Revista Colombiana de Estadística - Theorical Statistics
January 2023, volume 46, issue 1, pp. 81–91
http://doi.org/10.15446/rce.v46n1.104427

An Improved Estimator of Finite Population Variance Using two Auxiliary Variable SRS

Un estimador mejorado de varianza de población finita utilizando dos atributos auxiliares en SRS

Sunil Kumar[a], Monica Choudhary[b]

Department of Statistics, University of Jammu, Jammu, India

Abstract

In the present study, we explore the problem of estimation of finite population variance in simple random sampling (without replacement) by utilizing information of two auxiliary variables. A ratio cum exponential estimator has been proposed and its properties are studied to the first degree of approximation. To demonstrate the efficiency, members of the proposed estimator as well as other existing estimators are compared to the usual unbiased estimator. To study the performance, a simulation study is undertaken for both real and artificial population using R software. The suggested estimator is found to be more efficient than other existing estimators in terms of having minimum MSE.

***Key words*:** Simple random sampling; Variance estimation; Study variable; Auxiliary variable; Bias; Mean square error.

Resumen

En el presente estudio, exploramos el problema de la estimación de la varianza de una población finita en un muestreo aleatorio simple (sin reemplazo) utilizando la información de dos variables auxiliares. Se ha propuesto un estimador de razón cum exponencial y se estudian sus propiedades hasta el primer grado de aproximación. Para demostrar la eficiencia, los miembros del estimador propuesto, así como otros estimadores existentes, se comparan con el estimador insesgado habitual. Usando el software R, se lleva a cabo un análisis de simulación para respaldar las conclusiones teóricas. Se determina que el estimador sugerido es más eficiente que otros estimadores existentes según el análisis del estudio de simulación.

***Palabras clave*:** Muestreo aleatorio simple; Estimación de la varianza; Variable de estudio; Variable auxiliar; Sesgo; Error cuadrático medio.

[a]Professor. E-mail: sunilbhougal06@gmail.com
[b]Professor. E-mail: monicachoudhary68071@gmail.com

1. Introduction

In the theory of survey sampling, it is unanimously accepted that the appropriate use of auxiliary information improves the efficiency of the estimators of the population parameter under consideration for the variable of interest. The ratio, product, and regression estimators are well-known examples. Such estimators take advantage of the correlation between the auxiliary variable(s) and the study variable; vide Murthy (1967) and Das (1988). Isaki (1983) proposed ratio estimator, usually known as a classical estimator of the finite population variance. Kadilar & Cingi (2006) extended the idea of Isaki (1983) ratio estimator for population variance by utilizing the information available about the coefficient of variation and coefficient of kurtosis of the auxiliary variable under simple random sampling. Gupta & Shabbir (2008) gave a hybrid class of variance estimator. Subramani & Kumarapandiyan (2012) further developed the usual ratio-type variance estimators using lower and upper quartiles, inter-quartile range, quartile deviation, and quartile average of the auxiliary variables. Further, Subramani & Kumarapandiyan (2013) developed more efficient modified ratio-type estimators using median and coefficient of variation of the auxiliary variable. Some recent significant contributions in this direction include Bhushan et al. (2021) and Shabbir et al. (2022).

Variations may be seen in almost every sphere of human life. According to the law of nature, no two objects or people are exactly the same. For example, a doctor must have a thorough knowledge of the fluctuations in human blood pressure, body temperature, and pulse rate to prescribe properly. In addition, to ensure the productivity of the crop, an agriculturist must understand the different climatic factors that affect the climate. Many practical situations arise where the problem of estimation of population variance of the study variable is needed. Some modified and efficient class of estimators for population variance using auxiliary attributes are given by Bhushan et al. (2021) and Bhushan et al. (2022).

Usually, the ratio estimators do not work efficiently in situations when the relationship between the auxiliary and study variables is not linear. In such situations, the exponential product and ratio estimators proposed by Bahl & Tuteja (1991) are used. Bahl & Tuteja (1991) suggested new exponential ratio and product type estimators for estimating the population mean of the study variable using auxiliary information. In the present paper, we propose a ratio-cum-exponential estimator for population variance of the study variable using the information on an auxiliary variable(s) that is(are) correlated with the study variable and have shown that the suggested estimator is more efficient than the existing estimators.

Further, the paper is organized as follows: in section 2, we have discussed some existing estimators and used notations are defined. The proposed estimator along with its properties are discussed in section 3. In section 4, we compare the proposed estimator with the usual and other considered existing estimators and the efficiency conditions are developed. Section 5 provides a simulation study to check the effectiveness of the proposed estimator. At Last, the conclusion is discussed in section 6.

2. Notations and Existing Estimators

A sample of size n is selected from a finite population $P = (P_1, P_2, \ldots, P_N)$ of size N by using a simple random sampling without replacement to estimate the population variance. Let Y denote the study variable, and X and Z are the auxiliary variables. Here, we use the following notations:
$\bar{Y}$: the population mean of study variable.
$\bar{X}$ & $\bar{Z}$: the population mean of auxiliary variables.
S_y^2, S_x^2 and S_z^2: the population variances of the study variable Y and auxiliary variables X & Z, respectively.
s_y^2, s_x^2 and s_z^2: the sample variances of the study variable y, and auxiliary variables x and z, respectively.

We define expectations in order to get the mean square errors of suggested estimators.

$s_y^2 = S_y^2(1+e_y), s_x^2 = S_x^2(1+e_x)$ and $s_z^2 = S_z^2(1+e_z)$. $E(e_y) = E(e_x) = E(e_z) = 0$. $E(e_y^2) = f(\lambda_{400}-1)$, $E(e_x^2) = f(\lambda_{040}-1)$, $E(e_z^2) = f(\lambda_{004}-1)$, $E(e_y e_x) = f(\lambda_{220}-1), E(e_y e_z) = f(\lambda_{202}-1), E(e_z e_x) = f(\lambda_{022}-1)$

where $f = (\frac{1}{n} - \frac{1}{N})$, $g = \frac{n}{N-n}$, $\lambda_{rsq} = \dfrac{\mu_{rsq}}{\mu_{200}^{r/2}\mu_{020}^{s/2}\mu_{002}^{q/2}}$ and $\mu_{rsq} = \dfrac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r(X_i-\bar{X})^s(Z_i-\bar{Z})^q$, $(r, s$ and $q)$ are non-negative integer and μ_{200}, μ_{020} and μ_{002} are the second order moment, and λ_{rsq} is the moment ratio.

Some existing estimators are defined as:

Based on a finite population, the usual unbiased estimator of variance is defined as

$$T_1 = s_y^2 \tag{1}$$

Isaki (1983) proposed the ratio estimator using one auxiliary variable and is defined as

$$T_2 = s_y^2\left(\frac{S_x^2}{s_x^2}\right) \tag{2}$$

Yadav & Kadilar (2013) proposed transformed ratio estimator to estimate population variance, which is given as

$$T_3 = s_y^2\left(\frac{s_x^{*2}}{S_x^2}\right) \tag{3}$$

where $s_x^{*2} = \dfrac{NS_x^{*2} - ns_x^{*2}}{N-n}$.

Singh et al. (2011) proposed different type of ratio, product and generalized exponential estimator for estimating the population variance as

$$T_4 = s_y^2 \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) \tag{4}$$

$$T_5 = s_y^2\left[\alpha \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) + (1-\alpha)\left(\frac{s_z^2 - S_z^2}{S_z^2 + s_z^2}\right)\right] \tag{5}$$

where α is a real constant.

Singh et al. (2014) suggested a class of estimator as

$$T_6 = \frac{s_y^2}{2}\left(1 + \frac{S_x^2}{s_x^2}\right) \tag{6}$$

$$T_7 = \frac{s_y^2}{2}\left(1 + \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right) \tag{7}$$

$$T_8 = \frac{s_y^2}{2}\left(\frac{S_x^2}{s_x^2} + \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right) \tag{8}$$

Yasmeen et al. (2019) proposed generalized ratio cum exponential estimator using transformed auxiliary variables is given as

$$T_9 = s_y^2\left(\frac{s_x^{*2}}{S_x^2}\right)^{v_1} \exp\left(v_2\frac{s_z^{*2} - S_z^2}{s_z^{*2} + S_z^2}\right) \tag{9}$$

where, v_1 and v_2 are optimization constants.

Table 1: MSE's of the Existing Estimator

Estimators	Mean Square Error
T_1	$Var(T_1) = fS_y^4(\lambda_{400} - 1)$
T_2	$MSE(T_2) = fS_y^4\Big[(\lambda_{400} - 1) + (\lambda_{040} - 1) - 2(\lambda_{220} - 1)\Big]$
T_3	$MSE(T_3) = fS_y^4\Big[(\lambda_{400} - 1) + g^2(\lambda_{040} - 1) - 2g(\lambda_{220} - 1)\Big]$
T_4	$MSE(T_4) = fS_y^4\Big[\lambda_{400} + \frac{\lambda_{040}}{4} - \lambda_{220} + \frac{1}{4}\Big]$
T_5	$MSE(T_5)_{\min} = fS_y^4\Big[(\lambda_{400} - 1) + \frac{\alpha^2}{4}(\lambda_{040} - 1) + \frac{1-\alpha^2}{4}(\lambda_{004} - 1)$
	$-\alpha(\lambda_{220-1}) + (1-\alpha)(\lambda_{202} - 1) - \frac{\alpha(1-\alpha)}{2}(\lambda_{022} - 1)\Big]$
T_6	$MSE(T_6) = fS_y^4\Big[(\lambda_{400} - 1) + \frac{(\lambda_{040} - 1)}{4}(1 - 4C^*)\Big]$
T_7	$MSE(T_7) = fS_y^4\Big[(\lambda_{400} - 1) + \frac{3(\lambda_{040} - 1)}{16}(1 - 8C^*)\Big]$
T_8	$MSE(T_8) = fS_y^4\Big[(\lambda_{400} - 1) + \frac{3(\lambda_{040} - 1)}{16}(3 - 8C^*)\Big]$
T_9	$MSE(T_9)_{\min} = \frac{fS_y^4}{(1-BD)}\Big[(\lambda_{400} - 1) + \frac{(A-BC)^2}{(1-BD)}g^2(\lambda_{040} - 1)$
	$+\frac{1}{4}\frac{(C-AD)^2}{(1-BD)}g^2(\lambda_{004} - 1) - 2(A-BC)g(\lambda_{220} - 1)$
	$+\left(\frac{(A-BC)(C-AD)}{(1-BD)}\right)g^2(\lambda_{022} - 1) - (C-AD)g(\lambda_{202} - 1)\Big]$

where $\alpha = \dfrac{\lambda_{004} + 2(\lambda_{220} + \lambda_{202}) + \lambda_{022} - 6}{\lambda_{040} + \lambda_{004} + 2\lambda_{022} - 4}$, $C^* = \dfrac{(\lambda_{220} - 1)}{(\lambda_{040} - 1)}$, $g = \left(\dfrac{n}{N-n}\right)$

$$A = \frac{(\lambda_{220} - 1)}{g(\lambda_{040} - 1)}, B = \frac{(\lambda_{022} - 1)}{2(\lambda_{040} - 1)}, C = \frac{2(\lambda_{202} - 1)}{g(\lambda_{004} - 1)}, D = \frac{2(\lambda_{022} - 1)}{(\lambda_{004} - 1)}.$$

3. Proposed Estimator

It is known that exponential-type estimators perform better than comparable standard ratio and product-type estimators in terms of less mean square errors in a situation when the relationship between the auxiliary and study variables is not linear. As a result, the issue arises when the parameters that favor exponential-type estimators over standard estimators are not easily satisfied. The development of other effective estimators that would perform better than both the conventional exponential and standard estimators would be the obvious solution. Taking motivation from previous research and following the work of Yasmeen et al. (2019), we developed a ratio cum exponential estimator for estimating the finite population variance of study variables y by utilizing two auxiliary variables under simple random sampling, which is specified as

$$T_M = s_y^2\left(\frac{s_x^2}{S_x^2}\right)^{\alpha_0}\left(\frac{s_z^2}{S_z^2}\right)^{\alpha_1}\exp\left[\left(\frac{S_x^2 - s_x^2}{S_x^2}\right)\left(\frac{s_z^2 - S_z^2}{S_z^2}\right)\right] \tag{10}$$

where, α_0 and α_1 are two unknown constants and their values must be determined so that MSE can be minimized.

$$T_M = S_y^2(1+e_y)(1+e_x)^{\alpha_0}(1+e_z)^{\alpha_1}\exp(e_x e_z) \tag{11}$$

On simplifying and expanding the exponential function and ignoring the terms with a power greater than two, then we get

$$(T_M - S_y^2) = S_y^2\Big[e_y + \alpha_0 e_x + \alpha_1 e_z + \alpha_0\alpha_1 e_x e_z + \alpha_1 e_z e_y + \alpha_0 e_x e_y + e_x e_z + \frac{1}{2}\big\{(\alpha_1^2 - \alpha_1)e_z^2 + (\alpha_0^2 - \alpha_0)e_x^2\big\}\Big] \tag{12}$$

Taking Expectation on both side of equation (12), we get the bias of proposed estimator T_M as

$$B(T_M) = E(T_M - S_y^2) = fS_y^2\Big[\alpha_0\alpha_1(\lambda_{022}-1) + \alpha_1(\lambda_{202}-1) + \alpha_0(\lambda_{220}-1) + (\lambda_{022}-1) + \frac{1}{2}\big\{(\alpha_1^2-\alpha_1)(\lambda_{004}-1) + (\alpha_0^2-\alpha_0)(\lambda_{040}-1)\big\}\Big] \tag{13}$$

Squaring and taking expectation on both side of equation (12), the mean square error of T_M is given as

$$MSE(T_M) = E(T_M - S_y^2)^2 = fS_y^4\Big[(\lambda_{400}-1) + \alpha_0^2(\lambda_{040}-1) + \alpha_1^2(\lambda_{004}-1) + 2\alpha_0(\lambda_{220}-1) + 2\alpha_0\alpha_1(\lambda_{022}-1) + 2\alpha_1(\lambda_{202}-1)\Big] \tag{14}$$

The minimum mean square error of the proposed estimator is obtained by differentiating equation (14) with respect to unknown constant α_0 and α_1, respectively and equating to zero, then we get the optimum value of unknown constant as

$$\alpha_0 = \frac{a}{c} = \alpha_{0(opt)} \quad \text{and} \quad \alpha_1 = \frac{b}{c} = \alpha_{1(opt)}$$

where

$$a = \Big[\big\{(\lambda_{202}-1)(\lambda_{022}-1)\big\} - \big\{(\lambda_{220}-1)(\lambda_{004}-1)\big\}\Big]$$
$$b = \Big[\big\{(\lambda_{220}-1)(\lambda_{022}-1)\big\} - \big\{(\lambda_{202}-1)(\lambda_{004}-1)\big\}\Big]$$
$$c = \Big[\big\{(\lambda_{040}-1)(\lambda_{004}-1)\big\} - \big\{(\lambda_{022}-1)^2\big\}\Big]$$

After substituting the optimum value of α_0 and α_1, the minimum mean square error of T_M is given as

$$MSE(T_M)_{\min} = fS_y^4\Bigg[(\lambda_{400}-1) + \left(\frac{a}{c}\right)^2(\lambda_{040}-1) + \left(\frac{b}{c}\right)^2(\lambda_{004}-1) + 2\left(\frac{a}{c}\right)(\lambda_{220}-1) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022}-1) + 2\left(\frac{b}{c}\right)(\lambda_{202}-1)\Bigg] \quad (15)$$

4. Efficiency Comparisons

In this section, we find the conditions by comparing the MSE of proposed estimator with the variance of usual unbiased estimator S_y^2 and MSE of other considered existing estimators.

$$MSE(T_M)_{\min} < Var(T_1), \text{ iff}$$

$$\Bigg[\left(\frac{a}{c}\right)^2(\lambda_{040}-1) + \left(\frac{b}{c}\right)^2(\lambda_{004}-1) + 2\left(\frac{a}{c}\right)(\lambda_{220}-1) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022}-1) + 2\left(\frac{b}{c}\right)(\lambda_{202}-1)\Bigg] < 0$$

$$MSE(T_M)_{\min} < MSE(T_2), \text{ iff}$$

$$\Bigg[\left(\left(\frac{a}{c}\right)^2 - 1\right)(\lambda_{040}-1) + \left(\frac{b}{c}\right)^2(\lambda_{004}-1) + 2\left(1+\frac{a}{c}\right)(\lambda_{220}-1) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022}-1) + 2\left(\frac{b}{c}\right)(\lambda_{202}-1)\Bigg] < 0$$

$$MSE(T_M)_{\min} < MSE(T_3), \text{ iff}$$

$$\Bigg[\left(\left(\frac{a}{c}\right)^2 - g^2\right)(\lambda_{040}-1) + \left(\frac{b}{c}\right)^2(\lambda_{004}-1) + 2\left(\frac{a}{c}+g\right)(\lambda_{220}-1) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022}-1) + 2\left(\frac{b}{c}\right)(\lambda_{202}-1)\Bigg] < 0$$

$MSE(T_M)_{\min} < MSE(T_4)$, iff

$$\left[\frac{5}{4} + \left(\left(\frac{a}{c}\right)^2 - \frac{1}{4}\right)\lambda_{040} + \frac{a}{c}\left(\frac{a}{c} - 2\right) + \left(\frac{b}{c}\right)^2(\lambda_{004} - 1) + \left(\frac{2a}{c} + 1\right)\lambda_{220} + 2\left(\frac{ab}{c^2}\right)(\lambda_{022} - 1) + 2\left(\frac{b}{c}\right)(\lambda_{202} - 1)\right] < 0$$

$MSE(T_M)_{\min} < MSE(T_5)$, iff

$$\left[\left(\left(\frac{a}{c}\right)^2 - \frac{\alpha^2}{4}\right)(\lambda_{040} - 1) + \left(\left(\frac{b}{c}\right)^2 - \frac{1-\alpha^2}{4}\right)(\lambda_{004} - 1) + \left(\frac{2a}{c} - \alpha\right)(\lambda_{220-1}) + 2\left(\frac{ab}{c^2} - \frac{\alpha(1-\alpha)}{2}\right)(\lambda_{022} - 1) + 2\left(\frac{b}{c} + (1-\alpha)\right)(\lambda_{202} - 1)\right] < 0$$

$MSE(T_M)_{\min} < MSE(T_6)$, iff

$$\left[\left\{\left(\frac{a}{c}\right)^2 - \frac{1}{4} + C^*\right\}(\lambda_{040} - 1) + \left(\frac{b}{c}\right)^2(\lambda_{004} - 1) + 2\left(\frac{a}{c}\right)(\lambda_{220-1}) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022} - 1) + 2\left(\frac{b}{c}\right)(\lambda_{202} - 1)\right] < 0$$

$MSE(T_M)_{\min} < MSE(T_7)$, iff

$$\left[\left\{\left(\frac{a}{c}\right)^2 - \frac{3}{16} + -\frac{3}{2}C^*\right\}(\lambda_{040} - 1) + \left(\frac{b}{c}\right)^2(\lambda_{004} - 1) + 2\left(\frac{a}{c}\right)(\lambda_{220-1}) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022} - 1) + 2\left(\frac{b}{c}\right)(\lambda_{202} - 1)\right] < 0$$

$MSE(T_M)_{\min} < MSE(T_8)$, iff

$$\left[\left\{\left(\frac{a}{c}\right)^2 - \frac{9}{16} + -\frac{3}{2}C^*\right\}(\lambda_{040} - 1) + \left(\frac{b}{c}\right)^2(\lambda_{004} - 1) + 2\left(\frac{a}{c}\right)(\lambda_{220-1}) + 2\left(\frac{ab}{c^2}\right)(\lambda_{022} - 1) + 2\left(\frac{b}{c}\right)(\lambda_{202} - 1)\right] < 0$$

$MSE(T_M)_{\min} < MSE(T_9)$, iff

$$\left[\left(\frac{BD}{BD-1}\right)(\lambda_{400} - 1) + \delta_1(\lambda_{040} - 1) + \delta_2(\lambda_{004} - 1) + 2\left(\frac{a}{c} + (A - BC)g\right)(\lambda_{220-1}) + \delta_3(\lambda_{022} - 1) + \left(\frac{2b}{c} - (C - AD)g\right)(\lambda_{202} - 1)\right] < 0$$

where
$\delta_1 = \left\{\dfrac{a^2(1-BD)^2 - c^2(A-BC)^2}{c^2(1-BD)^2}\right\}$, $\delta_2 = \left\{\dfrac{4b^2(1-BD) - g^2c^2(C-AD)^2}{4c^2(1-BD)^2}\right\}$
and $\delta_3 = \left(\dfrac{2ab}{c^2} - \dfrac{(A-BC)(C-AD)g^2}{(1-BD)^2}\right)$.

If the above conditions hold true, then the proposed estimator is more efficient than the other considered estimators.

5. Simulation Study

This section describes how we performed a simulation to examine the performance of the proposed estimator T_M against the usual and other existing estimators using R software for both artificial and real-life data.

For an artificial population, we constructed a population of size $N = 100$ and took different sample sizes as $n = 30, 40, 50, 60$, and 70, using the non-linear model $Y = \exp(X) + \exp(Z) + rnorm(N, 0.05, 0.45)$, where $X = rnorm(N, 0.05, 0.45)$ and $Z = rnorm(N, 0.05, 0.45)$. As a result, we iterated over 100 times the average values of the statistics used to simulate the simulated data[1].

Table 2: MSE's of the proposed estimator for different value of α_0 and α_1.

		Sample size				
α_0	α_1	30	40	50	60	70
$\alpha_{0(opt)}$	$\alpha_{1(opt)}$	0.0009	0.0021	0.0001	0.0001	0.0001
0	0	0.0373	0.0240	0.0160	0.0107	0.0069
-1	1	155.2657	99.8137	66.5424	44.3616	28.5182
1	-1	155.2657	99.8137	66.5424	44.3616	28.5182
1	$\alpha_{1(opt)}$	5.0632	3.2549	2.1700	1.4466	0.9300
$\alpha_{0(opt)}$	0	4.9642	3.1913	2.1275	1.4183	0.9118

In Table 2, substitute the different values of α_0 and α_1 in equation (15) i.e., the minimum MSE of the estimator. As the sample size increases, the value of MSEs decreases. Various estimators develop when we use different values of constant, and the MSE of all these estimators is greater than our proposed estimator. Then, we can clearly say that our proposed estimator is more efficient than the other members of the estimator.

In Table 3, we compare the proposed estimator with the usual unbiased estimator and other conventional estimators. It is visible from Table 3 that the proposed estimator is better than all the other existing estimators. With the increase in the sample size, the MSEs of all the considered estimators are decreases, but the MSE of the proposed estimator T_M first increased and then decreased. The MSE of the estimators T_5 and T_3 have a greater MSE, which means the estimators T_5 and T_3 are less efficient than all the other estimators.

[1]rnorm() Random number generated from normal distribution.

Table 3: MSE's of the proposed estimator with respect to existing estimators for different sample sizes.

Estimator	Sample size				
	30	40	50	60	70
T_M	0.0009	0.0021	0.0001	0.0001	0.0001
T_1	0.0373	0.0240	0.01599	0.0107	0.0069
T_2	5.3999	3.4714	2.3143	1.5428	0.9918
T_3	2.3288	2.3183	2.3143	2.3149	2.3209
T_4	2.7190	1.7479	1.1653	0.7769	0.4994
T_5	7.8189	5.0264	3.3509	2.2340	1.4361
T_6	4.1249	2.6517	1.7678	1.1785	0.7576
T_7	2.0785	1.3361	0.8907	0.5938	0.3818
T_8	4.0541	2.6062	1.7375	1.1583	0.7446
T_9	5.1832	3.3321	2.2214	1.4809	0.9520

To test the proposed estimator's real-world applicability, we use the HBAT real-data set, which is a prominent maker of paper goods in the US. From this data set, we choose **Likely to Purchase** as a study variable (y), **Satisfaction**, and **Delivery Speed** as auxiliary variables (x and z), respectively. Here, the population size $N = 200$ and different samples are drawn to compare the performance of the proposed estimator to that of conventional and other existing estimators.

Table 4: MSE's of the proposed estimator with respect to existing estimators for different sample sizes.

Estimator	Sample size					
	100	110	120	130	140	150
T_M	0.0002	0.0003	0.0063	0.0002	0.0023	0.0005
T_1	0.0120	0.0098	0.0080	0.0065	0.0052	0.0040
T_2	3.4359	2.8112	2.2906	1.8501	1.4725	1.1453
T_3	3.4359	3.4369	3.4390	3.4425	3.4480	3.4564
T_4	1.7238	1.4104	1.1492	0.9282	0.7388	0.5747
T_5	3.5199	2.8799	2.3466	1.8954	1.5086	1.1733
T_6	2.5439	2.0814	1.6960	1.3698	1.0903	0.8480
T_7	1.2767	1.0445	0.8511	0.6874	0.5471	0.4256
T_8	2.5772	2.1087	1.7182	1.3877	1.1045	0.8591
T_9	3.2748	2.6796	2.1835	1.7638	1.4041	1.0923

From Table 4, It is envisaged that for different samples our proposed estimator is highly efficient than the usual and other existing estimators. As the sample size increases, the MSE of estimator T_M first increase then decrease, and again increases then decreases. Furthermore, the MSE of estimators T_1, T_2, T_4, T_5, T_6, T_7, T_8 and T_9 reduces as the sample size increases, whereas T_3 increases. The estimator T_3 is less efficient than all the other estimators. The performance of all estimators indicates $T_3 > T_5 > T_2 > T_9 > T_8 > T_6 > T_4 > T_7 > T_1 > T_M$.

6. Conclusion

In this article, we have proposed a ratio cum exponential estimator using two auxiliary variables under simple random sampling for the estimation of the finite population variance. The properties of the proposed estimator have been studied up to the first degree of approximation and we compared the results with respect to the usual unbiased estimator and other existing estimators. By using R software, both real and artificial generated populations are used for the simulation study. From the simulation results, we determine that our suggested estimator is more efficient than the usual and the other conventional estimators. Thus, we may recommend the use of our proposed estimator in a practical situation.

Acknowledgement

We are very grateful to the anonymous referee for his valuable comments that improved the quality of the paper.

[Received: April 2022 — Accepted: November 2022]

References

Bahl, S. & Tuteja, R. (1991), 'Ratio and product-type exponential estimators', *Journal of Information and Optimization Sciences* **12**(1), 159–164.

Bhushan, S., Kumar, A. & Kumar, S. (2021), 'Efficient classes of estimators for population variance using attribute', *International Journal of Mathematics in Operation Research* **22**(1), 74–92.

Bhushan, S., Kumar, A., Kumar, S. & Singh, S. (2022), 'Some modified classes of estimators for population variance using auxiliary attribute', *Pakistan Journal of Statistics* **38**(2), 235–252.

Bhushan, S., Kumar, A., Singh, S. & Kumar, S. (2021), 'An improved class of estimators of population mean under simple random sampling', *Philippine Statist* **70**(1), 33–47.

Das, A. K. (1988), *Contributions to the theory of sampling strategies based on auxiliary information*, Ph.D thesis submitted to BCKV, Mohanpur, Nadia, and West Bengal, India.

Gupta, S. & Shabbir, J. (2008), 'Variance estimation in simple random sampling using auxiliary information', *Hacettepe Journal of Mathematics and Statistics* **37**(1), 57–67.

Isaki, C. T. (1983), 'Variance estimation using auxiliary information', *Journal of the American statistical association* **78**(381), 117–123.

Kadilar, C. & Cingi, H. (2006), 'Ratio estimators for the population variance in simple and stratified random sampling', *Applied Mathematics and Computation* **173**(2), 1047–1059.

Murthy, M. N. (1967), *Sampling Theory and Methods*, Statistical Publishing Society, Barrackpore.

Shabbir, J., Gupta, S. & Masood, S. (2022), 'An improved general class of estimators for finite population mean in simple random sampling', *Communications in Statistics-Theory and Methods* **51**(11), 3508–3520.

Singh, H. P., Singh, A. K. & Solanki, R. S. (2014), 'Estimation of finite population variance using auxiliary information in sample surveys', *Statistica* **74**(1), 99–116.

Singh, R., Chauhan, P., Sawan, N. & Smarandache, F. (2011), 'Improved exponential estimator for population variance using two auxiliary variables', *Italian Journal of Pure and Applied Mathematics* **28**, 101–108.

Subramani, J. & Kumarapandiyan, G. (2012), 'Variance estimation using quartiles and their functions of an auxiliary variable', *International Journal of Statistics and Applications* **2**(5), 67–72.

Subramani, J. & Kumarapandiyan, G. (2013), 'Estimation of variance using known coefficient of variation and median of an auxiliary variable', *Journal of Modern Applied Statistical Methods* **12**(1), 58–64.

Yadav, S. & Kadilar, C. (2013), 'A class of ratio-cum-dual to ratio estimator of population variance', *Journal of Reliability and Statistical Studies* **6**(1), 29–34.

Yasmeen, U., Noor-ul Amin, M. & Hanif, M. (2019), 'Exponential estimators of finite population variance using transformed auxiliary variables', *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences* **89**, 185–191.