



An Evaluation of Recent Global Geopotential Models for Strip Area Project in Turkey

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ABSTRACT

The aim of this study is to present the evaluations based on comparisons of geoid heights that are computed from several global geopotential models (GGMs) and the GNSS/levelling data. In this application framework, differences between geoid heights obtained by GGMs and GNSS/levelling were computed. Then, the availability of geoid heights calculated by GGMs for engineering applications were investigated. The Konya-Polatli (Ankara) Express Train Project as a strip area project was chosen as the study area. The length of the project is approximately 210 km and consists of 110 benchmarks that belong to the Turkish National Triangulation Network. In this study a total of 69 GGMs were compared. In order to examine more detail, these models were classified as three groups based on CHAMP, GRACE and GOCE. Each group was evaluated separately and the results were obtained. According to results, the best five models were detected for geoid height differences ($N^{GNSS/lev} - N^{ggm}$) in terms of standard deviation. These are EIGEN-6c4, EIGEN-GRACE01s, EGM2008, EIGEN-6c3stat and EIGEN-6c2, respectively. Also, geoid heights were obtained using different parametric models. These parametric models were used in order to minimize the impact of the terms of bias, tilt etc. Generally, three, four, five and seven parametric models are used for the least-squares adjustment of the geoid height differences in the literature. Therefore, in this study the geoid heights were calculated for such different parametric models. After the geoid height values were computed from the parametric models, the best global geopotential models in terms of standard deviation were obtained as EIGEN-6c2, EIGEN-6c3stat, EGM2008, EIGEN-6c4 and EIGEN-GRACE01s, respectively.

Keywords: Global geopotential model, GNSS/levelling, Geoid undulations, Orthometric and ellipsoidal heights.

Evaluación de modelos geopotenciales globales recientes para un proyecto de área lineal en Turquía

RESUMEN

El propósito de este estudio es presentar las evaluaciones comparativas de alturas geoidales que fueron computadas a partir de varios Modelos Geopotenciales Globales (GGM, del inglés Global Geopotential Models) y la nivelación de información del Sistema Global de Navegación por Satélite. Luego se investigó la disposición para aplicaciones de ingeniería de las alturas geoidales calculadas por los modelos GGM. Se seleccionó el proyecto del Tren Expreso Konya-Polatli (Ankara) como el área de estudio por ser un terreno lineal. La longitud del proyecto es de 210 kilómetros y consiste de 110 puntos de referencia que pertenecen a la Red de Triangulación Nacional de Turquía. En este estudio se compararon 69 modelos GGM. Para un mejor examen, estos modelos se clasificaron en tres grupos basados en CHAMP (CHALLENGING Minisatellite Payload), GRACE (Gravity Recovery and Climate Experiment) y GOCE (Gravity field and steady-state Ocean Circulation Explorer). Cada grupo se evaluó por separado. De acuerdo con los resultados, se detectaron los cinco modelos mejores para las diferencias de alturas geoidales (NGNSS/LEV-NGGM) en términos de desviación estándar. Estos son EIGEN-6c4, EIGEN-GRACE01s, EGM2008, EIGEN-6c3stat, y EIGEN-6c2. También se obtuvieron las alturas geoidales a través de diferentes modelos paramétricos. Este mecanismo se utilizó para minimizar el impacto en términos de inclinación y declive. Generalmente, se utilizan tres, cuatro, cinco, y siete modelos paramétricos para el ajuste por mínimos cuadrados de las diferencias de alturas geoidales, según la literatura. Por lo tanto, en este estudio se calcularon las alturas geoidales con estos modelos paramétricos. Después de que se computaron los valores de altura geoidales desde los modelos paramétricos, se obtuvieron los mejores modelos geopotenciales globales en términos de desviación estándar, estos son el EIGEN-6c2, EIGEN-6c3stat, EGM2008, EIGEN-6c4 y EIGEN-GRACE01s, respectivamente.

Palabras clave: Modelo geopotencial global, nivelación GNSS, ondulaciones geoidales, alturas geoidales y ortométricas.

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1. Introduction

Determining the gravity field with as much accuracy as possible is significant for earth sciences such as geodesy and geophysics, and the gravity field is instrumental in many natural incidents related to earth dynamics, primarily mass transportation. In recent years, research conducted to determine the earth's gravity field has gained speed thanks to Low Earth Orbits (LEO). Various Global Geopotential Models (GGMs) have been released using in particular CHAMP (CHALLENGING Minisatellite Payload), GRACE (Gravity Recovery And Climate Experiment) and GOCE (Gravity field and steady-state Ocean Circulation Explorer) orbit data. These models have a positive impact on determining geoid changes.

Launched from Russia's Plesetsk spaceport on 15 July 2000, CHAMP was the first low earth satellite launched in order to determine gravity field. The CHAMP mission was carried out by the Potsdam (Germany) Earth Research Center (GeoForschungsZentrum-GFZ). The satellite is almost circular and located close to the pole, with 454 km starting height and 87.3° inclination degree. Although its work period was planned to be five years, it operated until 19 September 2010. The core aims to be reached through the CHAMP satellite are to specify long wavelength features of the static earth's gravity field (and partially time changes) with the greatest possible accuracy, mapping the global magnetic field, which means specifying the earth's primary magnetic field and its time changes, and to reveal ionosphere and troposphere profiles, respectively (Seeber 2003; Hofmann-Wellenhopf and Moritz 2005). CHAMP satellite was groundbreaking in specifying long wavelength components of the gravity field. Compared to GRIM5-S1 and EGM96S models produced by several observations and satellites, it has been found that gravity field resolution is higher specified with a couple of month's old CHAMP orbit tracking data (Reigber et al. 2003).

GRACE mission is the continuing part of the CHAMP satellite. Unlike CHAMP, it consists of two identical satellites following each other on the same orbit with 220 km ± 50 km distance between them. Both satellites were launched simultaneously from the Russian Plesetsk spaceport on 17 March, 2002. GRACE mission is a product of a joint project of DLR (Deutsches Zentrum für Luft- und Raumfahrt) and NASA (U.S. National Aeronautics and Space Administration). University of Texas, Center for Space Research (CSR) was assigned as the project principal. Like CHAMP, data are needed that are homogeneously distributed and surrounding the earth entirely, in order to obtain a sensitive projection of gravity potential. Therefore, the GRACE mission is close to the pole and almost circular. Initial orbit height was selected at around 500 km, and the orbit tendency 89°. Aim of GRACE mission is to determine the earth's high resolution global gravity field and time changes in this field. These satellites are also used in order to map Total Electron Content (TEC) as in the CHAMP satellite mission (Hofmann-Wellenhopf and Moritz 2005; Jäggi 2007). Resolution of the gravity field based on GRACE data is better than the previous ones. According to Tapley et al. (2004), it was found that improvements in the global gravity field were significant compared to GGM01S earth gravity model EGM96 determined by 110-day GRACE data.

GOCE mission is the core project of the Living Planet Programme of the European Space Agency (ESA). GOCE satellite is the last in the satellite series launched to determine gravity field. Orbit of GOCE satellite was launched from the Russian Plesetsk spaceport on 17 March 2009 as almost circular and solar-synchronized. Orbit height was selected as 250 km in order to get a stronger and more sensitive gravity signal. Monitoring and controlling the satellite were carried out by ESA/ESOC in collaboration with earth stations Kiruna in Sweden, and Svalbard in Norway. Main objective of GOCE is to participate in measuring the earth's gravity field and modelling the geoid with perfect accuracy and spatial resolution. Anticipated accuracies are determining gravity-field anomalies with 1mGal (10⁻⁵ ms⁻²), geoid with 1-2 cm accuracy, and reaching a spatial grid resolutions better than 100 km.

Many researchers have investigated the contribution of GGMs produced from CHAMP, GRACE, and GOCE satellites in local and global geoid models for various areas of the earth (Amos and Featherstone 2003; Kiamehr and Sjöberg 2005; Rodriguez-Caderot et al. 2006; Benahmed Dahoa et al. 2006; Ustun and Demirel 2006; Erol et al. 2009; Kotsakis and Katsambalos 2010; Yilmaz et al. 2010; Janak and Pitonak 2011; Gikas et al. 2013; Godah and Krynski 2013; Soycon 2014). Various methods and approaches have been suggested as a result of the above-mentioned research. One of the most frequently employed methods in the literature to determine the best GGM for an area's gravity field is comparing GGMs and revealing their performance through independent datasets (GNSS/levelling, gravity etc).

In this research, the geoid undulation values are calculated through GGMs (69 GGMs at the time of writing this paper) for a 210-km long strip project. Then these values are compared to accurate geoid undulation values calculated with GNSS/levelling method. As a result, performance of GGMs is revealed statistically and the best GGM is suggested. Moreover, investigation of possible maximum geoid accuracy to be obtained from GGMs is aimed at in this research employing only geographical latitude and longitude information of points without any geodetic measuring in the field.

2. Theoretical Background

2.1. Global Geopotential Models (GGMs) and GNSS/levelling

The sum of centrifugal and gravity forces on an object is defined as gravity force. Determining the earth's gravity field is the same as determining its potential. As this potential is harmonical out of masses that form the earth, spherical harmonic series are generally used for determining the gravity field (Kaula 1966; Heiskanen and Moritz 1984; Rummel et al. 2002; Seeber 2003; Hofmann-Wellenhopf and Moritz 2005).

Global Geopotential Models (GGMs) are defined as spherical harmonic coefficients representing the earth's gravity field in various wavelengths. These coefficients are obtained from satellite orbit deviation analyses, satellite altimeter data, gravity gradiometer data, and gravimeter data. GGMs are divided into three groups in this context (Vaniček and Featherstone 1998; Featherstone 2002).

- **Satellite-only models:** Coefficients of these GGMs are derived from orbit deviation analyses of artificial earth satellites. Degree of these models is low, and resolution is insufficient.
- **Combined models:** The combined GGMs are produced by combining gravity data derived from satellite data, terrestrial gravity observations, airborne gravimetry and satellite altimeter data in marine areas. Therefore, integrated models have higher degrees and produce more accurate results compared to satellite-only models.
- **Tailored models:** These models are generated as a result of improving harmonic coefficients of GGMs using special mathematical techniques within the first and second groups. The main aim of these models is to increase the degree of the model.

Improving measurement, calculation and evaluation techniques have contributed to constant improvement of GGMs. This improvement causes enhanced geoid resolution. In other words, when the degree of GGMs increased, errors of the deflection of the plumb line, gravity anomaly, and height anomaly are decreased, and the better the determination of the geoid (Wenzel 1998). GGMs point out the gravity field of the earth with spherical harmonic series. According to this the geoid height of a point the spherical geocentric coordinates of which are known might be calculated with Eq. (1) (Hofmann-Wellenhopf and Moritz 2005).

$$N_{ggm}(r, \theta, \lambda) = \frac{GM}{\gamma r} \sum_{n=2}^{n_{max}} \left(\frac{R}{r}\right)^n \sum_{m=0}^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \quad 1$$

where r , θ , λ are the spherical geocentric coordinates of the computation point: radial distance, co-latitude and longitude, respectively, GM is the gravitational constant (G) times mass (M) of the earth, γ is mean normal gravity of the reference ellipsoid, R is the mean earth's equatorial radius, \bar{C}_{nm} , \bar{S}_{nm} are fully normalized geopotential coefficients with degree n and order m , $\bar{P}_{nm}(\cos \theta)$ fully normalized associated Legendre functions, and n_{\max} is the maximum degree of the GGM.

In order to determine the optimum GGM for an area's gravity field, geoid heights derived from the GNSS/levelling method could be compared to the geoid heights calculated by GGMs (Amos and Featherstone 2003; Kılıçoğlu et al. 2009; Yılmaz et al. 2010; Hirt 2011; Guimarães et al. 2012). In many GNSS applications users feel a need for transformation between ellipsoidal height and orthometric height. The main reason of this is using orthometric heights in engineering, determined by levelling, and the difficulty of making levelling measurements for each point. Highly precision geoid models are needed for this type of implementation. Geoid height (undulation) values produced based on GNSS/levelling measurements are necessary to determine sensitively to this end (Doganalp and Selvi 2015). According to the GNSS/levelling method, geoid undulation value at a point is calculated with Eq. (2) (Heiskanen and Moritz 1984).

$$N^{GNSS/lev} = h - H \quad 2$$

Here N stands for geoid height (undulation), h for ellipsoidal height obtained from GNSS measurements, and H for orthometric height calculated after levelling measurements. To evaluate the quality of GGM-derived geoid undulation values (N^{GGM}) many independent $N^{GNSS/lev}$ values could be used that are spread over the project area. Therefore, absolute geoid height differences ΔN might be calculated with Eq. (3) (Kotsakis and Sideris 1999; Fotopoulos et al. 2003; Huang and Véronneau 2004; Gikas et al. 2013).

$$\Delta N = N^{GNSS/lev} - N^{GGM} = (h - H) - N^{GGM} \quad 3$$

3. Applications

3.1. Study area and data

Study field is the High-Speed Train route between Konya-Polatli (Ankara), which is 210 km long. There are 110 GNSS/levelling points (benchmarks) within the project field. Orthometric heights (H) of the points were determined with the geometric levelling method in the datum of the Turkey National Vertical Network (TUDKA). The geographic coordinates including ellipsoidal heights (h) were determined in static positioning mode and referred to the Turkish National Fundamental GPS Network (TUTGA). The orthometric and ellipsoidal height of the points within the study field are varying between 696 – 1198 m and 733 – 1234 m respectively. Also, the geoid heights are varying between 36.72 and 37.70 m (Fig. 1). The minimum, maximum and mean values of distance between sequential points of the route were observed as 1.05, 5.53 and 1.94 km respectively. SRTM-3 data and The Generic Mapping Tools (GMT) software were used to draw Figure 1 (Wessel and Smith 1998). The Shuttle Radar Topography Mission (SRTM) contains elevation data with 3 arc-second resolution and 16 m absolute height error (90 percent confidence level). These data are available with free of charge via the Internet for approximately 80 percent of the earth's land mass (Bildirici et al. 2009).

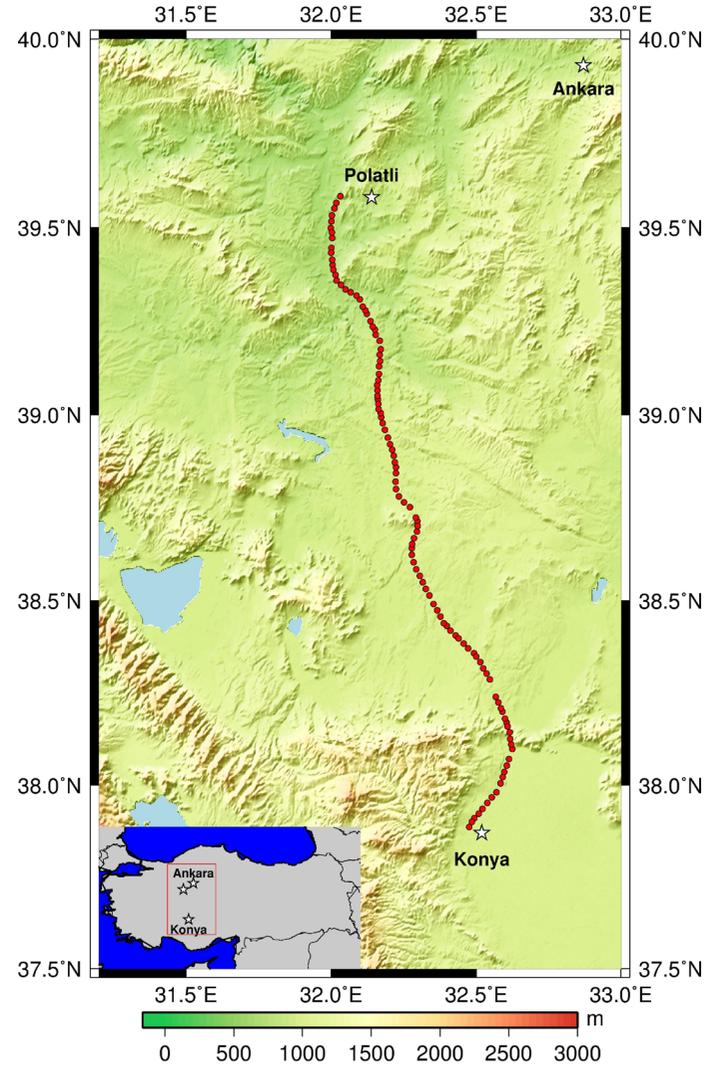


Fig. 1. Study area, Konya-Polatli express train project

3.2 Evaluation Procedure

A total of 69 models were used to investigate the quality of GGMs. In order to display the results from those models clearly, $GGMs$ are classified in three models derived from CHAMP, GRACE, and GOCE satellites. Spherical harmonic coefficients of all models were obtained from the International Centre for Global Earth Models (*ICGEM*) web page. In addition, geoid heights based on GGMs can be calculated using coefficients and software on this website. Within the scope of implementation, in order to include all of Turkey, separate grid networks (*grid step is 0.1° x 0.1°*) were established for each GGM. GRS80 ellipsoid parameters were used to establish grids. A correction needs to be applied on the difference between the true potential of the geoid and the normal potential of GRS80 when calculating geoid height, when using GRS80 as the chosen reference field. The reason for this correction is the difference between the GM value estimated for earth and the GM value for GRS80 ellipsoid. Therefore, the “zero-degree” term based on different GM values needs to be considered in geoid height computation (Smith 1998). In addition, while the shape of the geoid depends on the permanent tide system type (mean, zero and tide-free), the values of true gravity potential do not change when one system transform another system. Therefore, the tide-free geoid model is suggested to be used in conversion of the GNSS heights (Smith 1998). Thus, correction term (zero degree term) has been applied to all calculations within the context of this application, and tide-free model is used as the tide system. In each of the GGM grid nets established, geoid heights of project points are interpolated using a script.

GGMs based on CHAMP

In this section, the results on GGMs for 110 points of the study area produced based on CHAMP are addressed. Totally 17 CHAMP-based global geopotential models are used. Names, production years, degrees and the data used to produce them are given in Table 1. Geoid undulations within study area are estimated using these models. The statistics of the differences between the GNSS-based and the GGM-based geoid undulations are given in Table 2 and Figure 2.

Model	Year	Degree	Data	Reference
eigen-1	2002	119	S(Champ)	Reigber et al. 2003a
eigen-2	2003	140	S(Champ)	Reigber et al. 2003b
eigen-champ03sp	2003	140	S(Champ)	Reigber et al. 2004a
itg_champ01e	2003	75	S(Champ)	Flk et al. 2003
itg_champ01k	2003	70	S(Champ)	Flk et al. 2003
itg_champ01s	2003	70	S(Champ)	Flk et al. 2003
tum-1s	2003	60	S(Champ)	Gerlach et al. 2003
tum-2sp	2003	60	S(Champ)	Földvary et al. 2005
deos_champ-01c	2004	70	S(Champ)	Ditmar et al. 2006
eigen-champ03s	2004	140	S(Champ)	Reigber et al. 2004a
tum-2s	2004	70	S(Champ)	Wermuth et al. 2004
eigen-cg01c	2004	350	S(Champ,Grace),G,A	Reigber et al. 2006
eigen-cg03c	2005	350	S(Champ,Grace),G,A	Förste et al. 2005c
aiub-champ01s	2007	90	S(Champ)	Prange et al. 2009
aiub-champ03s	2010	100	S(Champ)	Prange 2011
eigen-champ05s	2010	150	S(Champ)	Flechtner et al. 2010
ulux_champ2013s	2013	120	S(Champ)	Weigelt et al. 2013

Table 1. Global geopotential models based on CHAMP (S: Satellite, G: Gravity, A: Altimetry)

Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$				Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$			
	Min.	Max.	Mean	Std. Dev.		Min.	Max.	Mean	Std. Dev.
aiub-champ03s	-1,998	-0,716	-1,555	0,279	itg_champ01e	-4,338	-1,205	-3,480	0,769
eigen-cg01c	-1,497	-0,407	-0,969	0,299	aiub-champ01s	-3,786	-0,507	-2,838	0,818
eigen-cg03c	-1,632	-0,406	-1,047	0,327	eigen-champ03sp	-4,174	-0,964	-3,262	0,820
eigen-champ05s	-3,038	-0,961	-2,467	0,465	itg_champ01k	-3,620	-0,289	-2,638	0,839
ulux_champ2013s	-2,391	-0,058	-1,591	0,638	eigen2	-4,168	-0,523	-3,056	0,911
eigen1	0,158	2,913	1,062	0,706	tum1s	-2,329	1,232	-1,359	0,994
eigen-champ03s	-3,924	-0,961	-3,082	0,720	tum2sp	-3,136	0,696	-1,974	1,005
deos_champ-01c	-3,927	-0,913	-3,104	0,724	tum2s	-3,066	0,840	-1,877	1,027
itg_champ01s	-3,620	-0,572	-2,750	0,746					

Table 2. Statistics of geoid height differences between GGM-derived (N^{GGM}) and GNSS/levelling ($N^{GNSS/lev}$) data, units in meters (based on CHAMP)

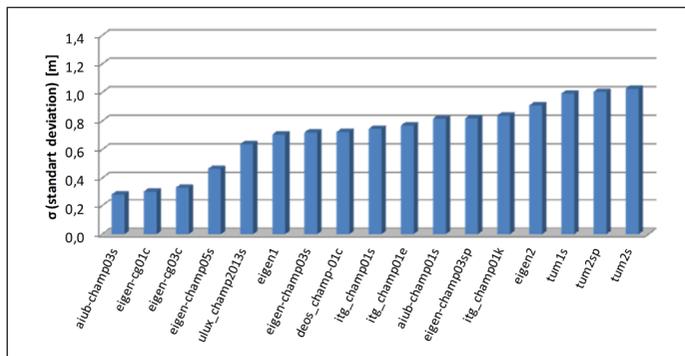


Fig. 2. The standard deviations of geoid undulation differences between GNSS/levelling and GGMs (based on CHAMP)

Table 2 and Figure 2 show the standard deviation values range between 0.28 – 1.03 m with regard to geoid undulation differences. Moreover, the best five models for CHAMP-based GGMs are aiub-champ03s, eigen-cg01c, eigen-cg03c, eigen-champ05s and ulux_champ2013s, respectively.

GGMs based on GRACE

In this section, 24 global geopotential model evaluations have been done using GRACE-based models. Information on GGMs is given in Table 3. Performance of GGMs determined after evaluation of 110 points within the project field is shown in Table 4. Moreover, the diagram displaying the differences between geoid heights calculated by GGMs and the known heights of GNSS/levelling points is given in Figure 3.

Model	Year	Degree	Data	Reference
eigen-grace01s	2003	140	S(Grace)	Reigber et al. 2003c
ggm01s	2003	120	S(Grace)	Tapley et al. 2003
ggm01e	2003	200	TEG4,S(Grace)	UTEX CSR 2003
eigen-grace02s	2004	150	S(Grace)	Reigber et al. 2005a
ggm02s	2004	160	S(Grace)	UTEX CSR 2004
ggm02e	2004	200	S(Grace),G,A	UTEX CSR 2004
itg-grace02s	2006	170	S(Grace)	Mayer-Gürr et al. 2006
eigen-gl04s1	2006	150	S(Grace,Lageos)	Förste et al. 2006
eigen-gl04c	2006	360	S(Grace,Lageos),G,A	Förste et al. 2006
itg-grace03	2007	180	S(Grace)	Mayer-Gürr et al. 2007
aiub-grace01s	2008	120	S(Grace)	Jäggi et al. 2008
ggm03s	2008	180	S(Grace)	Tapley et al. 2007
egm2008	2008	2190	S(Grace),G,A	Pavlis et al. 2008
eigen-5s	2008	150	S(Grace,Lageos)	Förste et al. 2008
eigen-5c	2008	360	S(Grace,Lageos),G,A	Förste et al. 2008
aiub-grace02s	2009	150	S(Grace)	Jäggi et al. 2009
ggm03c	2009	160	S(Grace),G,A	Tapley et al. 2007
eigen-51c	2010	359	S(Grace,Champ),G,A	Bruinsma et al. 2010
aiub-grace03s	2011	160	S(Grace)	Jäggi et al. 2011
gif48	2011	360	S(Grace),G,A	Ries et al. 2011
tonji-grace01	2013	160	S(Grace)	Shen et al. 2013
itsg-grace2014s	2014	200	S(Grace)	Mayer-Gürr et al. 2014
itsg-grace2014k	2014	200	S(Grace)	Mayer-Gürr et al. 2014
ggm05s	2014	180	S(Grace)	Tapley et al. 2013

Table 3. Global geopotential models based on GRACE (S: Satellite, G: Gravity, A: Altimetry)

Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$				Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$			
	Min.	Max.	Mean	Std. Dev.		Min.	Max.	Mean	Std. Dev.
eigen-grace01s	-1,004	-0,340	-0,768	0,131	aiub-grace03s	-1,421	-0,343	-0,919	0,304
egm2008	-1,212	-0,652	-1,041	0,137	tonji-grace01	-1,933	-0,752	-1,327	0,308
gif48	-1,306	-0,518	-1,060	0,157	eigen-grace02s	-1,773	-0,688	-1,147	0,316
aiub-grace01s	-1,614	-0,905	-1,357	0,158	ggm01c	-1,005	0,052	-0,561	0,323
eigen-51c	-1,394	-0,459	-1,097	0,167	itg-grace02s	-1,408	-0,275	-0,829	0,325
egm2008upto360	-1,433	-0,516	-1,177	0,173	ggm03c	-1,453	-0,268	-0,897	0,331
ggm02c	-0,953	-0,335	-0,722	0,175	eigen-5c	-1,461	-0,057	-0,808	0,339
ggm01s	-1,041	-0,329	-0,787	0,178	ggm05s	-1,692	-0,426	-0,873	0,390
eigen-5s	-1,483	-0,654	-1,015	0,193	itg-grace03	-1,824	-0,360	-1,018	0,397
aiub-grace02s	-1,616	-0,767	-1,194	0,202	eigen-gl04c	-1,420	-0,047	-0,747	0,485
eigen-gl04s1	-1,647	-0,724	-1,158	0,215	ggm03s	-1,148	2,392	1,184	1,119
itsg-grace2014k	-1,131	-0,184	-0,679	0,280	itsg-grace2014s	-2,428	1,483	-0,141	1,235
ggm02s	-3,333	-2,162	-2,636	0,287					

Table 4. Statistics of geoid height differences between GGM-derived (N^{GGM}) and GNSS/levelling ($N^{GNSS/lev}$) data, units in meters (based on GRACE)

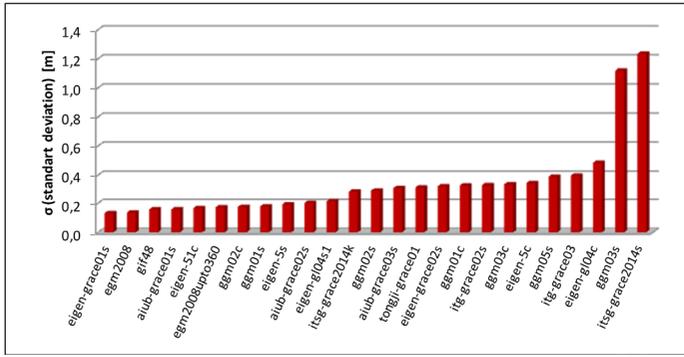


Fig. 3. The standard deviations of geoid undulation differences between GNSS/levelling and GGMs (based on GRACE)

Table 4 and Figure 3 show the standard deviation values range between 0.13 – 1.24 m according to geoid undulation differences. Lower standard deviation values are reached compared to CHAMP-based models. In addition, the best first five models for GRACE-based GGMs are eigen-grace01s, egm2008, gif48, aiub-grace01s and eigen-51c, respectively.

GGMs based on GOCE

In this section, 28 global geopotential models are evaluated based on the GOCE satellite. Information on GOCE-based GGMs is given in Table 5. Performance of GGMs within the study area is shown in Table 6. Moreover, the diagram displaying the standard deviation of differences between geoid heights calculated by GGMs, and the known heights of GNSS/levelling points is given in Figure 4.

Model	Year	Degree	Data	Reference
go_cons_gcf_2_dir_r1	2010	240	S(Goce)	Bruinsma et al. 2010
uo_cons_gcf_2_spw_r1	2010	210	S(Goce)	Migliaccio et al. 2010
go_cons_gcf_2_tim_r1	2010	224	S(Goce)	Pail et al. 2010a
goc01s	2010	224	S(Goce,Grace)	Pail et al. 2010b
go_cons_gcf_2_dir_r2	2011	240	S(Goce)	Bruinsma et al. 2010
go_cons_gcf_2_spw_r2	2011	240	S(Goce)	Migliaccio et al. 2011
uo_cons_gcf_2_tim_r2	2011	250	S(Goce)	Pail et al. 2011
go_cons_gcf_2_tim_r3	2011	250	S(Goce)	Pail et al. 2011
goc02s	2011	250	S(Goce,Grace,...)	Goiginger et al. 2011
eigen-6s	2011	240	S(Goce,Grace,Lageos)	Förste et al. 2011
go_cons_gcf_2_dir_r3	2011	240	S(Goce,Grace,Lageos)	Bruinsma et al. 2010
eigen-6c	2011	1420	S(Goce,Grace,Lageos),G,A	Förste et al. 2011
dgm-1s	2012	250	S(Goce,Grace)	Farahani et al. 2013
goc03s	2012	250	S(Goce,Grace,...)	Mayer-Gürr et al. 2012
eigen-6c2	2012	1949	S(Goce,Grace,Lageos),G,A	Förste et al. 2012
go_cons_gcf_2_tim_r4	2013	250	S(Goce)	Pail et al. 2011
litg-uo02	2013	240	S(Goce)	Schall et al. 2014
gogra02s	2013	250	S(Goce,Grace)	Yi et al. 2013
uo_cons_gcf_2_dir_r4	2013	260	S(Goce,Grace,Lageos)	Bruinsma et al. 2013
jyy_goc02s	2013	250	S(Goce)	Yi et al. 2013
uo_cons_gcf_2_spw_r4	2014	280	S(Goce)	Gatti et al. 2014
eigen-6c4	2014	2190	S(Goce,Grace,Lageos),G,A	Förste et al. 2014
go_cons_gcf_2_tim_r5	2014	280	S(Goce)	Brockmann et al. 2014
uo_cons_gcf_2_dir_r5	2014	300	S(Goce,Grace,Lageos)	Bruinsma et al. 2013
jyy_goc04s	2014	250	S(Goce)	Yi et al. 2013
gogra04s	2014	250	S(Goce,Grace)	Yi et al. 2013
eigen-6s2	2014	260	S(Goce,Grace,Lageos)	Rudenko et al. 2014
eigen-6c3stat	2014	1949	S(Goce,Grace,Lageos),G,A	Förste et al. 2012

Table 5. Global geopotential models based on GOCE (S: Satellite, G: Gravity, A: Altimetry)

Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$				Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$			
	Min.	Max.	Mean	Std. Dev.		Min.	Max.	Mean	Std. Dev.
eigen-6c4	-1.094	0.613	-0.936	0.122	goc03s	-1.461	-0.331	-0.968	0.313
eigen-6c3stat	-1.108	-0.606	-0.931	0.137	go_cons_gcf_2_tim_r3	-1.523	-0.239	-0.999	0.315
eigen-6c2	-1.121	-0.616	-0.934	0.144	jyy_goc04s	-1.276	-0.142	-0.765	0.333
eigen-6c	-1.158	-0.569	-0.943	0.159	gogra04s	-1.285	-0.145	-0.771	0.334
go_cons_gcf_2_tim_r5	-1.423	-0.580	-1.002	0.204	dgm-1s	-1.518	-0.323	-0.925	0.342
uo_cons_gcf_2_dir_r5	-1.410	-0.412	-1.018	0.208	litg-uo02	-1.508	-0.312	-0.957	0.348
go_cons_gcf_2_tim_r4	-1.418	-0.475	-0.922	0.234	go_cons_gcf_2_spw_r2	-1.478	-0.276	-0.914	0.357
go_cons_gcf_2_dir_r1	-1.425	0.484	-0.920	0.239	go_cons_gcf_2_spw_r1	-1.656	-0.412	-1.087	0.392
eigen-6s2	-1.462	-0.438	-0.930	0.250	eigen-6s	-1.612	-0.182	-0.921	0.401
go_cons_gcf_2_dir_r4	-1.464	0.439	-0.933	0.250	go_cons_gcf_2_dir_r2	-1.704	-0.242	-0.997	0.414
go_cons_gcf_2_spw_r4	-1.481	-0.407	-0.931	0.270	goc02s	-1.706	-0.202	-1.048	0.436
go_cons_gcf_2_dir_r3	-1.340	-0.367	-0.887	0.279	go_cons_gcf_2_tim_r2	-1.744	-0.265	-1.062	0.443
uogra02s	-1.218	-0.227	-0.722	0.290	uo01s	-1.567	-0.225	-0.939	0.477
jyy_goc02s	-1.214	-0.219	-0.717	0.291	go_cons_gcf_2_tim_r1	-1.612	-0.258	-0.989	0.487

Table 6. Statistics of geoid height differences between GGM-derived (N^{GGM}) and GNSS/levelling ($N^{GNSS/lev}$) data, units in meters (based on GOCE)

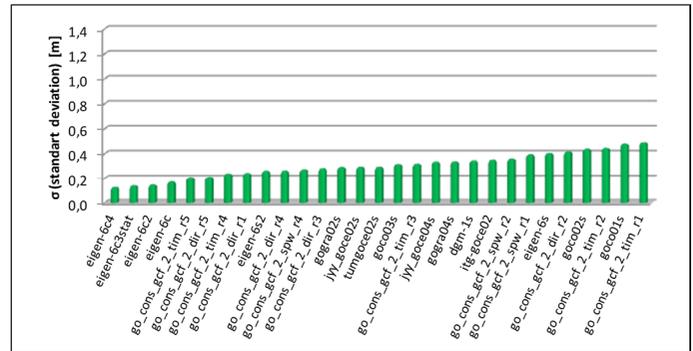


Fig. 4. The standard deviations of geoid undulation differences between GNSS/levelling and GGMs (based on GOCE)

Table 6 and Figure 4 show the standard deviation values range between 0.12 – 0.49 m with regard to geoid undulation differences. These values might be considered good compared to CHAMP and GRACE-based models. Moreover, the best first five models for GOCE-based GGMs are eigen-6c4, eigen-6c3stat, eigen-6c2, eigen-6c and go_cons_gcf_2_tim_r5, respectively.

Best GGMs in the study area

In order to summarize the results of all the implementations, five GGMs obtained from implementations conducted in three classes are evaluated. Table 7 contains the best five GGMs according to geoid undulation differences within every class. Figure 5 shows the performance of GGMs using the standard deviation values calculated in Table 7.

Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$				Model Name	$\Delta N = N^{GNSS/lev} - N^{GGM}$			
	Min.	Max.	Mean	Std. Dev.		Min.	Max.	Mean	Std. Dev.
eigen-6c4	-1.094	-0.613	-0.936	0.122	eigen-6c	-1.158	-0.569	-0.943	0.169
eigen-grace01s	-1.004	-0.340	-0.768	0.131	go_cons_gcf_2_tim_r5	-1.423	-0.580	-1.002	0.204
egm2008	-1.212	-0.652	-1.041	0.137	aiub-champ03s	-1.998	-0.716	-1.555	0.279
eigen-6c3stat	-1.108	-0.606	-0.931	0.137	eigen-cg01c	-1.497	-0.407	-0.969	0.299
eigen-6c2	-1.121	-0.616	-0.934	0.144	eigen-cg03c	-1.632	-0.406	-1.047	0.327
gif48	-1.306	-0.518	-1.060	0.157	eigen-champ05s	-3.038	-0.961	-2.467	0.465
aiub-grace01s	-1.614	-0.905	-1.357	0.158	ulux_champ2013s	-2.391	-0.058	-1.591	0.638
eigen-51c	-1.394	-0.459	-1.097	0.167					

Table 7. Statistics of geoid height differences between GGM-derived (N^{GGM}) and GNSS/levelling ($N^{GNSS/lev}$) data, units in meters.

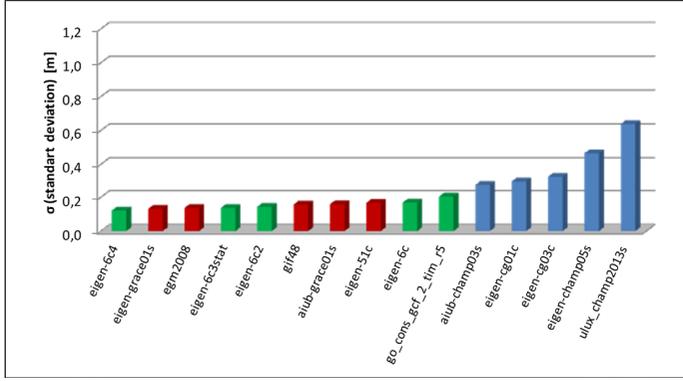


Fig. 5. The standard deviations of geoid undulation differences between GNSS/levelling and GGMs

Table 7 and Figure 5 show the best GGM for the study area is eigen-6c4. This model is followed by eigen-grace01s, egm2008, eigen-6c3stat and eigen-6c2, respectively. Considering the performance of these models, there is no large difference between them with regard to standard deviation values. In addition to this, general characteristics of models show that all the models except *eigen-grace01s* have a considerable degree, and gravity and altimeter data make positive contributions to the model (Table 8).

Differences between GNSS/levelling-based geoid heights and GGM-based geoid heights are influenced by datum inconsistencies and systematic errors. Therefore, different parametric models could be used in order to minimize these values (bias and tilt) (Kotsakis and Katsambalos 2010; Yilmaz et al. 2010). Generally, three, four, five and seven parametric models are used for the least-squares adjustment of the geoid height differences in the literature. Therefore, these parametric models were tested in this paper.

Model	Year	Degree	Data	Reference
eigen-6c4	2014	2190	S(Goce,Grace,Lageos),G,A	Förste et al. 2014
eigen-grace01s	2003	140	S(Grace)	Reigber et al. 2003c
egm2008	2008	2190	S(Grace),G,A	Pavlis et al. 2008
eigen-6c3stat	2014	1949	S(Goce,Grace,Lageos),G,A	Förste et al. 2012
eigen-6c2	2012	1949	S(Goce,Grace,Lageos),G,A	Förste et al. 2012

Table 8. The best five global geopotential models in the study area (S: Satellite, G: Gravity, A: Altimetry)

Geoid Heights Determination using Different Parametric Models

Systematic errors such as datum shifts and distortion among height systems might be minimized using the Least Squares Adjustment (LSA) model. If Eq. (3) is edited with this aim

$$h_i - H_i - N_i = \mathbf{a}_i^T \mathbf{x} + v_i \quad 4$$

is obtained (Kotsakis and Sideris 1999; Kiamehr and Sjöberg 2005). Here \mathbf{a}_i stands for known coefficients vector, \mathbf{x} unknown parameters vector, and v_i a residual random noise term. Four different parametric models (3, 4, 5 and 7-parameters) are used within the context of application. Mathematical statements for these models are as below, respectively (Kotsakis and Katsambalos 2010).

(Model 1)	$\Delta N = \cos \varphi_i \cos \lambda_i \cdot x_1 + \cos \varphi_i \sin \lambda_i \cdot x_2 + \sin \varphi_i \cdot x_3 + v_i$	5
(Model 2)	$\Delta N = \cos \varphi_i \cos \lambda_i \cdot x_1 + \cos \varphi_i \sin \lambda_i \cdot x_2 + \sin \varphi_i \cdot x_3 + x_4 + v_i$	6
(Model 3)	$\Delta N = \cos \varphi_i \cos \lambda_i \cdot x_1 + \cos \varphi_i \sin \lambda_i \cdot x_2 + \sin \varphi_i \cdot x_3 + \sin^2 \varphi_i \cdot x_4 + x_5 + v_i$	7
(Model 4)	$\Delta N = \cos \varphi_i \cos \lambda_i \cdot x_1 + \cos \varphi_i \sin \lambda_i \cdot x_2 + \sin \varphi_i \cdot x_3 + \frac{\cos \varphi_i \sin \varphi_i \cos \lambda_i}{w_i} \cdot x_4 + \frac{\cos \varphi_i \sin \varphi_i \sin \lambda_i}{w_i} \cdot x_5 + \frac{\sin^2 \varphi_i}{w_i} \cdot x_6 + x_7 + v_i$	8

where x_i transformation parameters between two datums, v_i residual random noise term, (φ_i, λ_i) geodetic coordinates of points, e first eccentricity of the reference ellipsoid, and $w_i = \sqrt{1 - e^2 \sin^2 \varphi_i}$. The matrix form is created as follows for the equations

$$\mathbf{a}_i = \begin{bmatrix} \cos \varphi_i \cos \lambda_i \\ \cos \varphi_i \sin \lambda_i \\ \sin \varphi_i \\ \cos \varphi_i \sin \varphi_i \cos \lambda_i / w_i \\ \cos \varphi_i \sin \varphi_i \sin \lambda_i / w_i \\ \sin^2 \varphi_i / w_i \\ 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad 9$$

Where \mathbf{a}_i is the coefficients vector and \mathbf{x} is unknown's vector for Model 4. For other parametric models, coefficients and unknowns vector might be formed as in Eq. (9). If the formed coefficients are expanded for each GNSS point, \mathbf{A} coefficients matrix (design matrix) would be obtained to be used in LSA (Eq. 10). Afterwards, unknown parameters can be solved with LSA.

$$\mathbf{A} = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \dots \mathbf{a}_i] \quad 10$$

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} (\mathbf{A}^T \Delta N) \quad 11$$

$$\mathbf{v} = \mathbf{A} \mathbf{x} - \Delta N \quad 12$$

Table 9 and Figure 6 show the statistical values obtained with 3, 4, 5, and 7-parameter LSA implementation of the best five GGMs reached as a result of application. Table 9 and Figure 6 also show the best results obtained from the eigen-6c2 model. Moreover, the 7-parameter transformation model gives results that are the most compatible with minimum standard deviation values when compared to other parametric models. Figures 7-11 show the geoid heights after implementing transformation and geoid heights obtained from GGMs with GNSS/levelling geoid heights.

GGMs		Model 1	Model 2	Model 3	Model 4
eigen-6c2	Min.	-0,178	-0,068	-0,078	-0,094
	Max.	0,368	0,064	0,063	0,043
	Std. Dev.	0,134	0,030	0,029	0,027
eigen-6c3stat	Min.	-0,171	-0,071	-0,084	-0,086
	Max.	0,353	0,063	0,060	0,052
	Std. Dev.	0,131	0,031	0,029	0,027
egm2008	Min.	-0,168	-0,074	-0,086	-0,095
	Max.	0,357	0,067	0,064	0,047
	Std. Dev.	0,132	0,034	0,032	0,029
eigen-6c4	Min.	-0,156	-0,075	-0,086	-0,097
	Max.	0,321	0,066	0,065	0,045
	Std. Dev.	0,118	0,034	0,032	0,030
eigen-grace01s	Min.	-0,159	-0,145	-0,145	-0,170
	Max.	0,178	0,164	0,164	0,171
	Std. Dev.	0,084	0,083	0,083	0,075

Table 9. Statistics of the differences between geoid heights obtained from GGMs and from GNSS/levelling data after applying the 3-, 4-, 5- and 7-parameter transformation, units in meters

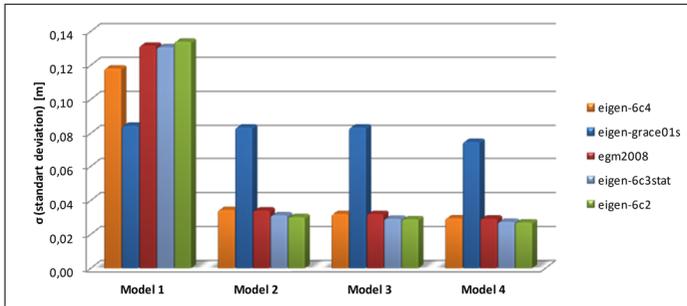


Fig. 6. The standard deviations of geoid undulation differences between GNSS/levelling and GGMs after transformations

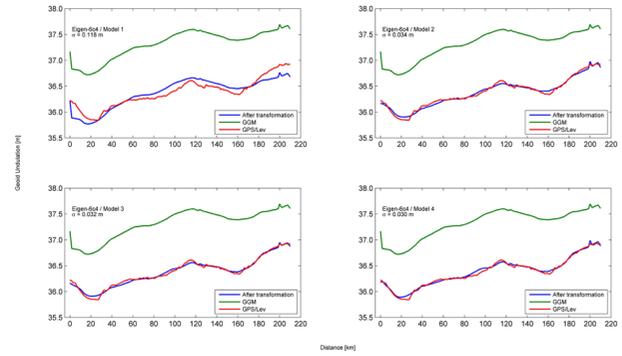


Fig. 10. Undulation values according to eigen-6c4 model

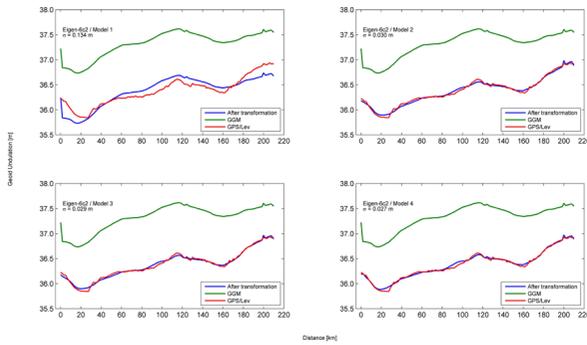


Fig. 7. Undulation values according to eigen-6c2 model

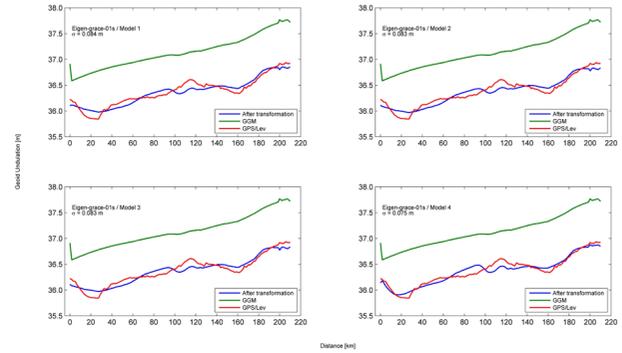


Fig. 11. Undulation values according to eigen-grace-01s model

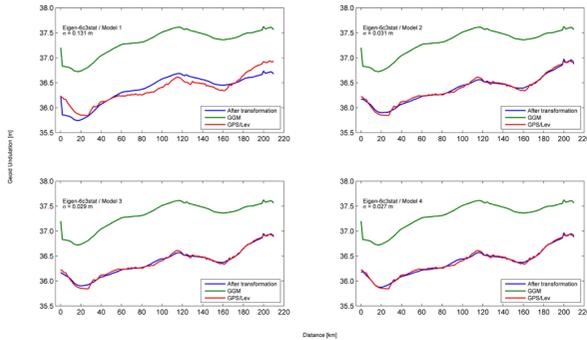


Fig. 8. Undulation values according to eigen-6c3stat model

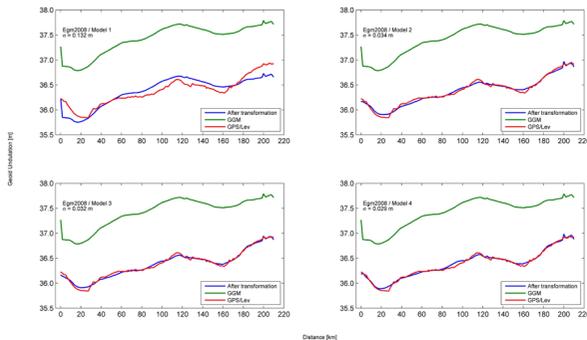


Fig. 9. Undulation values according to egm2008 model

When the Figures 7-11 were examined, it has seen that there were a jump for the first geoid height values calculated from GGMs and the datum shift. The reason for these effects is thought to be systematic errors. Therefore, different parametric models could be used in order to minimize these effects. If the Figures are reviewed, it can be seen that these effects are minimized when the degree of parametric transformation is increased.

The parametric models describing the systematic errors and datum shifts in the different height data sets are used in the study. When the application results and the figures were examined, the 7-parameter transformation model gave results that are the most compatible with minimum standard deviation values when compared to other parametric models. Also, results show that eigen-6c2, eigen-6c3stat, eigen-6c4 and egm2008 global geopotential models produce approximately ± 3 cm accuracy when compared to GNSS/levelling. Especially, 7-parameter transformation of eigen-6c2 and eigen-6c3stat models produces geoid height information with ± 2.7 cm accuracy for 110 points within this test field. This accuracy obtained from GGMs can be seen adequate for several engineering applications. When the contributions of the new satellites going to be launched in the near future are considered, the accuracies obtained from GGMs will increase with each passing day. Thus, the use of GGMs for determining values such as the geoid height, the height anomaly, the gravity field etc. will play an important role in the earth sciences.

4. Conclusion and Suggestions

The aim of this study is to investigate the performance of GGMs produced by CHAMP, GRACE, and GOCE as well as comparing GGMs with GNSS/levelling data, and eventually to determine the best model. Within the scope of this study, the 210-km long Konya-Ankara high-speed train project was selected in order to investigate GGMs performance. When the study area is examined, it can be stated that the route has a flat ground and there is not a sudden height-change structure. While the orthometric height values of the points in the study area range between 696 and 1198 m, the ellipsoidal height

733 - 1234 m. 110 GNSS/levelling (benchmarks) points were included in the study area. Geoid heights of these points were calculated for all GGMs using only latitude and longitude of the points. Then these values were compared to true geoid heights and the performances of GGMs were examined.

According to results, the standard deviation was obtained as approximately 12 cm after comparing with true values of the geoid heights obtained from the best match GGMs. The best GGMs are obtained eigen-6c4, eigen-grace01s, egm2008, eigen-6c3stat, and eigen-6c2, respectively. High degrees of the GGM of all the models except for eigen-grace01s along with its different data variety (gravity and altimeter) contributed to improving geoid height information obtained from the models.

Systematic errors in GGMs such as datum shifts and distortions can be minimized using different parametric models. Therefore, these errors are minimized using four different parametric (3, 4, 5, and 7-parametric) models in the scope of the implementation. When were examined the results after transformation, 7-parameter model compared to other models were produced the better results. According to 7-parameter model results, the best five GGMs are eigen-6c2, eigen-6c3stat, egm2008, eigen-6c4 and eigen-grace01s. Standard deviation value obtained from GGMs is approximately 3 cm for the first four models and 8 cm for the eigen-grace01s model. Standard deviation value obtained from the *eigen-grace01s* is significantly higher compared to the other models. This is most probably related to the data, including only satellite observations, and lower degree ($n_{max} = 140$) of the model.

According to the application results, it may be possible to use 3 cm-geoid height information for various engineering applications. It is obvious that geoid heights obtained from GGMs will be improved through developing technology and new satellite missions to be launched in the near future. It is also possible to conclude that after reaching 1-2 cm geoid height accuracy, height of points will be obtained with high accuracy using GGMs without the need for the levelling process in many engineering applications.

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