Solution and type curves for the seepage model of the water-bearing coalbed with leakage recharge

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ABSTRACT

To analyze the effects of the leakage recharge of the aquifer on the initial dewatering of coalbed methane wells, the mathematical seepage model of water in the coalbed considering the aquifer leakage was established by using the leakage coefficient according to the unsteady seepage theory. The model was solved after Laplace transform and the Stehfest numerical reverse inversion was used to obtain the solution in right space. Then, the log-log type curves of pressure and pressure derivative were created with new combinations of parameters. Based on the natural seepage mechanism, the influence of aquifer leakage on curve shape was judged. It is found that the radial flow ends earlier as the leakage coefficient increases. Moreover, it was proposed to obtain reservoir permeability, skin factor, and leakage coefficient by using type curve matching. The type curves are useful for quantitatively evaluating the level of leakage, thereby guiding the adjustment of the following production system for CBM wells.

Curvas de solución y tipo para el modelo de filtración de capas carboníferas acuíferas con recarga de fugas

RESUMEN

Este estudio estableció el modelo matemático de filtración de agua en una capa carbonífera al estimar la salida acuífera con el uso del coeficiente de fuga, de acuerdo con la teoría de filtración inestable, para analizar los efectos en la recarga de pérdida de fluidos de un acuífero en el drenado inicial para pozos de gas metano. El modelo se resolvió tras usar la transformación Laplace y la inversión numérica Stehfest para encontrar la respuesta en el lugar indicado. Luego, se creó la representación algebraica de la presión y la presión derivativa con nuevas combinaciones de parámetros. Se evaluó la influencia de la pérdida de fluido del acuífero en la forma de la curva con base al mecanismo físico de filtración. Se estableció que el flujo radial finaliza antes de que el coeficiente de pérdida de fluido se incremente. Además, se propone el uso de la curva tipo correspondiente para obtener la permeabilidad del reservorio, el factor de daño y el coeficiente de pérdida de fluido. Las curvas tipo son útiles para evaluar cuantitativamente el nivel de la pérdida de fluído, y de esta manera guiar el ajuste de un sistema de producción consecuente para pozos de gas metano de carbón.
1. Introduction

The coalbed methane (CBM) exists, in the form of adsorbed gas, in the coal reservoir. To recover the CBM, the coalbed is firstly dewatered and depressurized. Only when the coal seam pressure drops to the critical desorption pressure, can the methane be desorbed and diffuse from matrix pores to fracture network. In the Wyodak Anderson coalbed (Onsager and Cox, 2000; Cox and Onsager, 2002) of Fort Union Formation in the Powder River Basin, U.S., the 15# coalbed in Shouyang-Yangquan of the Qinshui Basin, China, and other coal measure strata (Meng 2010), water bodies exist in the upper or lower coalbed, in addition to the coalbed cleat system. The silty or muddy aquitard with a certain thickness of the coalbed and the aquifer has a certain permeability. During the coalbed dewatering, the pressure difference between the aquifer and the coalbed may drive the interlayer leakage recharge, and accordingly water production of the gas well may change, thus impressing the coalbed dewatering and depressurizing.

The scholars in China and abroad have frequently studied interlayer leakage of the reservoir. In the 1960s and 1970s, some scientist elsewhere investigated the reservoir seepage in the case of leakage (Katz and Tek, 1962; Russell and Prats, 1962). In China, Jia Yonghui established the dynamic model of double-layer reservoir with inter-layer leakage in 1997 (Jia 1997). Then proposed the double-layer dual-porosity model, double-layer triple-porosity model and triple-layer confined model with inter-layer leakage (Zhang et al., 2009; Chen et al., 2011; Huo et al., 2006). Sun Hedong, Gao Chengtai, et al. established the dynamic model of infinite triple-layer oil and gas reservoir with inter-layer leakage. Wu Yulu analyzed the causes for the leakage in the multi-layer gas reservoir and established the mathematical seepage model for the multi-layer gas reservoir with leakage [12]. For the models established in previous studies, all layers were perforated for exploitation, and horizontal radial seepage occurred in each layer, except for inter-layer leakage. However, when the coalbed is perforated for production, the aquifers in the roof and floor are selectively avoided, so horizontal radial flow only occurs in the coalbed. Therefore, the existing models for multi-layer reservoir with leakage are not suitable for the analysis of inter-layer leakage in the CBM wells. According to the unsteady seepage theory, the infiltration model for CBM wells considering the inter-layer leakage can be established to obtain the analytical solution of bottom hole flowing pressure under Laplace space. After the solution in true space is obtained by Stehfest numerical inversion, new type curves can be established. Then, these type curves are used to judge whether any leakage recharge exists in the coalbed during dewatering, so as to determine the formation parameters and leakage parameters and analyze the level of leakage recharge.

2. Methods

2.1. Physical Model

As shown in Figure 1, for a CBM well in the infinite formation center, only the coalbed is perforated for dewatering and gas production. Compared with the water content in the coalbed, the water body of the aquifer in the roof and floor can be regarded as infinite water body with a great volume (Su and Lin, 2009). Aquitard with low permeability exists between coalbed and water body. Based on the data statistics of the Powder River Basin in the United States, the vertical permeability of such aquitard is 0.003-0.03 md, suggesting a certain impermeable capacity (Onsager and Cox, 2000).

According to the research on the leakage process of the underground water by Hantush and Jacob, et al., the model is based on the following assumptions (Hantush and Jacob, 1955a; Hantush 1955b): 1) Each layer is homogeneous and isotropic, neglecting the effect of capillary pressure and gravity; 2) Each layer has an infinite boundary; 3) Only the coalbed is perforated, and the formation water in the coalbed flows into the wellbore in the form of horizontal radial flow; 4) The formation water in the floor recharges the coalbed through the aquitard in the manner of vertical seepage and then the inter-layer leakage occurs; 5) The compressibility of the rock in the aquitard is not considered; 6) Due to the significant volume, the water body in the aquifer can be regarded as the constant pressure recharge boundary for the coalbed water.

2.2. Mathematical Model and Solution

In the coalbed with leakage recharge, the basic mathematic equation for water seepage is expressed as (Streltsova, 1988):

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{1}{3.6\eta} \frac{\partial p}{\partial t} + \frac{\mu v_z}{kh}$$

where

$$\eta = \frac{k}{\phi \mu C_i}$$

The aquifer is regarded as the infinite water body, so the boundary between the aquifer and the aquitard can be treated as the constant pressure boundary. The flow rate recharged into the coalbed by the aquifer through the aquitard is

$$v_z = -\frac{k_{v,conf}}{\mu} \frac{\partial p}{\partial z} = \frac{k_{v,conf}}{\mu} \frac{P_{aq} - p}{h_{conf}}$$

Based on the definition of leakage coefficient

$$b = \frac{k_{v,conf}}{h_{conf}}$$

the dimensionless leakage coefficient is defined as:

$$b_D = \frac{k_{v,conf} r_w^2}{kh h_{conf}}$$

Equation (2) is substituted into Equation (1). According to the definitions of dimensionless pressure, dimensionless time and dimensionless radius, the dimensionless partial differential equation is obtained after simplification.

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} + b_D p_D$$

Initial condition: $p_D(r_D, t_D = 0) = 0$ ; Outer boundary condition: $p_D(r_D, t_D = 0) = 0$ ; Inner boundary condition:

$$C_D \left( \frac{\partial p_D}{\partial r_D} \right)_{r_D = 0} = 1$$

$$p_D = p_D - \left( \frac{\partial p_D}{\partial r_D} \right)_{r_D = 0} = 1$$
Solution and type curves for the seepage model of the water-bearing coalbed with leakage recharge

According to the definition of Laplace transform: $\overline{p_D}(z) = \int_{0}^{t} p_D(t)e^{-zt}dt$

Equation (4) is transformed to ordinary differential equation:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = Zf(Z)\overline{p_D}$$

where $f(Z) = \frac{Z + b_D}{Z}.$

Similarly, the expressions of the establish condition in Laplace space are as follows:

$$\overline{F_D}(r_D, 0) = 0$$

$$\overline{p_D}(\infty, Z) = 0$$

$$\overline{C_DZp_D} - \frac{\partial \overline{p_D}}{\partial r_D}\bigg|_{r_D = 0} = \frac{1}{Z}$$

$$\overline{p_wD} = \left(\overline{p_D} - \frac{S}{d_D}\overline{d_D}\right)_{r_D = 0} = 1$$

Equation (5) is zero-order Bessel equation with imaginary argument, and its Laplace spatial solution is

$$\overline{p_D}(Z) = \frac{1}{ZC_D} \left( S + \frac{K_0\left(\sqrt{Z}f(Z)\right)}{\sqrt{Z}\left[\sqrt{Z}f(Z)\right]}\right)^{-1}$$

Due to $S + \frac{K_0\left(\sqrt{Z}f(Z)\right)}{\sqrt{Z}\left[\sqrt{Z}f(Z)\right]} = \ln \frac{2}{\sqrt{\gamma\sqrt{Z}(Z)^{28}}} + \ln \gamma = \ln \frac{2}{\sqrt{\gamma\sqrt{Z}(Z)^{28}}}$, where $\gamma$ is the Euler constant and $\gamma = 1.781$, Equation (10) is deformed as:

$$\overline{p_D}(Z) = \frac{1}{ZC_D} \left[ \ln \frac{2}{\sqrt{\gamma\sqrt{Z}(Z)^{28}}} \right]^{-1}$$

3. Results and Discussion

In the traditional well testing analysis, the reservoir permeability, skin factor, and other formation parameters can be determined, with the measured data in type curve matching. For the coalbed with leakage recharge, the leakage coefficient should also be determined and the level of interlayer leakage recharge be analyzed. Next, considering the dewatering test with continuous production and water injection test with constant flow rate, the corresponding type curves are derived.

3.1. Type Curves of the Bottom hole Pressure in the Dewatering with Constant Production

The drawdown test is conducted to measure the change of the bottom hole fluid pressure with time in the process of dewatering with constant production. According to the ideas of Ramey, Gringarten, and Bourdet to make type curves (Liu, 2003). New type curves applicable to dewatering test analysis with constant production can be derived by establishing a new combination of parameters.

The dynamic change of the bottom-hole pressure tested is always susceptible to the wellbore storage effect. Even before the wellbore storage effect ends, the leakage effect of the aquifer has superseded the influence of after flow. To avoid the interference of wellbore storage effect, the well should be opened or shut in by using downhole control valve in the test process (Yang et al., 2015). The wellbore storage coefficient is minus and can be ignored, i.e. $C_D = 0$.

Let $Z' = Ze^{-28}$, then $Z = Z'e^{28}$ Equation (11) can be simplified as:

$$\overline{p_D}(Z)e^{28} = \frac{\ln \frac{2}{\sqrt{\gamma Z'} + b_De^{-28}}}{Z'}$$

It is proved below that $\overline{p_D}(Z)e^{28}$ is the Laplace transformation result of $p_D$ based on $t_De^{28}$.

$$\overline{p_D}(Z) = \int_{0}^{t_D} p_D(t)e^{-zt}dt,$$ so

$$\overline{p_D}(Z)e^{28} = \int_{0}^{t_D} p_D(t)e^{-Ze^{28}z}dt = \int_{0}^{t_D} p_D(t)e^{-Ze^{28}z}dt_{D}e^{28}$$

Equation (12) can be finally written as:

$$\overline{p_D}(Z) = \frac{\ln \frac{2}{\sqrt{\gamma Z'} + b_De^{-28}}}{Z'}$$

The solution in right space $p_D(t_De^{28})$ can be calculated by using Stehfest numerical inversion. The basic equation is [19]:

$$p_D(t_De^{28}) = \frac{\ln 2}{t_De^{28}} \sum_{i=1}^{N} V_i \overline{p_D}(Z_i)$$

where

$$Z_i = \frac{\ln 2}{t_De^{28}} i$$

$$V_i = (-1)^{N-1} \sum_{k=1}^{N} \binom{N}{k} \frac{K_2(2Kt)}{N-2-k} (K-1) (2K-i)$$

In the log-log coordinate system, a group of dimensionless pressure type curves (Figure 2) are prepared, with $p_D(t_De^{28})$ as ordinate against $t_De^{28}$ as abscissa and $b_De^{28}$ as the curve parameter. Each type curve corresponds to a value of $b_De^{28}$.

Similarly, in the log-log coordinate system, $p_D$ is calculated based on the definition of dimensionless pressure derivative $p_D' = \frac{dp_D}{dt_De^{28}}$. A group of pressure derivative type curves (Figure 3) are drawn, with $t_De^{28} \cdot p_D$ as ordinate against $t_De^{28}$ as abscissa and $b_De^{28}$ as the curve parameter. Each type curve corresponds to a value of $b_De^{28}$.

In Figure 3, when the inter-layer leakage does not happen, i.e. $b_De^{28} = 0$, the model only has radial seepage, and the derivative curve of the bottom-hole pressure is a horizontal line, which conforms to the study results of Gringarten, Bourdet, et al. (Yang et al., 2015).

According to the definition of $D_D$, the effect of aquitard thickness $h_{conf}$ and vertical permeability $k_v_{conf}$ on the formation water leakage
is comprehensively considered. From the underground fluid seepage law, we can see that the smaller the aquitard thickness $h_{\text{conf}}$ is, or the bigger the vertical permeability $k_{v_{\text{conf}}}$ of the aquitard is, the weaker the barrier action of the aquitard on the vertical flow of the formation water is and the stronger the effect of the formation water leakage on coalbed seepage is. Figures 2-3 indicates that the bigger the value of $b_D$ is (i.e. the bigger the value of $b_D e^{-2S}$), the earlier the leakage affects the coalbed water seepage and the earlier the coalbed radial flow stage ends.

In the actual dewatering and production of the coalbed, it is also proved that when the surrounding rock of the coalbed roof and floor has high permeability and strong leakage effect, desorption of CBM can only occur in the small range near the borehole, which is manifested as the deficiency of gas supply (Zhang et al., 2013).

According to the definition of dimensionless pressure

$$p_D = \frac{kh}{1.842 \times 10^{-3} B q u} \Delta p$$

It is shown that the smaller $p_D$ is, the higher the bottom-hole flowing pressure is. In other words, the inter-layer leakage may increase the bottom-hole pressure and affect desorption of methane. According to the characteristics of the type curves, the existence of leakage recharge in the aquifer during testing can be qualitatively judged.

Then, the formation parameters can be obtained by using the type curves shown in Figures 2-3.

### 3.1.1. Formation Parameters Matching by Using the Type Curves

First, the actual testing data obtained from the well site is processed. On the log-log coordinate with the same size of type curves, the curve is drawn with actual differential pressure $\Delta p = p_i - p_{wf}$ as ordinate and actual dewatering time $t$ as abscissa and then matched with the type curves in Figure 2.

Permeability can be obtained through the pressure matching value.

$$k = \frac{1.842 \times 10^{-3} B q u}{h} \left( \frac{p_D}{\Delta p} \right)_{\text{match}}$$

The skin coefficient can be obtained by time matching value.

$$S = \frac{1}{2} \ln \left[ \frac{ \Delta C t_{\text{diff}}^2 }{ 3.6 k t} \left( \frac{t_D e^{2S}}{t} \right)_{\text{match}} \right]$$

The leakage coefficient can be obtained by curve matching value.

$$b = \frac{kh}{r_w^2} \left( \frac{b_D e^{-2S}}{e^{-2S}} \right)_{\text{match}}$$

### 3.1.2. Formation Parameters Matching by Using the Type Curves of Pressure Derivative

Similarly, on the log-log coordinate with the same size of pressure derivative type curves, the curve is drawn with the product of actually measured differential pressure derivative $\frac{dp}{dt}$ against actual dewatering time $t$ as ordinate against actual dewatering time $t$ as abscissa, and then matched with the interpretation chart.

Permeability can be obtained through the pressure derivative matching value.

$$k = \frac{1.842 \times 10^{-3} B q u}{h} \left( \frac{p_D t_D e^{2S}}{\Delta p t} \right)_{\text{match}}$$

By time matching, the skin coefficient can be obtained and its calculation equation is same with Equation (19). By curve value matching, the leakage coefficient can be obtained and its calculation equation is same with Equation (20).

### 3.2. Type Curves of the Bottom-hole Pressure in the Water Injection Test with Constant Production

In the process of dewatering test, the bottom-hole pressure may drop below the critical desorption pressure, resulting in the gas and water (two-phase) flow, which may interfere the well test interpretation. In order to prevent such two-phase flow, the coalbed can be tested by using the method of water injection with constant flow rate. In the process of water injection, the added value of the bottom-hole pressure is

$$p_{wf} (\Delta t) - p_i = \frac{1.842 \times 10^{-3} B q u}{kh} p_D (t_D)$$
The differential pressure is transformed to \( \Delta p = p_{wf} - p_t \), then all the parameters may be determined and the effect or leakage on gas well production can be analyzed by using the type curves in Figure 2 and Figure 3 directly and matching of Equations (18)-(21).

4. Conclusions

(1) The mathematical model for inter-layer leakage in CBM wells is established. Under bottom hole open and shut-in condition, the true spatial solution of the mathematical model is obtained by using Laplace transformation and Stehfest reverse inversion.

(2) New combination of parameters is developed under the precondition of dewatering or water injection with constant production, respectively. On the log-log coordinate, the type curves of \( t_0 \ln(e^{28}) \) and \( t_0 e^{28} \) are drawn, respectively. So the existence of leakage recharge in the aquifer in the process of testing can be judged. The characteristics of the type curves show that the bigger the leakage efficient is, the earlier the leakage recharge is and the higher the bottom-hole pressure is.

(3) By type curve matching, the coalbed permeability, skin factor, and leakage coefficient can be determined. The curve matching contributes to quantitatively evaluating the level of leakage recharge, and judging the effect of the aquifer on the coalbed dewatering and production, so that subsequent dewatering and production can be adjusted timely and properly.

Explanation of Symbols

- \( P \) indicates the pressure, MPa; \( r \) indicates the radius, m; \( \eta \) indicates the pressure transmitting coefficient, cm²/s; \( t \) indicates the production time, h; \( \mu \) indicates the fluid viscosity, mPa·s; \( v_z \) indicates the seepage velocity of aquitard, m/s; \( k \) indicates the permeability of coalbed, mD; \( h \) indicates the coalbed thickness, m; \( \phi \) indicates the porosity, decimal; \( C_i \) indicates the comprehensive compressibility, m³/m³; \( b_{v,coef} \) indicates the permeability of aquitard, mD; \( p_{aq} \) indicates the aquifer pressure, with value equal to initial formation pressure, MPa; \( h_{conf} \) indicates the aquitard thickness, m; \( b \) indicates the leakage coefficient, mD/m; \( b_D \) indicates the dimensionless leakage coefficient, dimensionless; \( r_w \) indicates the borehole radius, m; \( p_D \) indicates the dimensionless pressure, dimensionless; \( f_D \) indicates the dimensionless radius, dimensionless; \( t_D \) indicates the dimensionless time, dimensionless; \( C_D \) indicates the dimensionless wellbore storage coefficient, dimensionless; \( P_{wD} \) indicates the dimensionless bottom-hole flowing pressure, dimensionless; \( S \) indicates the skin factor, dimensionless; \( Z \) indicates the Laplace transform variable based on \( t_D \), dimensionless; \( P_{wD} \) indicates the image function of Laplace transform about dimensionless bottom-hole pressure, dimensionless; \( k_0 \left( z^2 \right) \) indicates zero-order second class modified Bessel function, dimensionless; \( k_1 \left( z^2 \right) \) indicates first-order second class modified Bessel function, dimensionless; \( Y \) indicates Euler’s constant, \( \gamma = 1.781 \); \( Z' \) indicates the Laplace transform variable based on \( t_0 e^{28} \), dimensionless; \( i \) indicates the natural number, dimensionless; \( N \) indicates the incremental number; \( P_{D0} \) indicates the dimensionless pressure derivative, dimensionless; \( q \) indicates daily water production, m³/d; \( B \) indicates the water volume factor, m³/m³; \( \Delta p' \) indicates the producing pressure drop, MPa; \( P_i \) indicates the initial formation pressure, MPa; \( P_{wf} \) indicates the bottom-hole flowing pressure, MPa; \( \Delta p \) indicates the drawdown pressure derivative.

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References


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