



Inversion Technique of Physical Parameters Based on Regularization Extension Depth Constraint

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ABSTRACT

Through the regularization downward continuation of gravity and magnetic anomalies, the depth of the field source can be solved. However, due to the Gibbs effect, the horizontal resolving power of the field source is poor. In view of this, based on the depth of field source established by regularization downward continuation, this paper proposes a physical property parameter inversion method based on iterative continuation and anomaly separation, which can effectively improve the inversion accuracy of superimposed anomaly physical parameters, and provide a new idea for solving the physical parameters of superposition gravity and magnetic anomalies.

Keywords: regularization; iterative continuation; anomaly separation; physical parameters.

Técnica de inversión de parámetros físicos basada en la regularización de restricción de extensión de profundidad

RESUMEN

A través de la regularización de la gravedad y las anomalías magnéticas, se puede resolver la profundidad de la fuente del campo. Sin embargo, debido al efecto Gibbs, el poder de resolución horizontal de la fuente de campo es poco. En vista de esto, basado en la profundidad de la fuente de campo establecida por la continuación de la regularización hacia abajo, este artículo propone un método de inversión de parámetros de propiedades físicas basado en la continuación iterativa y la separación de anomalías, que puede mejorar efectivamente la precisión de inversión de los parámetros físicos de anomalías superpuestas, y proporcionar una nueva idea para resolver los parámetros físicos de la superposición de gravedad y anomalías magnéticas.

Palabras clave: regularización; continuación iterativa; separación de anomalías; parámetros físicos; inversión.

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Introduction

The downward continuation of the potential field can provide abundant information on the spatial distribution of the potential field and is an important part of the interpretation of gravity and magnetic anomalies (Zhu, Cao, & Lu, 2014; Deng, Liu, Song, & Chen, 2014). Luan Wengui adopted another regularization functional and deduced another response. According to the same regularization functional, another response formula was derived by Luan Wengui, which is suitable for understanding the distribution characteristics of the source potential field in the near field (Zhang, Dai, & Liu, 2014). C. W. Groetsch, P. Mauriello, D. Patella, Xu Deshu, Zeng Hualin, Wang Xuben, Gao Yongcai and other scholars have also studied the extension method. Through research and comparison, it shows that the regularization method has the best downward extension effect. But due to the influence of the Gibbs effect, on both sides of the main anomaly zone, there is often a “positive and negative alternating “ λ ” shaped weak anomaly zone with the main source distribution area as the center, which may be a separate anomaly or composite anomalies formed by the superposition with other anomalies, thus increasing the difficulty to explain the field source. To solve this problem, based on the gravity and magnetic data regularization downward extension, this paper puts forward downward gravity and magnetic property parameters inversion method of downward continuation and anomaly separation based on the iterative method, providing a new way of thinking for the comprehensive interpretation of the field source from the material properties and geometric parameters (Al Farajat et al., 2015; Li & Zhang, 2017; Mazgaj, Szular, & Szczurek, 2017).

Basic principle

Regularization extension technique

According to the Laplace equation and the boundary condition of the potential field, there is:

$$\begin{cases} \partial^2 U(x, z) / \partial x^2 + \partial^2 U(x, z) / \partial z^2 = 0 \\ U(x, z)_{x \rightarrow L} = 0 \\ U(x, z)_{z = \phi(x)} = f(x, z) \end{cases} \quad (1)$$

$U(x, z)$ is the potential field at the measuring point (x, z) , and $f(x, z)$ is the known field value on the measuring point; $\phi(x)$ is the height of the measuring point. By using the method of separating variables and the boundary condition, the general formula of the series decomposition of the field potential can be obtained (Boehm & Ulbrich, 2015).

$$f(x, z) = \sum_{i=0}^N a_i \cos(\lambda_i x) e^{\lambda_i z} \quad (2)$$

Set M as the number of points at the measuring point, and Δx is the distance between the measuring points, there are

$$x = (i-1) \cdot \Delta x$$

$$\lambda_i = i \cdot \pi / (M-1) \cdot \Delta x$$

This result is the series decomposition method of the traditional downward continuation (Raknes & Arntsen, 2014).

In order to overcome the oscillatory effect, the series is still taken as the basic form of the field value solution. In solving the continuation function, a decomposition function with a relatively smooth variation is selected and the error of fitting it to the original field is minimized on the original observation profile, so that the analytical function of regularized downward continuation series after correction can be obtained (Zhang et al., 2015; Fontchastagner, Lubin, Mezani, & Takorabet, 2018).

$$f(x, z) = \frac{1}{2} a_0 + \sum_{i=1}^N a_i \cos(\lambda_i x) e^{\lambda_i z} v_i \quad (3)$$

v_i is called the regularization factor, which is $1 / (1 + \alpha \lambda_i^2 e^{2\lambda_i z})$, a function related to the frequency λ_i , continuation depth z and coefficient α . It corrects the frequency of the series term of the continuation field, and it is proved by practice that as long as the regularization function is properly selected, it cannot only guarantee the non-singularity of the potential field in the process of continuation, so as to obtain the complete field value distribution in the lower half space, but also the Gibbs effect cannot be very significant.

Iterative continuation technique

It is often necessary to extend downwards to the top of different depth layers to separate the anomalies produced by different depth layer sources on the ground. The stability and depth of continuation directly affect the result of continuation. To deal with the instability of potential field downward continuation and the limited depth of downward continuation by the classical *FFT* method, Xu Shizhe put forward a downward continuation method in spatial domain, which is implemented based on stable *FFT* upward continuation. The principle is simple and the downward continuation is stable. The basic idea of the downward continuation of the iterative method is to assume that there are two planes, ΓA and ΓB , representing the ground surface and the plane of a certain depth under the ground, respectively. Between the two, there is no field source distribution, that is, there is a passive space, so that the downward continuation iterative solution of the known depth based on upward continuation can be realized according to the following diagram (Chen, 2015).

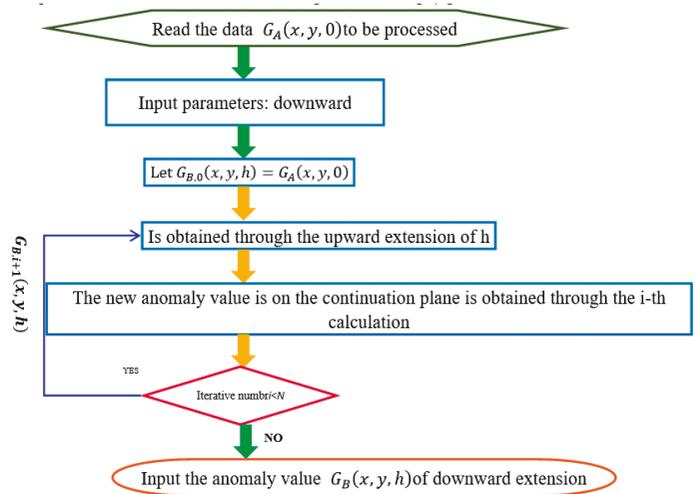


Figure 1. Implementation diagram of downward extension through the iterative method

Inversion of physical parameter of gravity and magnetic anomaly

Assuming the thickness of a certain depth of the underground layer as h , the layer is divided into MN ($MN = M \times N$) small vertical prisms of the same size with a finite extension, and the coordinate of the center point of the small prisms is (x_{0j}, y_{0j}) and residual density is ρ_j ($j = 1, 2, 3, \dots, MN$); its side length is a and b respectively; the top depth of the prism is h and the height is dh . Thus, the formula for the frequency spectrum $\tilde{g}_j(u, v)$ of the gravity anomaly $g_j(x, y)$ generated by the ground (x, y) of each prism in the frequency domain is as follows:

$$\tilde{g}_j(u, v) = 2\pi G \rho_j \cdot \frac{4}{uv} \sin\left(\frac{au}{2}\right) \sin\left(\frac{bv}{2}\right) \cdot \frac{e^{-hr}}{r} \cdot (1 - e^{-dhr}) \cdot e^{-i(ux_{0j} + vy_{0j})} \quad (4)$$

In the upper formula, u, v represent the wave number of x, y respectively; G is the gravitational constant; and $r = \sqrt{u^2 + v^2}$. The frequency spectrum $\tilde{g}(u, v)$ is made Fourier inversion to obtain the gravity anomaly $g(x, y)$ in the space domain.

According to the superposition property, the frequency spectrum $\tilde{g}(u, v)$ of gravity anomaly $g(x, y)$ produced by the whole depth layer at the ground (x, y) is as follows:

$$\tilde{g}(u, v) = 2\pi G \cdot \frac{4}{uv} \sin\left(\frac{au}{2}\right) \sin\left(\frac{bv}{2}\right) \cdot \frac{e^{-dhr}}{r} \cdot (1 - e^{-dhr}) \sum_{j=1}^{MN} \rho_j \cdot e^{-i(ux_0 + vy_0)} \quad (5)$$

Formula 5 is the forward formula of field source gravity anomaly with known physical parameters.

If the gravity anomaly $g(x, y)$ generated by the depth layer in the top surface is known, $h = 0$ (if $h \neq 0$, that is, the buried depth is not zero, and it can be extended to the top interface first, and then:

$$\tilde{\rho}(u, v) = \sum_{j=1}^{MN} \rho_j \cdot e^{-i(ux_0 + vy_0)} \quad (6)$$

Thus:

$$\tilde{g}(u, v) = 2\pi G \cdot \frac{4}{uv} \sin\left(\frac{au}{2}\right) \sin\left(\frac{bv}{2}\right) \cdot \frac{(1 - e^{-dhr})}{r} \cdot \tilde{\rho}(u, v) \quad (7)$$

The following formula can be deduced from the above:

$$\tilde{\rho}(u, v) = \frac{\tilde{g}(u, v)}{2\pi G \cdot \frac{4}{uv} \sin\left(\frac{au}{2}\right) \sin\left(\frac{bv}{2}\right) \cdot \frac{1 - e^{-dhr}}{r}} \quad (8)$$

Thus, by Fourier inverse transformation, the residual density values of each point (x, y) on the top surface of the depth layer can be obtained:

$$\rho(x, y) = F^{-1} \left[\frac{\tilde{g}(u, v)}{2\pi G \cdot \frac{4}{uv} \sin\left(\frac{au}{2}\right) \sin\left(\frac{bv}{2}\right) \cdot \frac{1 - e^{-dhr}}{r}} \right] \quad (9)$$

Formula 9 is the inversion formula for calculating the relative physical parameters of the gravity anomaly of the known field source. The apparent density corresponding to the gravity anomaly can be calculated by the formula.

For the magnetic source anomaly, the gravity anomaly of the magnetic source can be obtained first, and the magnetic source density or magnetization intensity can be inverted by the same method. In view of the space, its calculation formula is not deduced in this paper.

Gravity anomaly regularized continuation depth constraint inversion

Regularized continuation depth constrained inversion of single model gravity and magnetic anomalies

In this paper, a 101×101 mesh is designed, the distance between points and that between lines is all 1 (unit: meter), that is, the measuring area is $100 \times 100 \text{m}^2$. The specific parameters of the sphere model are shown in Table 1.

Table 1. Specific parameters of a single sphere model

Spherical center coordinate (m)	Sphere radius (m)	Residual Density (g/cm^3)	Magnetization intensity (A/m)	Magnetization inclination	Magnetization deviation angle
(50,50,20)	5	0.5	100	45°	0°

The corresponding gravity anomalies (Δg) and magnetic anomalies (ΔT) are derived from the forward calculation of the above model, as shown in Figure 2:

Figure 3 is a section of the regularization result of the above magnetic anomalies, which shows that the center of the anomaly is consistent with the depth of the given model. However, due to the influence of the Gibbs effect,

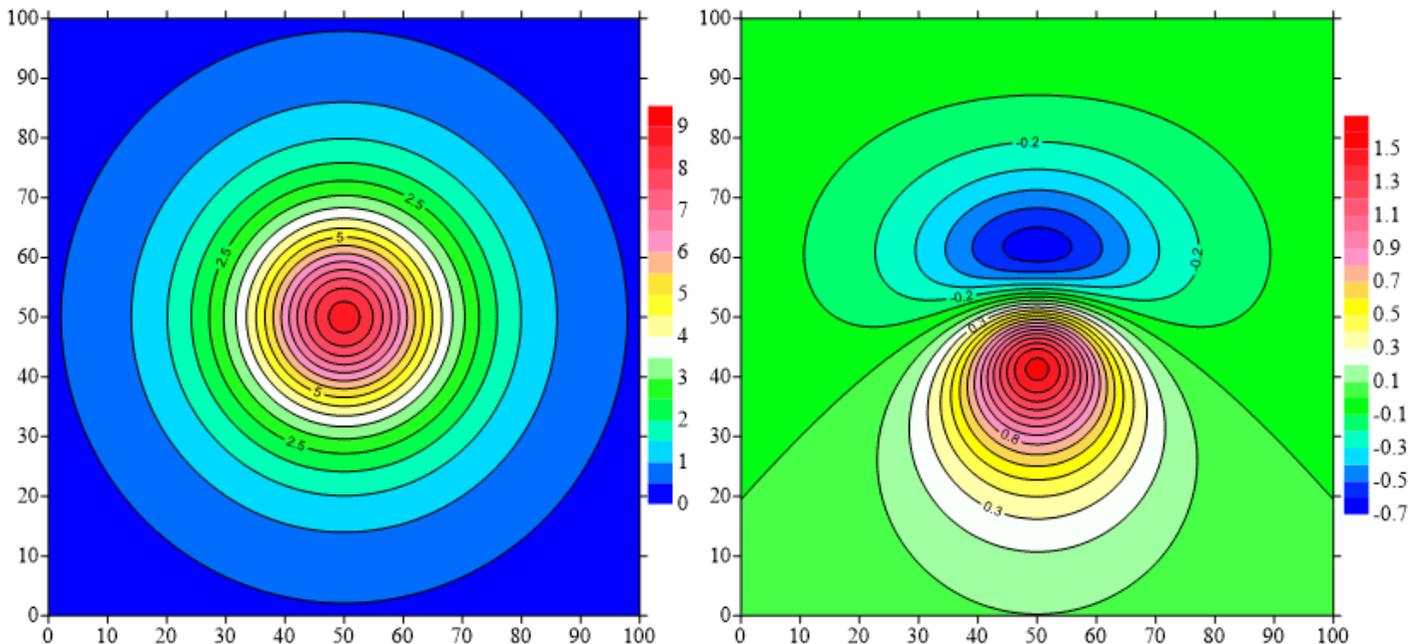


Figure 2. Gravity anomaly (left) and magnetic anomaly (right) of the sphere model

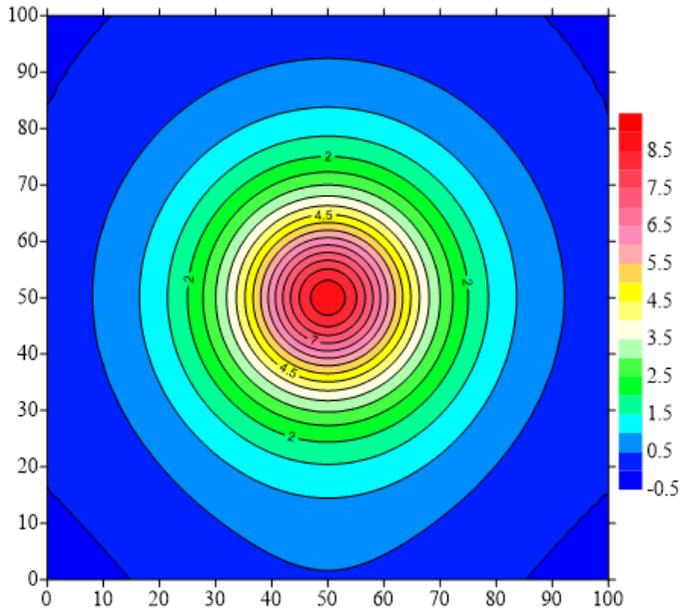


Figure 3. Magnetic source gravity anomaly of sphere model

on both sides of the main anomaly zone, there is often a “positive and negative alternating “) (“ shaped weak anomaly zone with the main source distribution area as the center, which may be a separate anomaly or form composite anomalies by the superposition with other anomalies, thus increasing the difficulty to explain the field source.

Regularization continuation depth constrained inversion of superimposed gravity and magnetic anomalies

In order to carry out theoretical calculation and verification, a 201×201 mesh is designed in this paper. The distance between points and that between lines is all 5 (unit: meter, that is, the range of measuring area is $1000 \times 1000m^2$). The model consists of three vertical prisms, in which the column centers of model 1 and model 3 are on the same vertical line, and the specific parameters of each column model are shown in Table 2:

According to the relevant parameters of the above model, the gravity anomaly and magnetic anomaly (unit: nT) of the model can be obtained by forward modeling (unit: mGal). The result is shown in Figure 6.

In order to extend this method to the processing of magnetic anomalies effectively, the magnetic anomalies’ magnetic source gravity is solved first, and the magnetic anomalies are also solved according to the gravity anomaly process. Figure 7 shows the magnetic source gravity anomaly of the magnetic source model.

The anomaly of the main section of the model is selected for regularized downward continuation, so that the depth of the field source is obtained respectively. It can be seen from the diagram that the depth of the regularization extension has a high accuracy.

The regularization is used to establish the depth of the field source for iterative continuation. Considering that the basic requirement of iterative continuation is that the two depth interfaces are passive field spaces, the idea of this paper is to carry out the continuation of shallow source depth first; then, the shallow-source anomaly is separated by cutting method, and the cutting radius is established according to the anomaly range of shallow-source anomaly. (the idea of this paper is to use Laplace operator to establish abnormal boundary and then the radius is taken as $(L+D)/2$ according to the anomaly boundary.)

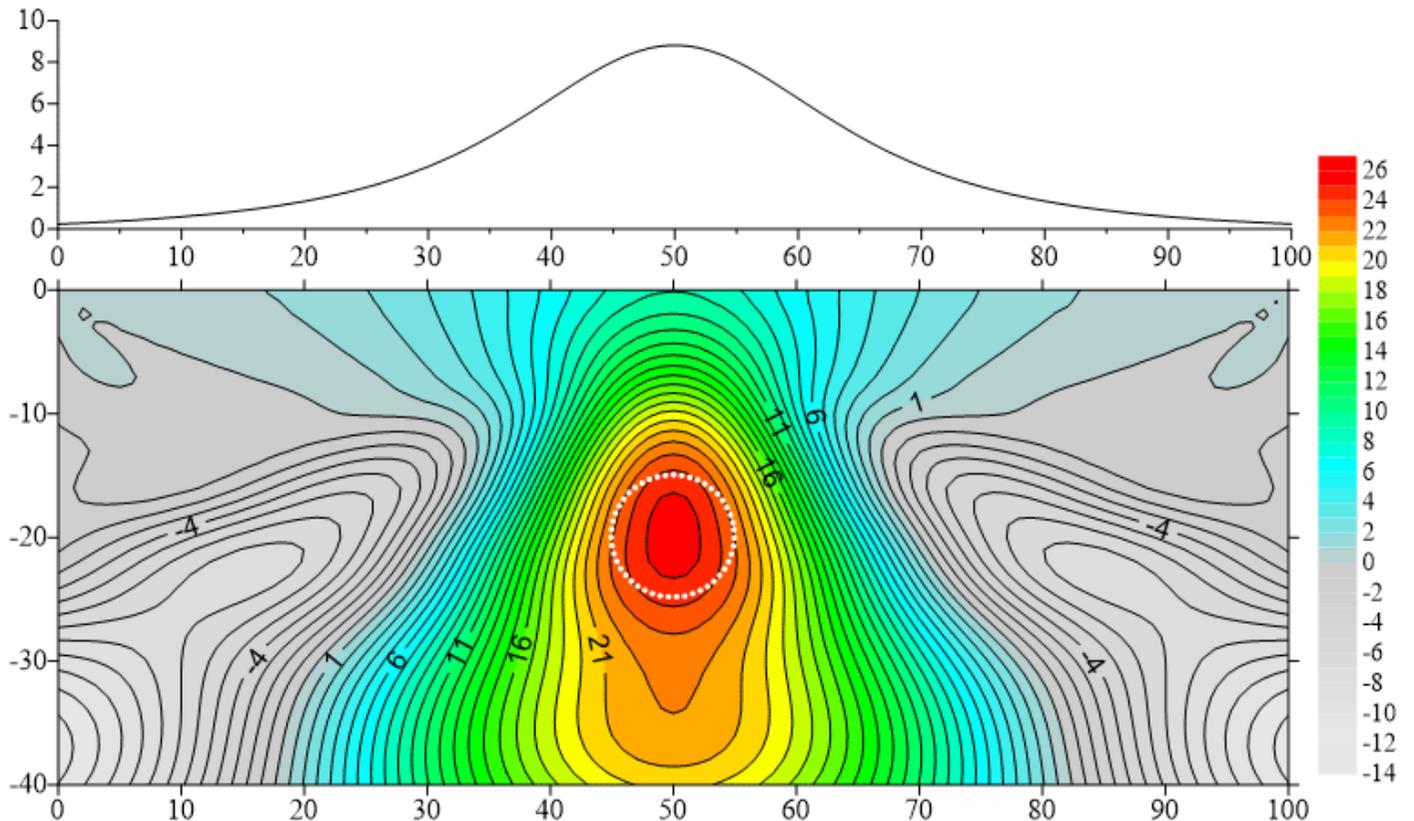


Figure 4. The principal curve of magnetic source gravity anomaly and its regularization downward extension fracture surface

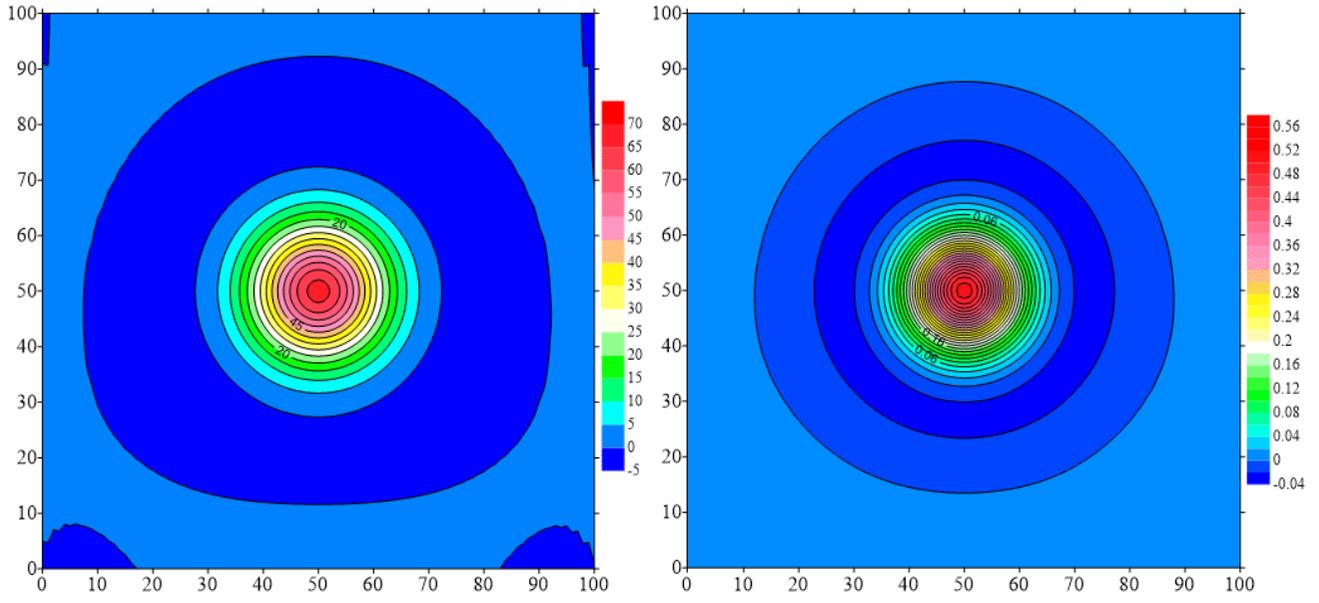
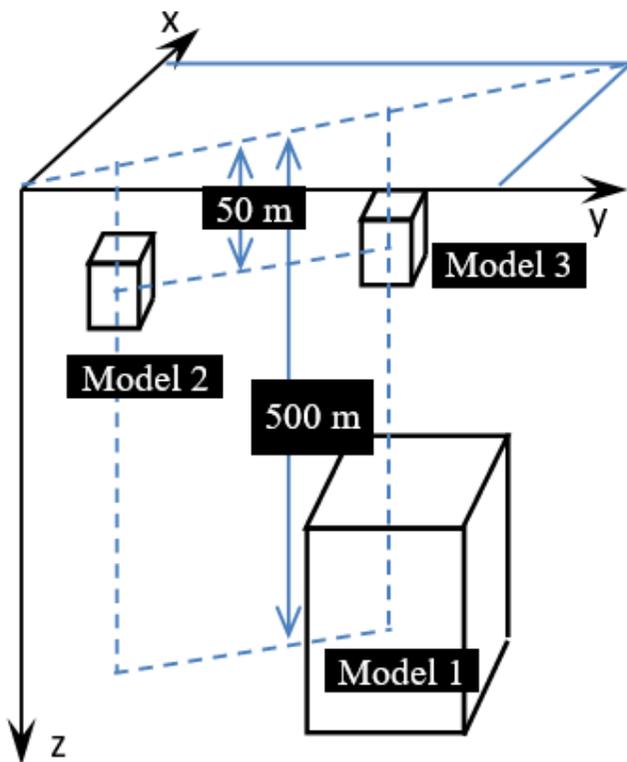


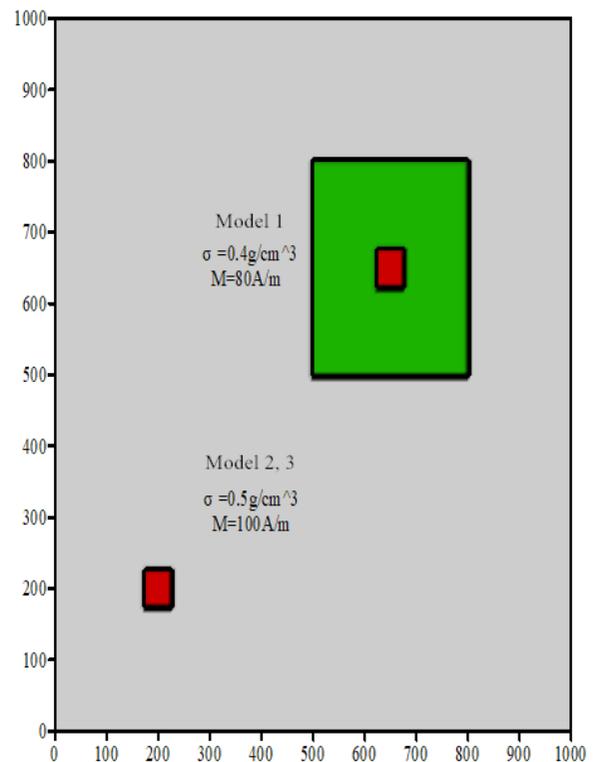
Figure 5. Magnetic source gravity anomaly (left) and apparent density inversion results (right)

Table 2. Specific parameters of the combined model

Parameters cylinder	Cylindrical center coordinate (m)	Cylinder length (m)	Cylinder width (m)	Cylinder height (m)	Density (g/cm ³)	Magnetization intensity (A/m)	Declination	Magnetic dip
1	(650,650,500)	300	300	100	0.4	80	0°	45°
2	(200,200,50)	50	50	50	0.5	100	0°	45°
3	(650,650,50)	50	50	50	0.5	100	0°	45°



(a) Spatial position of the model



(b) Model plane position and parameters

Figure 6. Model and its forward modelling anomaly

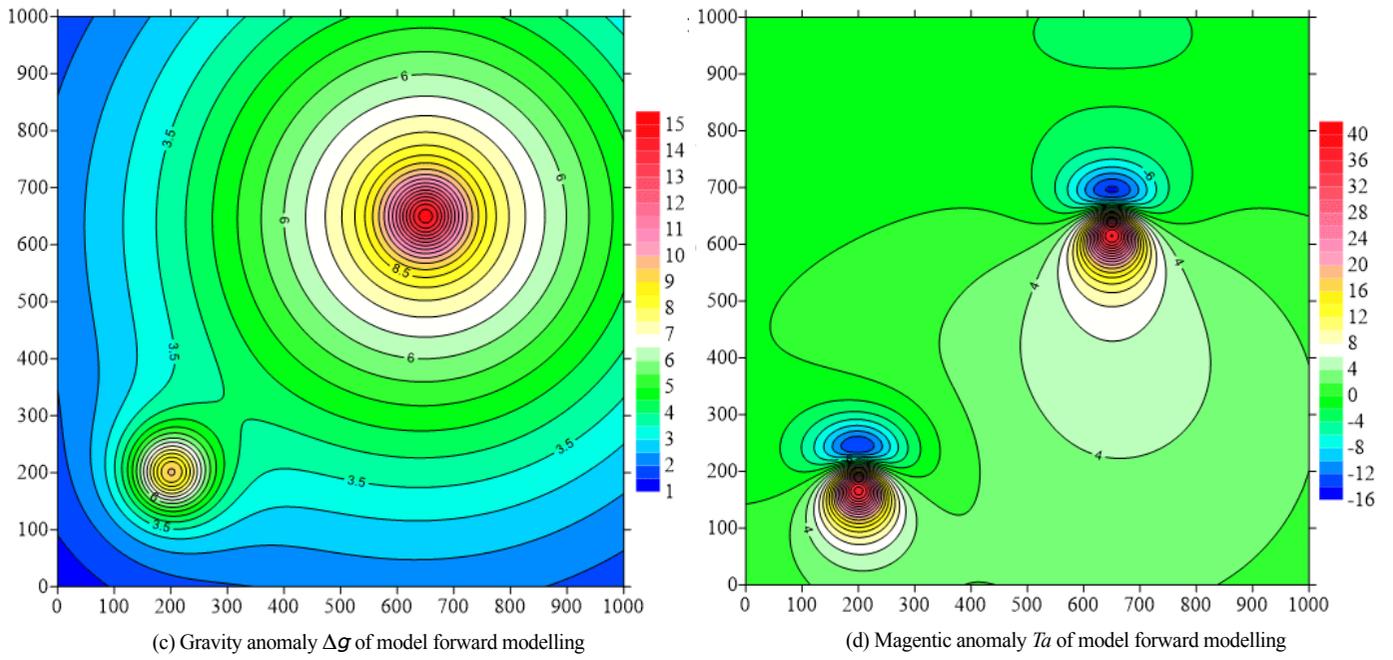


Figure 6. Model and its forward modelling anomaly

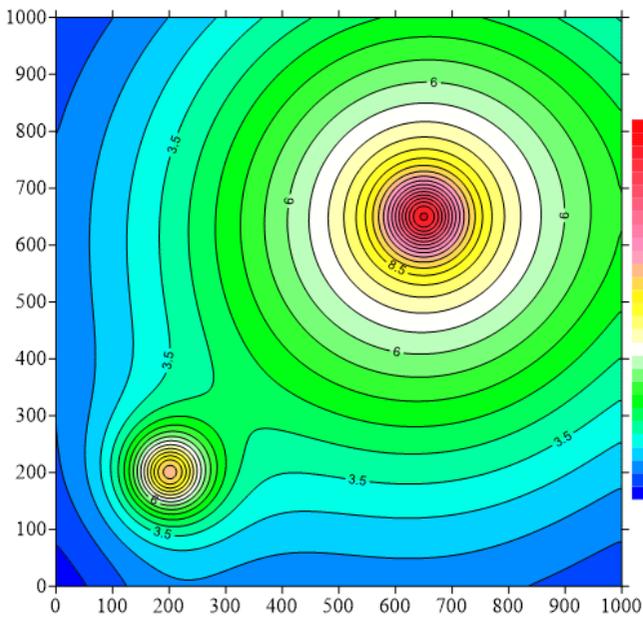


Figure 7. Magnetic source gravity of the model

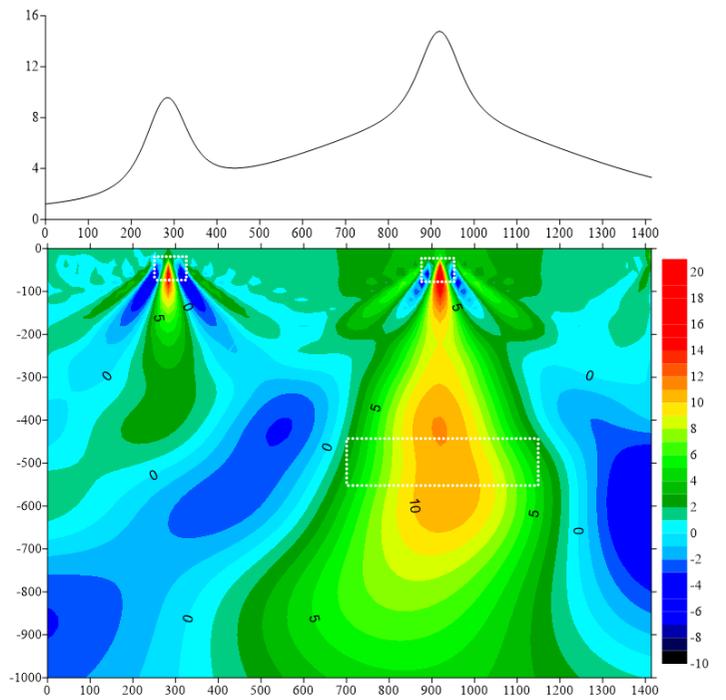


Figure 8. Anomaly curve of diagonal section of magnetic source gravity anomaly and its regularized extension

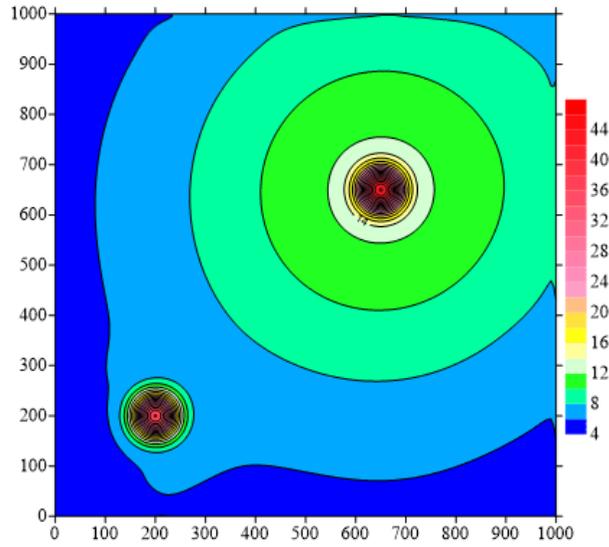


Figure 9. Magnetic source gravity's anomaly value after the downward extension of 50 m

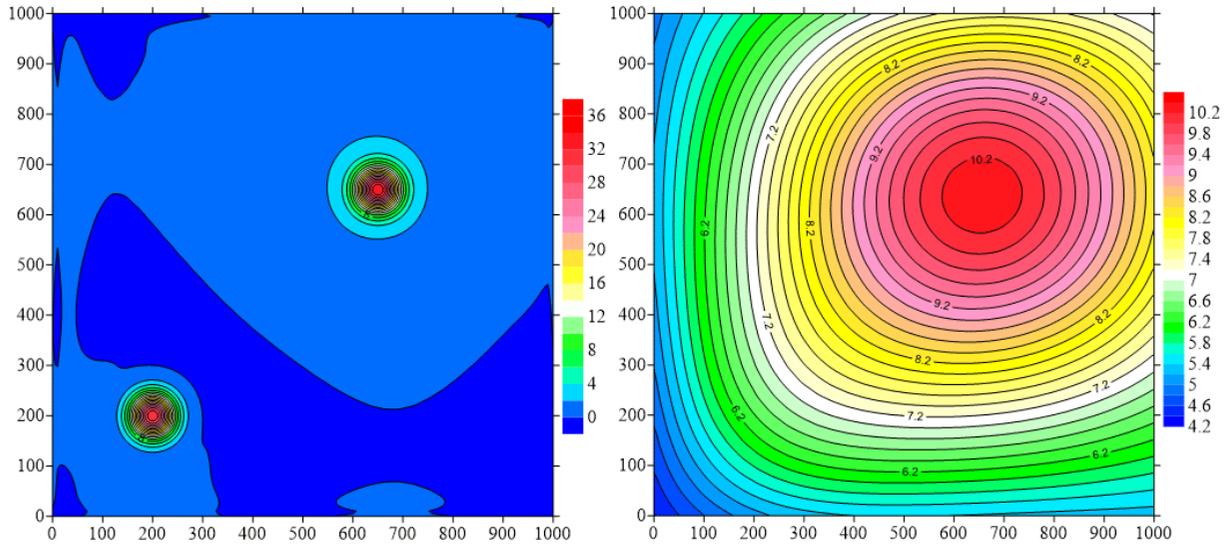


Figure 10. Shallow anomaly (left) and deep anomaly (right) obtained by cutting 50 m depth anomaly

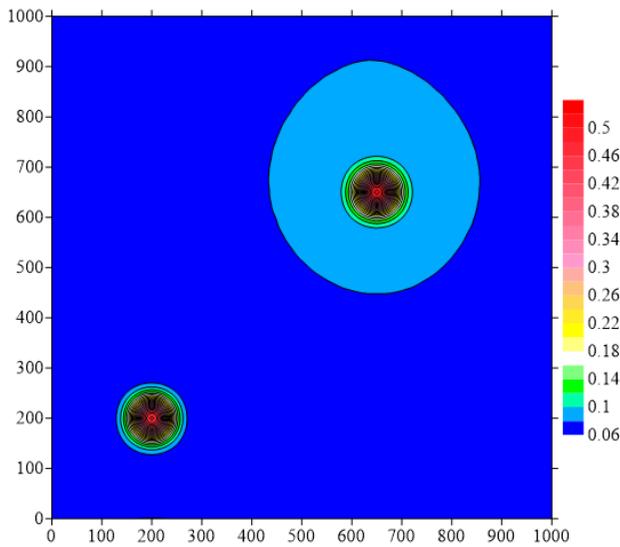


Figure 11. The density value obtained by the shallow anomaly inversion

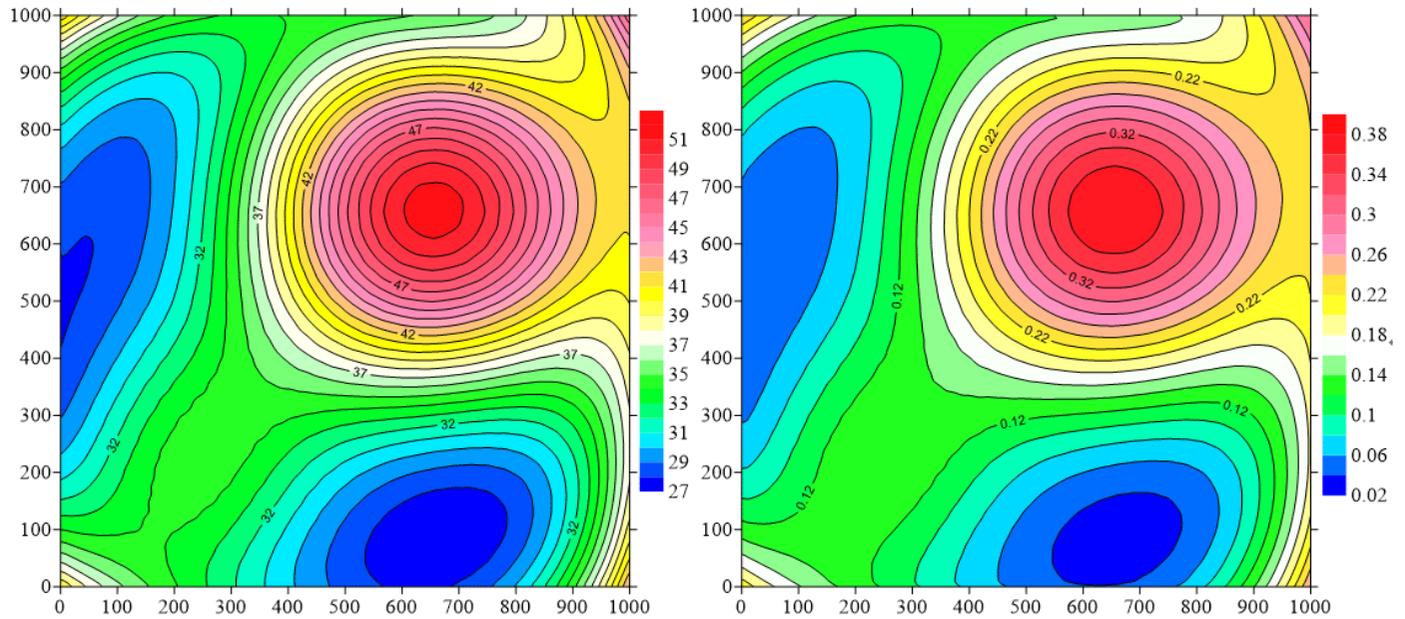


Figure 12. Abnormal value of the extension of 450 m of depth anomaly (left) and its inversion density (right)

Conclusion

Based on the potential field theory and regularization theory, a method for inversion of gravity and magnetic physical property parameters of downward continuation and anomaly separation based on iterative method is proposed. The following points should be pointed out:

- 1) The inversion effect based on single model depth constraint is better;
- 2) The lateral superposition model can obtain higher lateral resolution anomalies in the process of iterative downward continuation. Therefore, the inversion of physical parameters has a higher accuracy, while the vertical superposition anomaly and physical properties are related to the separation accuracy of anomalies.
- 3) The study of high accuracy anomaly separation is the key to the inversion of depth constrained physical parameters of complex geological models.

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References

- Al Farajat, M., Schaefer, B., Al Hassanat, H., Al Atteyat, N., Al Jahed, N., & Khataibeh, J. (2015). Using GIS and geophysics in selecting suitable basins with freshwater aquifers for an efficient exploration strategy - A case study from Petra-region, Jordan. *Earth Sciences Research Journal*, 19(1): 39-50.
- Boehm, C. & Ulbrich, M. (2015). A semismooth newton-cg method for constrained parameter identification in seismic tomography. *Siam Journal on Scientific Computing*, 37(5), S334-S364.
- Chen, Y. (2015). Iterative Deblending With multiple constraints based on shaping regularization. *IEEE Geoscience & Remote Sensing Letters*, 12(11), 2247-2251.
- Deng, C. L., Liu, H. J., Song, B. H., & Chen, J. (2014). Density parameters inversion for structures based on regularization method. *Journal of Guilin University of Technology*.
- Fontchastagner, J., Lubin, T., Mezani, S., & Takorabet, N. (2018). Design optimization of an axial-field eddy-current magnetic coupling based on magneto-thermal analytical model. *Open Physics*, 16(1), 21-26.
- Li, C. & Zhang, F. (2017). AVA inversion based on the L1-norm-based likelihood function and the Total Variation Regularization constraint. *Geophysics*, 82(3), 1-54.
- Mazgaj, W., Szular, Z., & Szczurek, P. (2017). Calculations of magnetic field in dynamo sheets taking into account their texture. *Open Physics*, 15(1), 1034-1038.
- Raknes, E. B. & Arntsen, B. (2014). Time-lapse full-waveform inversion of limited-offset seismic data using a local migration regularization. *Geophysics*, 79(3), WA117-WA128.
- Zhang, F., Dai, R., & Liu, H. (2014). Seismic inversion based on L1-norm misfit function and total variation regularization. *Journal of Applied Geophysics*, 109, 111-118.
- Zhang, Y., Yan, J., Li, F., Chen, C., Mei, B., Jin, S., & Dohm, J. H. (2015). A new bound constraints method for 3-D potential field data inversion using Lagrangian multipliers. *Geophysical Journal International*, 201(1), 267-275.
- Zhu, Z. Q., Cao, S. J., Lu, G. Y. (2014). 3D gravity inversion with bound constraint based on hyper-parameter regularization. *The Chinese Journal of Nonferrous Metals*, 24(10), 2601-2608.