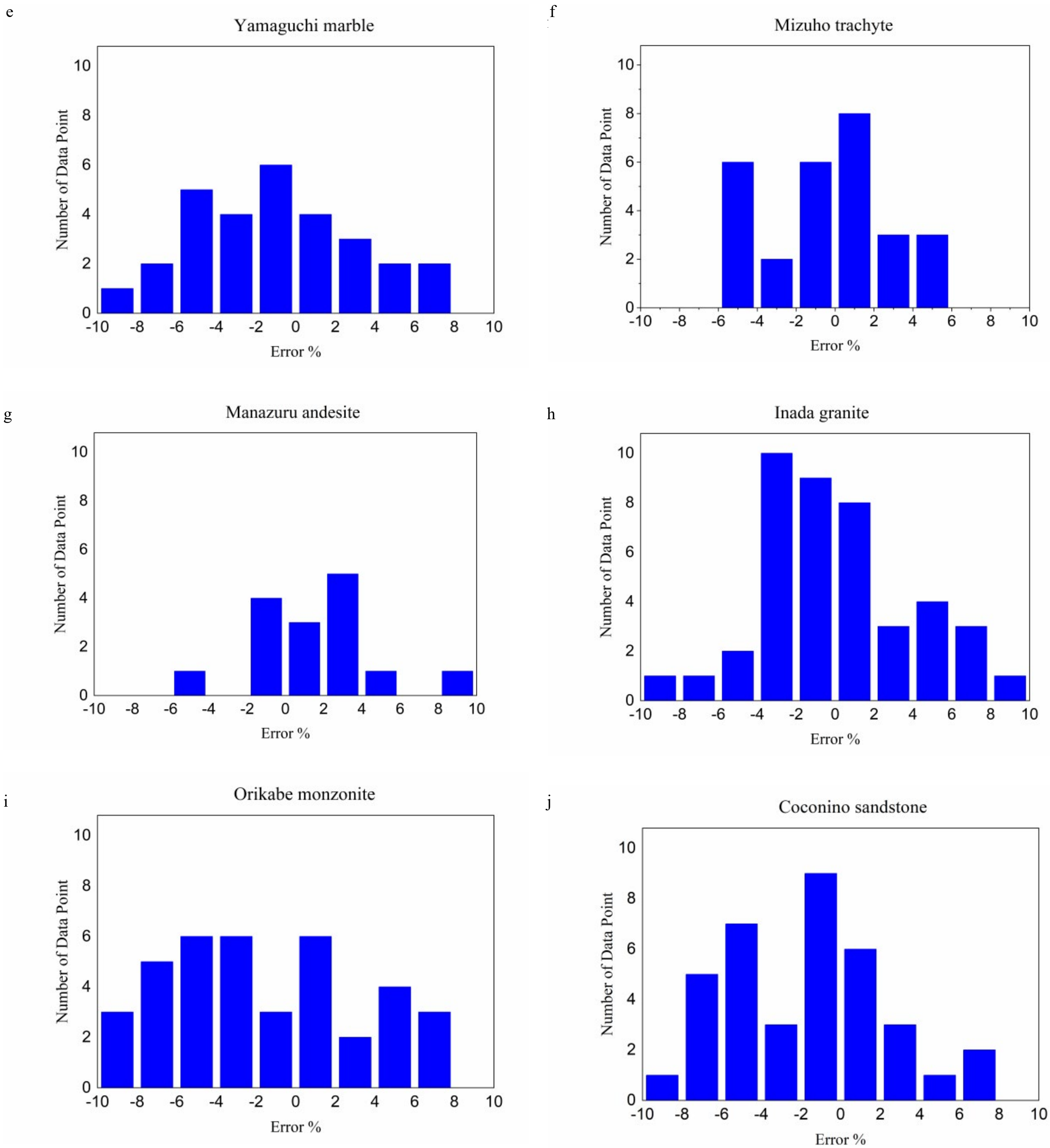


Figure 3 Error Percentage Distribution as Histogram for Ten Rock Types.

Validation on Haigh- Westergaard Coordinates Stress Space (ξ, ρ, θ)

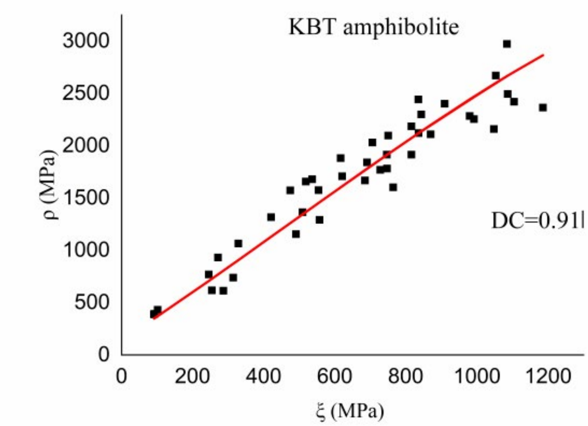
The proposed failure criterion is validated with the help of Haigh-Westergaard coordinates stress space (ξ, ρ, θ). The non-linear failure criterion generates a curve in stress space (ξ, ρ, θ), so validation can be done with the coefficient of determination (DC). To validate the proposed non-linear failure criterion here, we compare it with experimental triaxial test data of ten rock types, as shown in Figure 4.

Equation 9 is the failure criterion in Haigh-Westergaard coordinates. (ξ, ρ, θ). Here ρ is a dependent variable, ξ is an independent variable, and θ depends on rock type. Figure 4 compares the proposed 3D failure criterion with triaxial compression test data. Scatter data on the plot indicate the triaxial test data, where the solid red line indicates the failure criterion by the equation (9). Validation of the proposed 3D failure criterion is done with the help of DC with the reliability prediction of by the failure criterion.

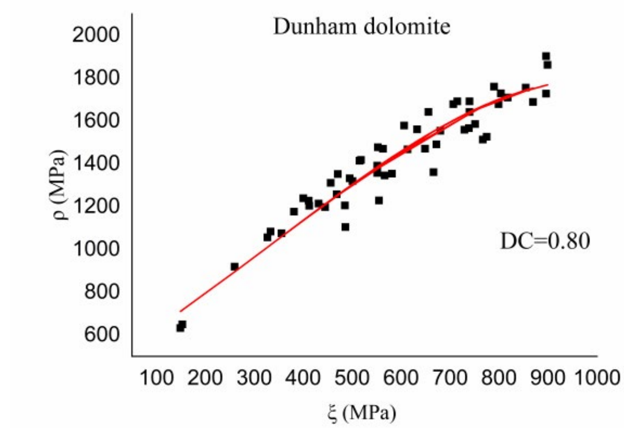
$$DC = 1 - \frac{\sum_{i=1}^{n}(\rho_i^{calc} - \rho_i^{test})^2}{\sum_{i=1}^{n}(\rho_i^{test} - \bar{\rho}_{test})^2} \tag{24}$$

Where ρ_i^{calc} i-th calculated value data, ρ_i^{test} is i-th tested data on the triaxial test, ρ_i^{test} is the mean value of tested data by the triaxial test. DC=1 means it is an ideal case where test data agree with the zero misfits. The desirable value of the DC must be near one. Desirable values of DC are for Inada granite (DC=0.95), Orikabe monzonite (DC=0.94), and KTB amphibolite (DC=0.91). DC for the rest of the rock types (dunham dolomite, yamaguchi marble, mizuho trachyte, solnhofen limestone, shirahama sandstone, manazuru andesite, coconino sandstone) in the range of 0.7 < DC < 0.9 shown in Figure 4. This DC value near one suggests that the proposed non-linear failure criterion shows better results in Haigh-Westergaard coordinates.

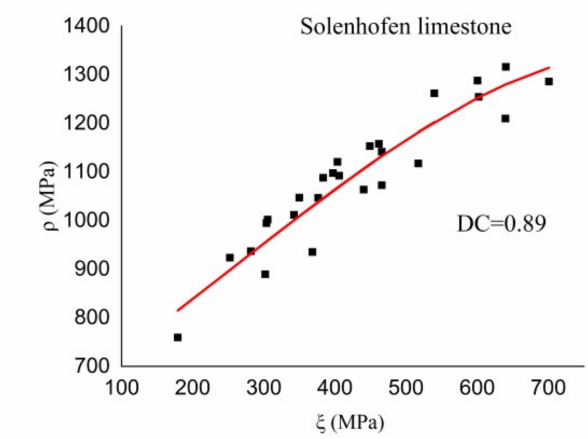
a



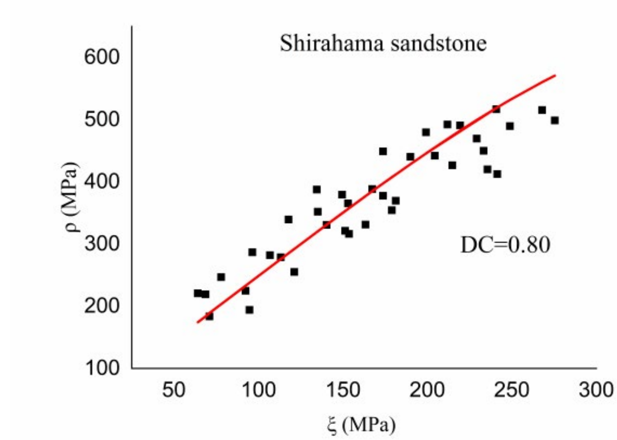
b



c



d



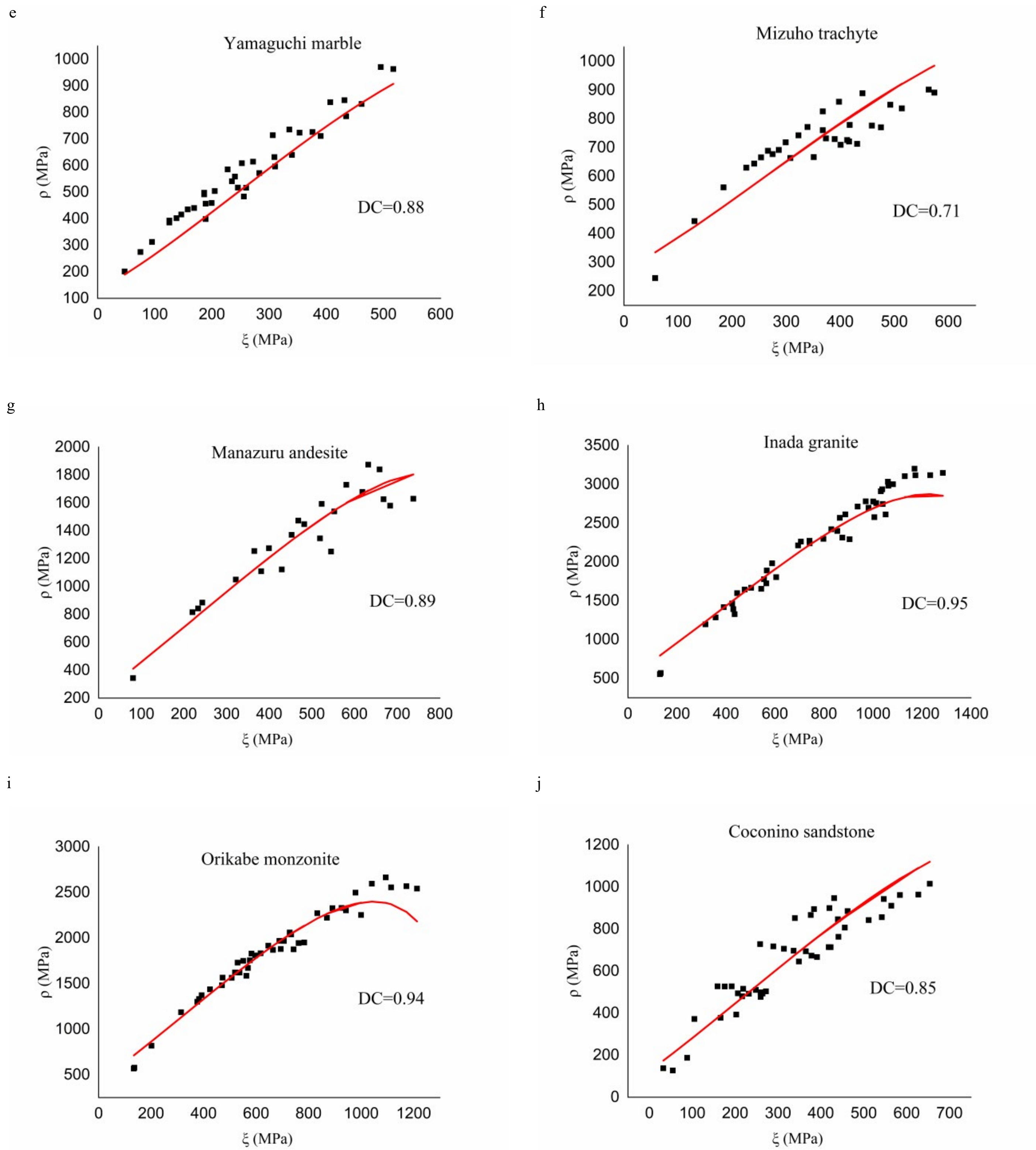


Figure 4. Validation of The New 3D Failure Criterion for the Ten Rock Types

Sensitivity Analysis on Variables of New 3D Criterion

Values of a, b_1, b_2, d_1, and d_2 are selected from non-linear regression analysis as shown in Table 2. There is a minimum RMSE value of σ_1 for all the optimum values of parameters. The influence of a single parameter on the value of RMSE(MPa) for σ_1 is discussed in this section. For the shirahama sandstone, values of parameter-driven from the regression are a = 299.3, b_1 = 0.0075, b_2 = 0.9390, d_1 = 0.02235,and d_2 = 0.2675. If the value of one parameter a change from 200 to 320 for shirahama sandstone, it has been observed that with no changes on the remaining parameter (b_1,b_2, d_1, and d_2) shown in Figure 5 (i).

In Figure 5 (i), the RMSE decreases from 66.83 MPa to 14.36 MPa when it increases from 200 to 300 and then increases again to 19.70 MPa when a value is more significant than 320. Hence, the best-fit value for the shirahama sandstone a is 300 with the minimum RMSE value (14.36 MPa). Figures 5(ii), 5(iii), 5(iv), and 5(v) show the influence of the parameter (b_1, b_2, d_1, and d_2) gradually on RMSE (MPa) of σ_1 when remain parameters are static and given in Table 2 for the shirahama sandstone. Hence best-fit value for b_1 is 0.0055 with minimum RMSE (9.33 MPa); is d_1 0.8 minimum RMSE (11.63MPa); b_2 is 0.0225 minimum RMSE (14.36 MPa), and d_2 is 0.27 minimum RMSE (14.35MPa) exclusively. Best fitting value of parameters a, b_1,b_2, d_1, and d_2 with minimum RMSE given by optimization methods like gradient descent and stochastic gradient descent in machine learning and deep learning.

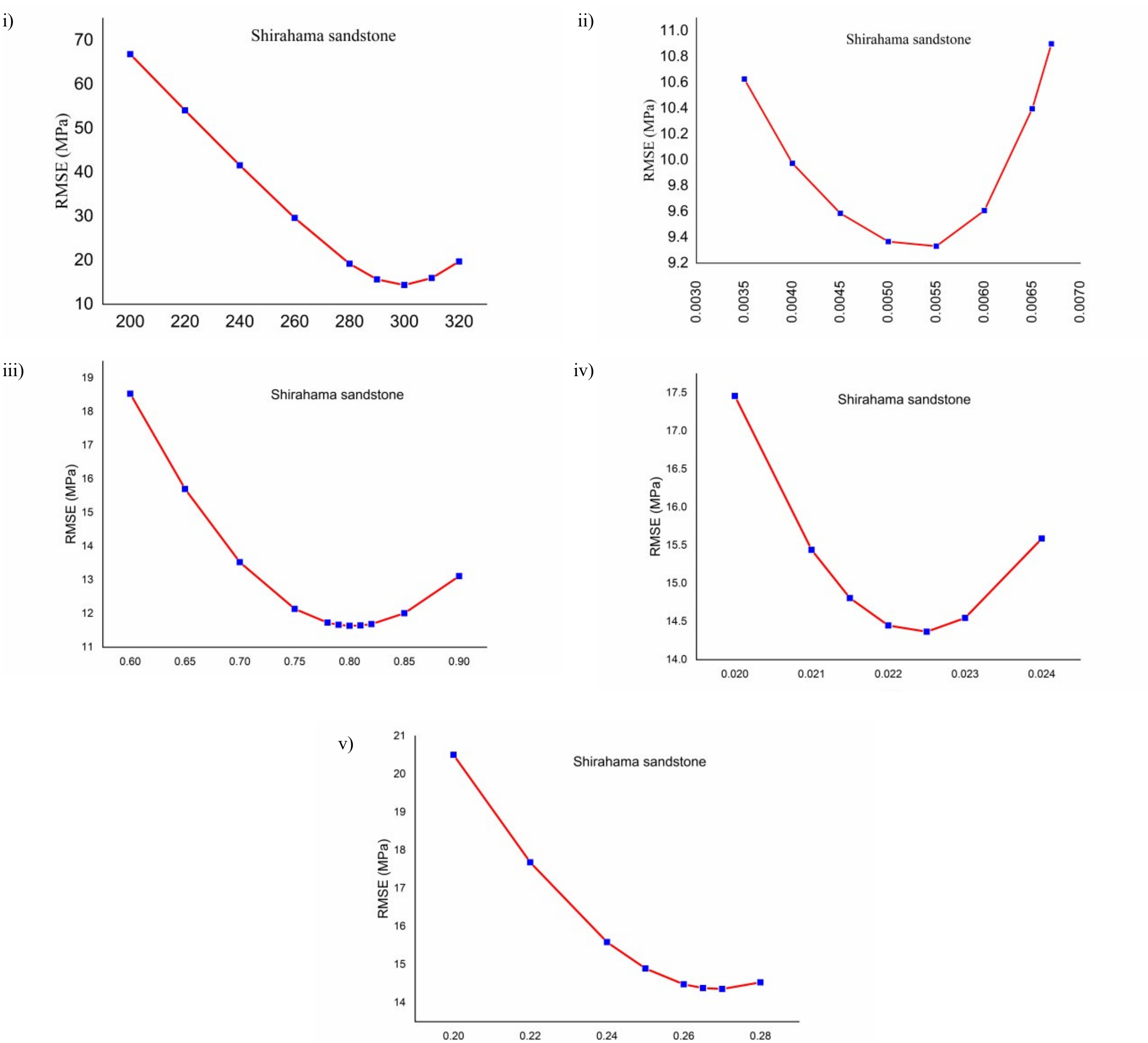


Figure 5. Sensitivity Analysis of Parameters a, b_1,b_2, d_1, and d_2 of New Non-linear Trigonometric Criterion for Failure Prediction in RMSE.

Comparisons of the New 3D Non-linear Criterion with Linear Regression, Polynomial Regression, and Existing Criteria.

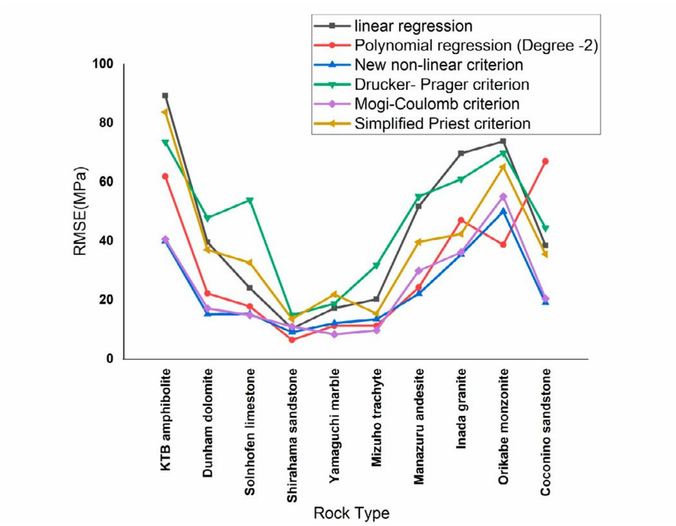


Figure 6. Comparisons of RMSE for by New 3D Non-linear Failure Criterion with Linear, Second-degree Polynomial, and Existing Criteria: Dracker-Prager, Mogi-Coulomb, and Simplified Priest Criterion.

The prediction of principal stress (σ_1) is made with the help of the combination of another two principal stresses (σ_2 and σ_3) by the failure criterion discussed above. Maximum principal stress at the failure using the New non-linear criterion given in Figure 2 and by Priest Criterion is given as shown in APPENDIX I. Maximum principals stress is not directly driven by the Drucker-Prager criterion and Mogi-Coulomb criterion. The equation for the Drucker-Prager and Mogi-Coulomb criterion is difficult to determine the maximum principal stress as a dual result of the quadratic equation of the principal stresses. The solution of the Drucker- Prager and Mogi-Coulomb criterion is done with the help of the Haigh-Westergaard coordinates (ξ, ρ, θ) and then converted into the principal stresses for the comparison of the all-failure criterion which was discussed above. APPENDIX II and APPENDIX III gradually show the Drucker- Prager and Mogi-Coulomb criterion result. Comparisons of the RMSE value with linear regression and second-degree polynomial regression and with existing failure criteria are given in Table 3 and Figure 6 for all rock types. RMSE in MPa for the linear regression, the Drucker-Prager criterion, and the Priest criterion are higher for all the rock types than the New non-linear criteria, Mogi-Coulomb criterion, and polynomial equation. RMSE by New non-linear criteria, second-degree polynomial, and Mogi-Coulomb criteria gave the lowest value among the ten-rock type. Seven rocks have lower RMSE by the New non-linear criterion than RMSE by the Mogi-Coulomb criterion, as shown in Figure 6. The minimum RMSE value by Mogi-Coulomb is 8.36 MPa for yamaguchi marble, and the maximum is 55.51 MPa for orikabe monzonite. The maximum RMSE by the New non-linear criterion is 50.04 for orikabe monzonite, and the minimum is 9.08 for shirahama sandstone.

Table 3. Comparison of New 3D Non-linear Criterion with Linear, Polynomial Criterion, and Existing Failure Criterion for RMSE of σ1 (MPa)

Rock Type	Number of Data Points	RMSE (Linear criterion)	RMSE (Polynomial criterion -degree 2)	RMSE (New non-linear criterion)	RMSE (Drucker-Prager criterion)	RMSE (Mogi-Coulomb criterion)	RMSE (Simplified Priest criterion)
KTB amphibolite	42	89.31	61.94	40.03	73.63	40.64	83.76
Dunham dolomite	54	39.62	22.29	15.16	47.88	17.22	37.03
Solnhofen limestone	26	24.10	17.83	15.32	53.93	14.92	32.67
Shirahama sandstone	38	10.39	6.49	9.08	14.89	11.06	13.58
Yamaguchi marble	38	17.26	11.31	12.15	18.86	8.36	21.96
Mizuho trachyte	31	20.27	11.31	13.53	31.84	9.74	15.47
Manazuru andesite	23	51.73	24.31	22.14	55.10	29.98	39.7
Inada granite	46	69.79	47.06	35.47	61.04	36.16	42.46
Orikabe monzonite	44	73.86	38.75	50.04	69.94	55.15	65.16
Coconino sandstone	48	38.50	67.05	19.047	44.50	20.51	35.57

4. Summary and Conclusions

This study develops three criteria based on the true-triaxial compressive test data as a relationship of intermediates principal stresses. These criteria are developed by considering intermediate principal stress for the prediction of failure of rock in the compressive nature. The first criterion is developed as a trigonometric function of stresses as discussed, the second one is the linear relationship of stresses, and the third is the second-degree polynomial relationship among principal stresses. These trigonometric non-linear and second-degree polynomial criteria develop a concave surface on 3D space of (σ_1, σ_2, σ_3) whenever linear criterion develops plane surface. These 3D failure criteria are not reported in the previous literature.

Failure criterion for linear and polynomial regression (second-degree) has four and six material parameters, respectively. Rock failure prediction by linear criterion has poor results compared to exiting criterion as regression is poor fitting with triaxial compression test data. RMSE value for the failure prediction is improved by second-degree regression criterion with the best fitting with actual data for shirahama sandstone, yamaguchi marble, mizuho trachyte, and orikabe monzonite considering six material parameters. RMSE for some rock types like KTB amphibolite (61.94 MPa RMSE), inada granite (47.06 MPa RMSE), and coconino sandstone (67.05 MPa RMSE) is higher due to the overfitting of data with six material parameters. Linear criterion is under fitted on data, and second-degree polynomial is overfitted on data, so best fitting on data is done with new non-linear data with five material parameters.

The material parameters in the proposed 3D failure criteria are obtained from the linear and non-linear regression analysis of conventional triaxial compression test data. A minimum of five to six triaxial testing data are required for regression analysis. The number of data points and their value affects the accuracy of regression analysis. The quality of the data point and the quantity of data used for the regression analysis play an important role in accuracy. The proposed new 3D failure criterion as a trigonometric function is validated with triaxial tested data points for the ten rock types. Comparing these new 3D failure criteria is done with existing criteria like the Priest criterion, the Drucker-Prager criterion, and the Mogi-Coulomb criterion. It has been shown that the New 3D non-linear criterion and Mogi-Coulomb performance gave better results among all these criteria. The RMSE by the new 3D non-linear criterion is lower than the Mogi-Coulomb criterion for seven rock types: KTB amphibolite - 40.03 MPa, Dunham dolomite - 15.16 MPa, shirahama sandstone - 9.08 MPa, manazuru andesite - 22.14 MPa, inada granite- 35.47, coconino sandstone - 19.047 MPa. Mogi-Coulomb has the best RMSE for solnhofen limestone - 14.92 MPa, yamaguchi marble - 8.36 MPa, and Mizuho trachyte- 13.53 MPa. The desirable DC values are close to one in certain rock types (0.95 for inada granite, 0.94 for orikabe monzonite, and 0.91 for KTB amphibolite), while others fall within the range of 0.7 to 0.9. The proposed non-linear failure criterion performs well in Haigh-Westergaard coordinates, effectively predicting the failure behavior of different rocks.

The Mogi-Coulomb criterion is related to octahedral shear stress and means effective stress that becomes difficult to solve. The Mogi-Coulomb solution gave dual results, making it challenging to choose one for the best prediction of principal stress. The number of parameters affecting the accuracy of the failure prediction. This new non-linear criterion considers five parameters that may reduce the accuracy. So, selecting the proper value of these parameters is challenging as per sensitivity analysis. However, sensitivity analysis suggests that all the parameters simultaneously influence the RMSE value. A limitation of this new 3D failure criterion is that the input parameters must be optimized initially to achieve higher accuracy. There must be a minimum RMSE for the best fitting of the parameters. Prediction should further improve on optimizing all five parameters, that is, concerning trigonometric function with machine learning tools. However, before applying the new failure criterion, significant research is required on calibrating constants with triaxial compression test results as material parameter is changed with rock type. It is advisable in further research to develop a criterion that contains a minimum material parameter that is not dependent on rock types. Although new 3D non-linear and Mogi-Coulomb criteria gave the best result, 3D non-linear criteria are recommended for simplicity for the solution of the equation. A linear criterion is the simplified extended Mohr-Coulomb criterion as linear relationship principles stress whenever a second-degree polynomial criterion adds complexity with overfitting on the data. The non-linear 3D criterion is further helpful in applying wellbore stability and sand production prediction with derived variables from the triaxial compression test on the in-situ rock.

References

Aadnoy, B., & Looyeh, R. (2019). *Petroleum rock mechanics: drilling operations and well design*. Gulf Professional Publishing.

Al-Ajmi, A. M., & Zimmerman, R. W. (2005). Relation between the Mogi and the Coulomb failure criteria. *International Journal of Rock Mechanics and Mining Sciences*, *42*(3), pp.431-439 doi.org/10.1016/j.ijrmms.2004.11.004

Bou-Hamdan, K. F. (2022). A 3D semianalytical model for simulating the proppant stresses and embedment in fractured reservoir rocks. *Journal of Porous Media*, *25*(6). doi.org 10.1615/JPorMedia.2022041752

Brannon, R. M., Fossum, A. F., & Strack, O. E. (2009). *KAYENTA: theory and user's guide* (No. SAND2009-2282). Sandia National Laboratories (SNL), Albuquerque, NM, and Livermore, CA (United States). doi.org/10.2172/984159

Bui, D., Nguyen, T., Nguyen, T., & Yoo, H. (2023). Formation damage simulation of a multi-fractured horizontal well in a tight gas/shale oil formation. *Journal of Petroleum Exploration and Production Technology*, 13(1), 163–184. https://doi.org/10.1007/S13202-022-01544-8/TABLES/6

Chang, C., & Haimson, B. (2000). True triaxial strength and deformability of the German Continental Deep Drilling Program (KTB) deep hole amphibolite. *Journal of Geophysical Research: Solid Earth*, *105*(B8), 18999-19013. doi.org/10.1029/2000JB900184

Colmenares, L. B., & Zoback, M. D. (2002). A statistical evaluation of intact rock failure criteria constrained by polyaxial test data for five different rocks. *International Journal of Rock Mechanics and Mining Sciences*, *39*(6), 695-729. doi.org/10.1016/S1365-1609(02)00048-5

Culshaw, M. G. (2015). The ISRM suggested methods for rock characterization, testing and monitoring: 2007–2014. In: Ulusay, R (ed.). *Bulletin of Engineering Geology and the Environment*, *74*(4), 1499-1500. DOI: 10.1007/978-3-319-007713-0

Drucker, D. C., & Prager, W. (1952). Soil mechanics and plastic analysis or limit design. *Quarterly of applied mathematics*, *10*(2), 157-165. https://www.jstor.org/stable/43633942

Ewy, R. T. (1999). Wellbore-stability predictions by use of a modified Lade criterion. *SPE Drilling & Completion*, *14*(02), 85-91. doi.org/10.2118/56862-PA

Ghassemi, A. (2017). Application of rock failure simulation in design optimization of the hydraulic fracturing. *Porous Rock Fracture Mechanics,* 3-23. https://doi.org/10.1016/B978-0-08-100781-5.00001-4

Haimson, B., & Chang, C. (2000). A new true triaxial cell for testing mechanical properties of rock, and its use to determine rock strength and deformability of Westerly granite. *International Journal of Rock Mechanics and Mining Sciences*, *37*(1-2), 285-296. doi.org/10.1016/S1365-1609(99)00106-9

Hoek, E., & Brown, E.T. (1997). Practical estimates of rock mass strength. *International journal of rock mechanics and mining sciences*, *34*(8), 1165-1186. doi.org/10.1016/S1365-1609(97)80069-X

Hoek, E., Carranza-Torres, C., & Corkum, B. (2002). Hoek-Brown failure criterion-2002 edition. *Proceedings of NARMS-Tac*, *1*(1), 267-273.

Jiang, H., & Yang, Y. (2020). A three- dimensional Hoek–Brown failure criterion based on an elliptical Lode dependence. *International Journal for Numerical and Analytical Methods in Geomechanics*, *44*(18), 2395-2411. doi.org/10.1007/s00603-014-0691-9

Kim, M., & Lade, P. (1984). Modelling rock strength in three dimensions. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 21(1), 21-33. https://doi.org/10.1016/0148-9062(84)90006-8

Lee, Y. K., Pietruszczak, S., & Choi, B. H. (2012). Failure criteria for rocks based on smooth approximations to Mohr–Coulomb and Hoek–Brown failure functions. *International Journal of Rock Mechanics and Mining Sciences*, 56, 146–160. https://doi.org/10.1016/J.IJRMMS.2012.07.032

Lian, J., Sharaf, M., Archie, F., & Münstermann, S. (2013). A hybrid approach for modelling of plasticity and failure behaviour of advanced high-strength steel sheets. *International Journal of Damage Mechanics*, *22*(2), 188-218. doi.org/10.1177/1056789512439319

Ma, X., & Haimson, B. C. (2016). Failure characteristics of two porous sandstones subjected to true triaxial stresses. *Journal of Geophysical Research: Solid Earth*, *121*(9), 6477-6498. 10.1002/2016JB012979

Mahetaji, M., Brahma, J., & Sircar, A. (2020). Pre-drill pore pressure prediction and safe well design on the top of Tulamura anticline, Tripura, India: a comparative study. *Journal of Petroleum Exploration and Production Technology,* 10(3). https://doi.org/10.1007/s13202-019-00816-0

Mahetaji, M., Brahma, J., & Vij, R. K. (2023). A new extended Mohr-Coulomb criterion in the space of three-dimensional stresses on the in-situ rock. *Geomechanics and Engineering*, 32(1), 49–68. https://doi.org/10.12989/GAE.2023.32.1.049

Mogi, K. (1971). Fracture and flow of rocks under high triaxial compression. *Journal of Geophysical Research*, *76*(5), 1255-1269. doi.org/10.1029/JB076i005p01255

Mogi, K. (2006). *Experimental rock mechanics* (Vol. 3). CRC Press.

Ottosen, N. S. (1977). A failure criterion for concrete. *Journal of the Engineering Mechanics Division*, *103*(4), 527-535. doi.org/10.1061/JMCEA3.0002248

Pan, X. D., & Hudson, J. A. (1988). A simplified three dimensional Hoek-Brown yield criterion. *ISRM International Symposium*. ISRM-IS-1988-011

Priest, S. D. (2005). Determination of shear strength and three-dimensional yield strength for the Hoek-Brown criterion. *Rock Mechanics and Rock Engineering*, *38*(4), 299-327. doi.org/10.1007/s00603-005-0056-5

Shvarts, A., & van Helden, G. (2022). Embodied learning at a distance: From sensory-motor experience to constructing and understanding a sine graph. *Mathematical Thinking and Learning*, 1-29. doi.org/10.1080/10986065.2021.1983691

Singh, M., Raj, A., & Singh, B. (2011). Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of intact rocks. *International Journal of Rock Mechanics and Mining Sciences*, *48*(4), 546-555. doi.org/10.1016/j.ijrmms.2011.02.004

Takahashi, M., & Koide, H. (1989). Effect of the intermediate principal stress on strength and deformation behavior of sedimentary rocks at the depth shallower than 2000 m. *ISRM international symposium*. ISRM.

Yi, X., Ong, S., & Russell, J. E. (2006). Quantifying the effect of rock strength criteria on minimum drilling mud weight prediction using polyaxial rock strength test data. *International Journal of Geomechanics*, *6*(4), 260-268. doi.org/10.1061/(ASCE)1532-3641(2006)6:4(260)

Yi, X., Valkó, P. P. & Russell, J. E. (2005). Effect of rock strength criterion on the predicted onset of sand production. *International Journal of Geomechanics*, *5*(1), 66-73. doi.org/10.1061/(ASCE)1532-3641(2005)5:1(66)

Yi, X., Valkó, P. P., & Russell, J. E. (2005). Effect of rock strength criterion on the predicted onset of sand production. *International Journal of Geomechanics*, *5*(1), 66-73. doi.org/10.1061/(ASCE)1532-3641(2005)5:1(66)

Zhang, J. J. (2019). *Applied petroleum geomechanics*. Gulf Professional Publishing.

Zhang, L. (2008). A generalized three-dimensional Hoek–Brown strength criterion. *Rock mechanics and rock engineering*, *41*(6), 893-915. doi.org/10.1007/s00603-008-0169-8

Zhang, L., & Zhu, H. (2007). Three-dimensional Hoek-Brown strength criterion for rocks. *Journal of Geotechnical and Geoenvironmental Engineering*, *133*(9), 1128-1135. doi.org/10.1061/(ASCE)1090-0241(2007)133:9(1128)