**Revista Colombiana de Estadística - Applied Statistics**
January 2025, volume 48, issue 1, pp. 215–237
http://doi.org/10.15446/rce.v48n1.118163

# A Class of Ratio Cum Product Estimators with Non-Response and Measurement Error Using ORRT Models: A Double Sampling Scheme

## Una clase de estimadores de relación con producto con falta de respuesta y error de medición utilizando modelos ORRT: un esquema de muestreo doble

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**Abstract**

The current study employs the ratio-cum-product estimator to estimate the population mean of a sensitive study variable, aiming to overcome issues related to non-response and measurement error in the context of double sampling. The characteristics of the proposed class of estimators are computed up to the first order of approximation. A comparative analysis is conducted to assess the performance of the suggested estimator amongst the class of estimator and the estimator proposed by Kumar & Zhang (2023). Additionally, theoretical findings are supported by conducting two model based simulation study by using an hypothetical population. The simulation results demonstrates that the proposed ratio-cum-product estimator under double sampling exhibits the lowest mean squared error among Kumar & Zhang (2023) estimator and all classes of suggested estimator which indicates their superior performance in estimating the population mean of a sensitive variable. As a result, the suggested estimator offers a valuable tool for estimating the population mean in surveys conducted across various agriculture, environmental studies, market research and health surveys.

***Key words*:** Sensitive variable; Non-response; Measurement error; Double sampling; Optional Randomized Response Technique (ORRT).

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**Resumen**

El presente estudio emplea el estimador de razón-producto para estimar la media poblacional de una variable de estudio sensible, con el objetivo de superar los problemas relacionados con la falta de respuesta y el error de medición en el contexto del muestreo doble. Las características de la clase de estimadores propuesta se calculan hasta el primer orden de aproximación. Se realiza un análisis comparativo para evaluar el desempeño del estimador sugerido entre la clase de estimador y el estimador propuesto por Kumar & Zhang (2023). Además, los hallazgos teóricos se respaldan mediante la realización de un estudio de simulación utilizando una población creada artificialmente. Los resultados de la simulación demuestran que el estimador de razón-producto propuesto bajo muestreo doble exhibe el error cuadrático medio más bajo entre el estimador de Kumar & Zhang (2023) y todas las clases de estimador sugerido, lo que indica su desempeño superior en la estimación de la población. media de una variable sensible. Como resultado, el estimador sugerido ofrece una herramienta valiosa para estimar la media poblacional en encuestas realizadas en diversos estudios agrícolas, ambientales, de mercado y de salud.

***Palabras clave*:** Variable sensible; Falta de respuesta; Error de medición; Doble muestreo; Técnica de Respuesta Aleatoria Opcional (ORRT).

# 1. Introduction

The tendency of survey respondents to produce responses that are considered as socially acceptable or desired, rather than representing their genuine beliefs or behaviors, is referred to as social desirability bias. In survey research, it is not always feasible to directly observe such variables due to their delicate nature. Respondents may feel uncomfortable or embarrassed sharing private information with the interviewer, especially when the questions touch on sensitive topics like corruption, criminal activity, abortion, drug addiction, and more. As a result, maintaining privacy and ensuring respondents trust becomes crucial for obtaining accurate and reliable survey data. When conducting social surveys that involve sensitive characteristics, optional randomized response models provide respondents with the choice to either disclose the true response or provide a scrambled response. If a respondent does not perceive the question as sensitive, they have the option to report the true response. In order to cope with refusals on sensitive variables, Warner, S. L. (1965) introduced a randomized response technique (RRT) was limited to binary variables. Further, survey statisticians including Eichhorn, B. H. and Hayre, L. S. (1983), Gupta, S., Gupta, B. and Singh, S. (2002), Gupta, S. N. and Thornton, B. and Shabbir, J. and Singhal, S. (2006), Saha, A. (2008), and Diana, G. and Perri, P. F. (2011), have examined the population mean of sensitive variables where the study variable is sensitive in nature. The most commonly used statistical sampling technique that involves taking two samples from a population to estimate population parameters with increased efficiency and accuracy is Double sampling. This technique is particularly useful when a complete census of a population is not feasible or too expensive, but a less efficient sampling technique would lead to inaccurate results. It is used in various fields such as market research, public health, and social sciences. It was first identified by Neyman, J. (1934) and then numerous authors have used auxiliary data to work on various aspects of two-phase sampling, including Pradhan, B. (2014), Singh, B. and Choudhury, S. (2012), Zaman, T. and Kadilar, C. (2019), Shabbir, J. and Ahmed, S. and Sanaullah, A. and Onyango, R. (2021), Kumar, S. and Kour, S. P. (2021), Dash, P. and Sunani, K. (2022) and so on.

When the respondents refuse to answer the sensitive question then this result, a high rate of non-response which may badly affect the estimates of population parameters. So, for estimating sensitive variables in survey research several authors likewise Zhang, Q., Khalil, S. and Gupta, S. (2021), Kumar, S. and Kour, S. P. (2021), Kumar & Kour (2022), Kumar & Zhang (2023), Kumar & Joorel (2023), Azeem & Salam (2024) developed the mean estimator for estimating the population mean of sensitive variable in the simultaneous presence of non-response and measurement error by utilizing ORRT models.

This paper aims to develop a ratio-cum-product estimator in the presence of non-response and measurement error at the same time under Double sampling and by utilized two auxiliary variable(s). This paper is structured as follows. Section 2 describes the optional randomized response technique and existing estimators. The proposed estimator along with its properties are discussed in Section 3. In Section 4, we compare the proposed estimator with the other conventional estimators and the efficiency conditions are developed. Section 5 provides a simulation study to check the effectiveness of the proposed estimator. Finally, the brief summary or conclusion is discussed in Section 6.

## 2. Notations and Existing Estimators

Let us consider $\xi = \xi_1, \xi_2, \ldots, \xi_N$ be a finite population of size $N$ units and from $\xi$, a sample of size $n$ is taken by using simple random sampling without replacement (SRSWOR). Let $Y$ be a sensitive study variable and $X_1$ and $X_2$ be two non sensitive auxiliary variable(s) i.e. ($\overline{Y}$, $\overline{X}_1$ and $\overline{X}_2$) and ($S_y^2$, $S_{x_1}^2$ and $S_{x_2}^2$) are the respective mean and variances of $Y$, $X_1$ and $X_2$. Assume $S_1$ and $S_2$ be two scrambling variables with means( $\overline{S}_1$, $\overline{S}_2$) and variances ($S_{S_1}^2$, $S_{S_2}^2$), respectively. Let $W$ be the probability that respondent find the question sensitive. If a respondent feels the question is sensitive then he or she is prompted to report a scrambled response otherwise, a correct response is reported.

To tackle the of problem non-response in sensitive surveys, Hansen, M. H. and Hurwitz, W. N. (1946) technique has been modified by Zhang, Q., Khalil, S. and Gupta, S. (2021), Kumar, S. and Kour, S. P. (2021), Kumar & Kour (2022) to collect sensitive information from the respondents. In this technique, the respondent gives direct answer in first phase then ORRT model is used to get answer from a sub-group of non-respondents in the second phase.

Therefore, ORRT model in the second phase is given as

$$Z = \begin{cases} Y & \text{with probability 1-W} \\ S_1 Y + S_2 & \text{with probability W,} \end{cases} \tag{1}$$

with mean $E(Z) = E(Y)$ and variance $Var(Z) = S_y^2 + S_{S_2}^2 W + S_{S_1}^2 (S_y^2 + \overline{Y}^2)W$. The RRT model is $Z = (S_1 Y + S_2)J + Y(1-J)$, where $J \sim$ Bernoulli$(W)$ with $E(J) = W, Var(J) = W(1-W)$ and $E(J^2) = Var(J) + E^2(J) = W$. And the expectation and variance of randomized mechanism is $E_R(Z) = (\overline{S}_1 W + 1 - W)Y + \overline{S}_2 W$ and $V_R(Z) = (Y^2 S_{S_1}^2 + S_{S_2}^2)W$.

Let us take a transformation of the randomized response be $\overline{y}_i$ whose expectation under the randomization mechanism is the true response $y_i$ and is given as

$$\widehat{y}_i^* = \frac{z_i - \overline{S}_2 W}{\overline{S}_1 W + 1 - W},$$

with $E_R(\widehat{y}_i{}^*) = y_i$ and $V_R(\widehat{y}_i{}^*) = \frac{V_R(z_i)}{(\overline{S}W+1-W)^2} = \frac{(y_i^2 S_{S_1}^2 + S_{S_2}^2)W}{(\overline{S}_1 W + 1 - W)^2} = \tau_i$.

From previous discussions, we assume that out of '$n$' sample units, only $n_1$ units provide response on first call and remaining $n_2 = (n - n_1)$ units do not respond. Then a sub-sample of $n_s (= n_2/k(k > 1))$ units are taken from non-responding units $n_2$, respectively. Then, a modified version of Hansen and Hurwitz estimator suggested by Zhang, Q., Khalil, S. and Gupta, S. (2021) and Kumar & Kour (2022) is given by

$$\widehat{\overline{y}}^* = w_1 \overline{y}_1^* + w_2 \widehat{\overline{y}}_2^*$$

with mean $E(\widehat{\overline{y}}^*) = \overline{Y}$ and variance

$$Var(\widehat{\overline{y}}^*) = \lambda S_y^2 + \lambda^* S_{y(2)}^2 + \frac{W_2 k}{n}\left[\frac{\{(S_{y(2)}^2 + \overline{y}_{(2)}^2) S_{S_1}^2 + S_{S_2}^2\}W}{(\overline{S}_1 W + 1 - W)^2}\right].$$

Similarly, one can write the estimator for $X_1$ and $X_2$ as

$$\overline{x}_1^* = w_1 \overline{x}_1^* + w_2 \overline{x}_{12}^*$$

and

$$\overline{x}_2^* = w_1 \overline{x}_2^* + w_2 \overline{x}_{22}^*$$

with $E(\overline{x}_1^*) = \overline{X}_1,\ E(\overline{x}_2^*) = \overline{X}_2$ and $Var(\overline{x}_1^*) = \lambda S_{x_1}^2 + \lambda^* S_{x_{1(2)}}^2,\ Var(\overline{x}_2^*) = \lambda S_{x_2}^2 + \lambda^* S_{x_{2(2)}}^2$.

In addition, let $U_i = y_i - Y_i,\ V_i = x_{1i} - X_{1i}$ and $W_i = x_{2i} - X_{2i}$ be the measurement error for the study variable $(Y)$ and auxiliary variables $(X_1,\ X_2)$ in the population. Let $P_i = z_i - Z_i$ represent the relative measurement error associated with the sensitive variables $(Z)$ in the face-to-face interview phase. These measurement errors are considered to be random and uncorrelated, with mean zero and variances $S_u^2,\ S_v^2,\ S_w^2,\ S_{v(2)}^2,\ S_{w(2)}^2$ and $S_p^2$, respectively.

In the context of non-response and measurement error simultaneously, the variances of $\widehat{\overline{y}}$, are given by

$$Var(\widehat{\overline{y}}^{**}) = \lambda(S_y^2 + S_u^2) + \lambda^*(S_{y(2)}^2 + S_p^2) + \kappa,$$

$$Var(\overline{x}_1^{**}) = \lambda(S_{x_1}^2 + S_v^2) + \lambda^*(S_{x_{1(2)}}^2 + S_{v(2)}^2)$$

and

$$Var(\overline{x}_2^{**}) = \lambda(S_{x_2}^2 + S_w^2) + \lambda^*(S_{x_{2(2)}}^2 + S_{w(2)}^2)$$

where $\kappa = \frac{W_2 k}{n}\left[\frac{\{(S_{y(2)}^2 + \overline{y}_{(2)}^2)S_{S_1}^2 + S_{S_2}^2\}W}{(\overline{S}_1 W + 1 - W)^2}\right]$.

Kumar & Zhang (2023) suggest a ratio-cum-product type estimators when the auxiliary variable(s) $\overline{X}_1$ and $\overline{X}_2$ are known under measurement error and non-response using ORRT model is given by

$$\widehat{T}_{sszd}^{**} = \left[\alpha^*\widehat{\overline{y}}^{**} + \beta_{yx_1}^{**}(\overline{x}_1' - \overline{x}_1^{**}) + \beta_{yx_2}^{**}(\overline{x}_2' - \overline{x}_2^{**})\right]\left(\frac{\overline{x}_1'}{\overline{x}_1^{**}} + \frac{\overline{x}_1^{**}}{\overline{x}_1'}\right)\left(\frac{\overline{x}_2'}{\overline{x}_2^{**}} + \frac{\overline{x}_2^{**}}{\overline{x}_2'}\right),$$

where $\beta_{yx_1}^{**} = s_{yx_1}^{**}/s_{x_1}^{**2}$ is the estimate of the population regression coefficient $\beta_{yx_1} = S_{yx_1}^{**}/S_{x_1}^{**2}$, $\beta_{yx_2}^{**} = s_{yx_2}^{**}/s_{x_2}^{**2}$ is the estimate of the population regression coefficient $\beta_{yx_2} = S_{yx_2}^{**}/\widehat{S}_{x_2}^{**2}$ and $\alpha^*$ be a finite quantity.

The bias and mean squared error of $\widehat{T}_{sszd}^{**}$ is given by

$$\begin{aligned} Bias(\widehat{T}_{sszd}^{**}) = {} & \overline{Y}(4\alpha - 1) - 4\beta_{yx_1}\left[(\lambda' - \lambda)\left(\frac{\alpha_{03}^*}{\alpha_{02}^*} - \frac{\alpha_{12}^*}{\alpha_{11}^*}\right) - \left(\frac{\alpha_{03}^*}{\alpha_{02}^*} - \frac{\alpha_{12}^*}{\alpha_{11}^*}\right)\right. \\ & \left. + \lambda^*\left(\frac{\alpha_{03(2)}^*}{\alpha_{02(2)}^*} + \frac{\alpha_{12(2)}^*}{\alpha_{11(2)}^*}\right)\right] + 4\beta_{yx_2}\left[(\lambda' - \lambda)\left(\frac{\mu_{03}^*}{\mu_{02}^*} + \frac{\mu_{12}^*}{\mu_{11}^*}\right)\right. \\ & \left. - \left(\frac{\mu_{03}^*}{\mu_{02}^*} - \frac{\mu_{12}^*}{\mu_{11}^*}\right) + \lambda^*\left(\frac{\mu_{03(2)}^*}{\mu_{02(2)}^*} + \frac{\mu_{12(2)}^*}{\mu_{11(2)}^*}\right)\right] \\ & + 2\alpha R_1(B - \lambda' S_{x_1}^2) + 2\alpha R_2(C - \lambda' S_{x_2}^2) \end{aligned}$$

and

$$\begin{aligned} MSE(\widehat{T}_{sszd}^{**}) = {} & \overline{Y}^2(4\alpha - 1)^2 + 16\alpha^2 A + 16\beta_{yx_1}^2 B + 16\beta_{yx_2}^2 C - 16\lambda'\beta_{yx_1}^2 S_{x_1}^2 \\ & - 16\lambda'\beta_{yx_2}^2 S_{x_2}^2 - 32\alpha\beta_{yx_1} D + 32\lambda'\alpha\beta_{yx_1}\rho_{yx_1} S_y S_{x_1} \\ & + 32\lambda'\alpha\beta_{yx_2}\rho_{yx_2} S_y S_{x_2} + 32\beta_{yx_1}\beta_{yx_2} E \\ & + 32\lambda'\beta_{yx_1}\beta_{yx_2} S_{x_1} S_{x_2} - 32\alpha\beta_{yx_2} F + 16\alpha^2\kappa \end{aligned} \tag{2}$$

which is optimum when

$$\widehat{\alpha}_{dopt}^{**} = \frac{\frac{1}{4}\overline{Y}^2 + \beta_{yx_1}(D - \lambda'\rho_{yx_1} S_y S_{x_1}) + \beta_{yx_2}(F\lambda'\rho_{yx_2} S_y S_{x_2})}{\overline{Y}^2 + A + \kappa}.$$

Putting the value of $\widehat{\alpha}^{**}_{dopt}$ in (2), we get the minimum MSE of the proposed estimator as

$$\begin{aligned} MSE_{\min}(\widehat{T}^{**}_{sszd}) = {} & \overline{Y}^2(4\widehat{\alpha}^{**}_{dopt}-1)^2 + 16\widehat{\alpha}^{**2}_{dopt}A + 16\beta^2_{yx_1}B + 16\beta^2_{yx_2}C \\ & - 16\lambda'\beta^2_{yx_1}S^2_{x_1} - 16\lambda'\beta^2_{yx_2}S^2_{x_2} - 32\widehat{\alpha}^{**}_{dopt}\beta_{yx_1}D \\ & + 32\lambda'\widehat{\alpha}^{**}_{dopt}\beta_{yx_1}\rho_{yx_1}S_yS_{x_1} + 32\lambda'\widehat{\alpha}^{**}_{dopt}\beta_{yx_2}\rho_{yx_2}S_yS_{x_2} \\ & + 32\beta_{yx_1}\beta_{yx_2}E + 32\lambda'\beta_{yx_1}\beta_{yx_2}S_{x_1}S_{x_2} \\ & - 32\widehat{\alpha}^{**}_{dopt}\beta_{yx_2}F + 16\widehat{\alpha}^{**2}_{dopt}\kappa, \end{aligned} \quad (3)$$

where $A = \left[\lambda(S^2_y + S^2_u) + \lambda^*(S^2_{y(2)} + S^2_{p(2)})\right]$, $B = \left[\lambda(S^2_{x_1} + S^2_v) + \lambda^*(S^2_{x_{1(2)}} + S^2_{v(2)})\right]$, $C = \left[\lambda(S^2_{x_2} + S^2_w) + \lambda^*(S^2_{x_{2(2)}} + S^2_{w(2)})\right]$, $D = \left(\lambda\rho_{yx_1}S_yS_{x_1} + \lambda^*\rho_{yx_{1(2)}}S_{y(2)}S_{x_{1(2)}}\right)$, $E = \left[\lambda\rho_{x_1x_2}S_{x_1}S_{x_2} + \lambda^*\rho_{x_1x_{2(2)}}S_{x_{1(2)}}S_{x_{2(2)}}\right]$, $F = \left(\lambda\rho_{yx_2}S_yS_{x_2} + \lambda^*\rho_{yx_{2(2)}}S_{y(2)}S_{x_{2(2)}}\right)$.

In the coming section, we establish a ratio-product type exponential estimator. The proposed estimator will be beneficial in the case of exponential type data and for estimating the mean of a sensitive variable.

# 3. Proposed Estimator in Double Sampling

When the population mean of $\overline{X}_1$ and $\overline{X}_2$ are assumed to be unknown then double sampling technique is used. This double sampling strategy is as follows that in the first phase a large sample of fixed size $n'$ is taken from $\xi$ to evaluate $X_1$ and $X_2$ in order to find estimates of $\overline{X}_1$ and $\overline{X}_2$ and in the second phase fixed-size $n$ sub-sample is taken from $n'$ to observe $Y$ only, so that $(n < n')$. Therefore inspiring from Kumar, S. and Kour, S. P. (2021) and Kumar & Zhang (2023), we propose following class of ratio-product estimators for estimating population mean $\overline{Y}$ of the sensitive study variable in presence of non-response and measurement error under Double sampling. Under this situation, the population means of $\overline{X}_1$ and $\overline{X}_2$ are not known as

$$T_d = w_0\widehat{\overline{y}}^{**}\left(\frac{\overline{x}'_1}{\overline{x}^{**}_1}\right)^{\alpha}\left(\frac{\overline{x}^{**}_2}{\overline{x}'_2}\right)^{\eta}\exp\left\{\frac{\alpha_1(\overline{x}'_1 - \overline{x}^{**}_1)}{\overline{x}'_1 + \overline{x}^{**}_1}\right\}\exp\left\{\frac{\alpha_2(\overline{x}'_2 - \overline{x}^{**}_2)}{\overline{x}'_2 + \overline{x}^{**}_2}\right\} + w_1(\overline{x}^{**}_1 - \overline{x}'_1) + w_2(\overline{x}^{**}_2 - \overline{x}'_2). \quad (4)$$

To obtain the bias and MSE of $T_d$, we write

$\widehat{\overline{y}}^{**} = \overline{Y}(1 + \widehat{e}^{**}_0)$, $\overline{x}^{**}_1 = \overline{X}_1(1 + e^{**}_1)$, $\overline{x}'_1 = \overline{X}_1(1 + e'_1)$, $\overline{x}^{**}_2 = \overline{X}_2(1 + e^{**}_2)$,

$\overline{x}'_2 = \overline{X}_2(1 + e'_2)$ such that

$E(\widehat{e}^{**}_0) = E(e^*_1) = E(e^{**}_2) = E(e'_1) = E(e'_2) = 0$ and

$E(\widehat{e}^{**2}_0) = \frac{1}{\overline{Y}^2}(A + \kappa)$, $E(e^{**2}_1) = \frac{1}{\overline{X}^2_1}B$, $E(e^{**2}_2) = \frac{1}{\overline{X}^2_2}C$,

$E(e'^2_1) = \lambda'\frac{(S^2_{x_1} + S^2_v)}{\overline{X}^2_1}$, $E(e'^2_2) = \lambda'\frac{(S^2_{x_2} + S^2_w)}{\overline{X}^2_2}$,

$E(\widehat{e}_0^{**}e_1^{**}) = \frac{1}{\overline{Y}\overline{X}_1}D,\ E(\widehat{e}_0^{**}e_2^{**}) = \frac{1}{\overline{Y}\overline{X}_1}F,\ E(\widehat{e}_0^{**}e_1') = \lambda'\frac{\rho_{yx_1}S_yS_{x_1}}{\overline{Y}\overline{X}_1},\ E(\widehat{e}_0^{**}e_2') = \lambda'\frac{\rho_{yx_2}S_yS_{x_2}}{\overline{Y}\overline{X}_2},\ E(e_1^{**}e_2^{**}) = \frac{1}{\overline{X}_1\overline{X}_2}E,$

$E(\widehat{e}_1^{**}e_2') = \lambda'\frac{\rho_{x_1x_2}S_{x_1}S_{x_2}}{\overline{X}_1\overline{X}_2},\ E(\widehat{e}_2^{**}e_2') = \lambda'\frac{S_{x_2}^2S_w^2}{\overline{X}_2^2}$, where $\lambda = \left(\frac{1}{n} - \frac{1}{N}\right),\ \lambda' = \left(\frac{1}{n'} - \frac{1}{N}\right), (\lambda - \lambda') = \left(\frac{1}{n} - \frac{1}{n'}\right)$ and $\lambda^* = \frac{(k-1)W_2}{n}$.

Expanding the right hand side of (4), multiplying out and neglecting terms of $e's$ having power greater than two, we have

$$\begin{aligned}
T_d = \overline{Y}\Bigg[& w_0\big\{1 + \widehat{e}_0^{**} + \alpha(e_1' - e_1^{**}) - \eta(e_2' - e_2^{**}) + \frac{1}{2}\alpha_1(e_1' - e_1^{**}) - \frac{1}{2}\alpha_2(e_2' - e_2^{**}) \\
&+ \alpha(e_1' - e_1^{**})\widehat{e}_0^{**} - \eta(e_2' - e_2^{**})\widehat{e}_0^{**} + \frac{1}{2}\alpha_1(e_1' - e_1^{**})\widehat{e}_0^{**} - \frac{1}{2}\alpha_2(e_2' - e_2^{**})\widehat{e}_0^{**} \\
&+ \frac{1}{2}(\alpha^2(e_1' - e_1^{**})^2 - \alpha(e_1'^2 - e_1^{**2})) + \frac{1}{2}(\eta^2(e_2' - e_2^{**})^2 - \eta(e_2'^2 - e_2^{**2})) \\
&+ \frac{1}{8}\big((\alpha_1(e_1' - e_1^{**}) - \alpha_2(e_2'^2 - e_2^{**2}))^2 - 2\alpha_1(e_1'^2 - e_1^{**2}) + 2\alpha_2(e_2'^2 - e_2^{**2})\big) \\
&- \alpha\eta(e_1' - e_1^{**})(e_2' - e_2^{**}) + \frac{1}{2}\alpha\alpha_1(e_1' - e_1^{**})^2 - \frac{1}{2}\eta\alpha_1(e_1' - e_1^{**})(e_2' - e_2^{**}) \\
&- \frac{1}{2}\alpha\alpha_2(e_1' - e_1^{**})(e_2' - e_2^{**}) + \frac{1}{2}\eta\alpha_2(e_2' - e_2^{**})^2\big\} \\
&+ \frac{w_1}{R_1}(e_1^{**} - e_1') + \frac{w_2}{R_2}(e_2^{**} - e_2')\Bigg]
\end{aligned}$$

or

$$\begin{aligned}
(T_d - \overline{Y}) = \overline{Y}\Bigg[& w_0\big\{1 + \widehat{e}_0^{**} + (\alpha + \frac{\alpha_1}{2})(e_1' - e_1^{**} + (\widehat{e}_0^{**}e_1' - \widehat{e}_0^{**}e_1^{**})) \\
&- (\eta + \frac{\alpha_2}{2})(e_2' - e_2^{**} + (\widehat{e}_0^{**}e_2' - \widehat{e}_0^{**}e_2^{**})) \\
&+ (\frac{1}{2}\alpha^2 + \frac{1}{2}\alpha\alpha_1 + \frac{1}{8}\alpha_1^2)(e_1' - e_1^{**})^2 + (\frac{1}{2}\eta^2 + \frac{1}{2}\eta\alpha_2 + \frac{1}{8}\alpha_2^2) \\
&(e_2' - e_2^{**})^2 - (\eta(\alpha + \frac{1}{2}\alpha_1) + \frac{1}{2}(\alpha + \frac{1}{2}\alpha_1))(e_1' - e_1^{**})(e_2' - e_2^{**}) \\
&- \frac{1}{2}(\alpha + \frac{1}{2}\alpha_1)(e_1'^2 - e_1^{**2}) + \frac{1}{2}(\eta + \frac{1}{2}\alpha_2)(e_2'^2 - e_2^{**2}) \\
&+ \frac{w_1}{R_1}(e_1^{**} - e_1') + \frac{w_2}{R_2}(e_2^{**} - e_2')\big\}\Bigg],
\end{aligned} \tag{5}$$

where $\theta_1 = (\alpha + \frac{1}{2}\alpha_1),\ \theta_2 = (\eta + \frac{1}{2}\alpha_2),\ \theta_3 = \left[\frac{\alpha(\alpha+1)}{2} + \frac{\alpha\alpha_1}{2} + \frac{\alpha_1(\alpha_1+2)}{8}\right]$ and $\theta_4 = \left[\frac{\eta(\eta-1)}{2} + \frac{\eta\alpha_2}{2} + \frac{\alpha_2(\alpha_2-2)}{8}\right]$.

Taking expectation of both sides of (5) we get the bias of $T_d$ to the first degree of approximation as

$$B(T_d) = \overline{Y}\left[w_0\left\{1 - \theta_1\frac{D^*}{\overline{Y}\overline{X}_1} + \theta_2\frac{F^*}{\overline{Y}\overline{X}_2} - \theta_1\theta_2\frac{E^*}{\overline{X}_1\overline{X}_2} + \theta_3\frac{B^*}{\overline{X}_1^2} + \theta_4\frac{C^*}{\overline{X}_2^2}\right\} - 1\right]$$
$$= \overline{Y}(w_0 H_6^* - 1).$$

Squaring both sides of (5) and neglecting terms $e's$ having power greater than two we have

$$\begin{aligned}(T_d - \overline{Y})^2 = \overline{Y}^2 \Bigg[ & 1 + w_0^2\Big\{1 + 2\widehat{e}_0^{**} + 2\theta_1(e_1' - e_1^{**}) - 2\theta_2(e_2' - e_2^{**}) + \widehat{e}_0^{**2} \\ & + 4\theta_1(\widehat{e}_0^{**}e_1' - \widehat{e}_0^{**}e_1^{**}) - 4\theta_2(\widehat{e}_0^{**}e_2' - \widehat{e}_0^{**}e_2^{**}) - 4\theta_1\theta_2(e_1' - e_1^{**}) \\ & + (\theta_1^2 + \alpha^2 + \alpha\alpha_1 + \frac{1}{4}\alpha_1^2)(e_1' - e_1^{**})^2 + (\theta_2^2 + \eta^2 + \eta\alpha_2 + \frac{1}{4}\alpha_2^2) \\ & (e_2' - e_2^{**})^2 - \theta_1(e_1'2 - e_1^{**2}) + \theta_2(e_2'2 - e_2^{**2})\Big\} + \frac{w_1^2}{R_1^2}(e_1^{**} - e_1')^2 \\ & + \frac{w_2^2}{R_2^2}(e_2^{**} - e_2')^2 + 2w_0w_1\frac{1}{R_1}\{e_1^{**} - e_1' + \widehat{e}_0^{**}e_1^{**} - \widehat{e}_0^{**}e_1' \\ & - \theta_1(e_1'^2 - e_1^{**2}) + \theta_2(e_2'^2 - e_2^{**2}) + 2w_0w_2\frac{1}{R_2} \\ & \{e_2^{**} - e_2' + \widehat{e}_0^{**}e_2^{**} - \widehat{e}_0^{**}e_2' - \theta_1(e_1^{**} - e_1') + \theta_2(e_2^{**} - e_2')^2\} \\ & + 2w_1w_2\frac{1}{R_1R_2}(e_1^{**} - e_1')(e_2^{**} - e_2') - 2w_0\{1 + \widehat{e}_0^{**} \\ & + \theta_1(e_1' - e_1^{**} + \widehat{e}_0^{**}e_1' - \widehat{e}_0^{**}e_1^{**}) - \theta_2(e_2' - e_2^{**} + \widehat{e}_0^{**}e_2' - \widehat{e}_0^{**}e_2^{**}) \\ & + (\frac{1}{2}\alpha^2 + \frac{1}{2}\alpha\alpha_1 + \frac{1}{8}\alpha_1^2)(e_1' - e_1^{**})^2 + (\frac{1}{2}\eta^2 + \frac{1}{2}\eta\alpha_2 + \frac{1}{8}\alpha_1^2) \\ & (e_2' - e_2^{**})^2\} - \theta_1\theta_2(e_1' - e_1^{**})(e_2' - e_2^{**}) + \frac{1}{2}\theta_1(e_1^{**2} - e_1'^2) \\ & - \frac{1}{2}\theta_2(e_2^{**2} - e_2'^2) - 2\frac{w_2}{R_2}(e_2^{**} - e_2')\Bigg],\end{aligned}$$

where $\theta_1 = (\alpha + \frac{1}{2}\alpha_1)$, $\theta_2 = (\eta + \frac{1}{2}\alpha_2)$, $\theta_3 = \left[\frac{\alpha(\alpha+1)}{2} + \frac{\alpha\alpha_1}{2} + \frac{\alpha_1(\alpha_1+2)}{8}\right]$ and

$\theta_4 = \left[\frac{\eta(\eta-1)}{2} + \frac{\eta\alpha_2}{2} + \frac{\alpha_2(\alpha_2-2)}{8}\right]$.

Taking expectation of both sides of (5), we get the MSE of $\widehat{T}_d^{**}$ to the first degree of approximation as

$$MSE(\widehat{T}_d^{**}) = \overline{Y}^2\big[1 + w_0^2H_0^* + w_1^2H_1^* + w_2^2H_2^* + 2w_0w_1H_3^* + 2w_0w_2H_4^* + 2w_1w_2H_5^* - 2w_0H_6^*\big], \quad (6)$$

where

$$H_0^* = \left[1 + \frac{1}{\overline{Y}^2}(A+K) - 4\theta_1 \frac{D^*}{\overline{Y}\overline{X}_1} + 4\theta_2 \frac{F^*}{\overline{Y}\overline{X}_2} - 4\theta_1\theta_2 \frac{E^*}{\overline{X}_1\overline{X}_2} + (\theta_1^2 + 2\theta_3)\frac{B^*}{\overline{X}_1^2} + (\theta_2^2 + 2\theta_4)\frac{C^*}{\overline{X}_2^2}\right],$$

$$H_1^* = \frac{B^*}{R_1^2\overline{X}_1^2},$$

$$H_2^* = \frac{C^*}{R_2^2\overline{X}_2^2},$$

$$H_3^* = \frac{1}{R_1}\left[\frac{D^*}{\overline{Y}\overline{X}_1} - \theta_1\frac{B^*}{\overline{X}_1^2} + \theta_2\frac{E^*}{\overline{X}_1\overline{X}_2}\right],$$

$$H_4^* = \frac{1}{R_2}\left[\frac{F^*}{\overline{Y}\overline{X}_2} - \theta_1\frac{E^*}{\overline{X}_1\overline{X}_2} + \theta_2\frac{C^*}{\overline{X}_2^2}\right],$$

$$H_5^* = \frac{E^*}{R_1R_2\overline{X}_1\overline{X}_2}$$

and

$$H_{6^*} = \left[1 - \theta_1\frac{D^*}{\overline{Y}\overline{X}_1} + \theta_2\frac{F^*}{\overline{Y}\overline{X}_1} + \theta_1\theta_2\frac{E^*}{\overline{X}_1\overline{X}_2} + \theta_3\frac{B^*}{\overline{X}_1^2} + \theta_4\frac{C^*}{\overline{X}_2^2}\right],$$

$$B^* = \left[(\lambda - \lambda')(S_{x_1}^2 + S_v^2) + \lambda^*(S_{x_{1(2)}}^2 + S_{v(2)}^2)\right],$$

$$C^* = \left[(\lambda - \lambda')(S_{x_2}^2 + S_w^2) + \lambda^*(S_{x_{2(2)}}^2 + S_{w(2)}^2)\right],$$

$$D^* = \left[(\lambda - \lambda')\rho_{yx_1}S_yS_{x_1} + \lambda^*\rho_{yx_{1(2)}}S_{y(2)}S_{x_{1(2)}}\right],$$

$$E^* = \left[(\lambda - \lambda')\rho_{x_1x_2}S_{x_1}S_{x_2} + \lambda^*\rho_{x_1x_{2(2)}}S_{x_{1(2)}}S_{x_{2(2)}}\right],$$

$$F^* = \left[(\lambda - \lambda')\rho_{yx_2}S_yS_{x_2} + \lambda^*\rho_{yx_{2(2)}}S_{y(2)}S_{x_{2(2)}}\right].$$

Putting $\frac{MSE(\widehat{T}_d^{**})}{\partial w_i} = 0$, $i = 0, 1, 2$, we have

$$\begin{bmatrix} H_0^* & H_3^* & H_4^* \\ H_3^* & H_1^* & H_5^* \\ H_4^* & H_5^* & H_2^* \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} H_6^* \\ 0 \\ 0 \end{bmatrix}.$$

Solving (6) we get the optimum values of $(w_0, w_1, w_2)$ as

$$w_{0(opt)} = \frac{\Delta_0^*}{\Delta^*}, w_{1(opt)} = \frac{\Delta_1^*}{\Delta^*}, w_{2(opt)} = \frac{\Delta_2^*}{\Delta^*}.$$

where

$$\Delta^* = H_0^*(H_1^*H_2^* - H_5^{*2}) - H_3^*(H_2^*H_3^* - H_4^*H_5^*) + H_4^*(H_3^*H_5^* - H_1^*H_4^*),$$

$$\Delta_0^* = H_6^*(H_1^*H_2^* - H_5^{*2}),$$

$$\Delta_1^* = -H_6^*(H_2^*H_3^* - H_4^*H_5^*)$$

and

$$\Delta_2^* = H_6^*(H_3^* H_5^* - H_1^* H_4^*).$$

Thus, the resulting minimum MSE of $\widehat{T}_d^{**}$ is given by

$$MSE_{\min}(\widehat{T}_d^{**}) = \overline{Y}^2 \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right]. \tag{7}$$

Now, we established the following theorem

**Theorem 1.** *The* $MSE(\widehat{T}_d^{**})$ *is greater than or equal to the minimum MSE of* $\widehat{T}_d^{**}$ *i.e.*

$$\begin{aligned} MSE(\widehat{T}_d^{**}) &\geq MSE_{\min}(\widehat{T}_d^{**}) \\ &= \overline{Y}^2 \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right], \end{aligned}$$

with equality holds if $w_i = w_{i(opt)}$, $i = 0, 1, 2$, where $w_{i(opt)}$, $i = 0, 1, 2$, is given in (14).

For different values of $(\alpha, \eta, \alpha_1, \alpha_2)$ a large number of estimators can be generated from the proposed class of estimators $\widehat{T}^{**}$ for the population mean $\overline{Y}$ and is presented in Table 1. The properties of the members of the proposed class of estimators can be studied from the suggested class of estimators.

TABLE 1: Some particular members of the class of proposed estimator $\widehat{T}_d^{**}$.

| $\alpha$ | $\eta$ | $\alpha_1$ | $\alpha_2$ | Estimator(s) | Min. Mean Squared Error |
|---|---|---|---|---|---|
| $\alpha$ | $\eta$ | 0 | 0 | $\widehat{T}_{(1d)}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{1d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(1)}^{*}\Delta_{0(1)}^{*}}{\Delta_{(1)}^{*}}\right]$ |
| 0 | 0 | $\alpha_1$ | $\alpha_2$ | $\widehat{T}_{(2d)}^{**}=w_0\widehat{\bar{y}}^{**}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{2d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(2)}^{*}\Delta_{0(2)}^{*}}{\Delta_{(2)}^{*}}\right]$ |
| $\alpha$ | 0 | 0 | $\alpha_2$ | $\widehat{T}_{(3d)}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{3d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(3)}^{*}\Delta_{0(3)}^{*}}{\Delta_{(3)}^{*}}\right]$ |
| 0 | $\eta$ | $\alpha_1$ | 0 | $\widehat{T}_{(4d)}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_2'}{\bar{x}_2^{**}}\right)^{\eta}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{4d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(4)}^{*}\Delta_{0(4)}^{*}}{\Delta_{(4)}^{*}}\right]$ |
| 0 | 0 | 0 | 0 | $\widehat{T}_{(5d)}^{**}=w_0\widehat{\bar{y}}^{**}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{5d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(5)}^{*}\Delta_{0(5)}^{*}}{\Delta_{(5)}^{*}}\right]$ |
| 0 | 0 ($w_0=1$) | 0 | 0 | $\widehat{T}_{(6d)}^{**}=\widehat{\bar{y}}^{**}+w_1(\bar{x}_1^{**}-\bar{x}_1')+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{6d}^{**})=\overline{Y}^2\left[(A+\kappa)-\frac{(C^*D^{*2}-2D^*E^*F^*+B^*F^{*2})}{(B^*C^*-E^{*2})}\right]$ |
| 0 | 0 ($w_0=1$, $w_2=0$) | 0 | 0 | $\widehat{T}_{(7d)}^{**}=\widehat{\bar{y}}^{**}+w_1(\bar{x}_1^{**}-\bar{x}_1')$ | $MSE_{\min}(\widehat{T}_{7d}^{**})=\left[(A+\kappa)+\frac{D^{*2}}{B^*}\right]$ |
| 0 | 0 ($w_0=1$, $w_1=0$) | 0 | 0 | $\widehat{T}_{(8d)}^{**}=\widehat{\bar{y}}^{**}+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{8d}^{**})=\left[(A+\kappa)-\frac{F^{*2}}{C^*}\right]$ |
| 0 | 0 ($w_2=0$) | 0 | 0 | $\widehat{T}_{(9d)}^{**}=w_0\widehat{\bar{y}}^{**}+w_1(\bar{x}_1^{**}-\bar{x}_1')$ | $MSE_{\min}(\widehat{T}_{9d}^{**})=\left[\frac{\widehat{\bar{y}}^{**}[(A+\kappa)B-D^2]}{B^*\widehat{\bar{y}}^{**}+\{(A+\kappa)B^*-D^{*2}\}}\right]$ |
| 0 | 0 ($w_1=0$) | 0 | 0 | $\widehat{T}_{(10d)}^{**}=w_0\widehat{\bar{y}}^{**}+w_2(\bar{x}_2^{**}-\bar{x}_2')$ | $MSE_{\min}(\widehat{T}_{10d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(10)}^{*}\Delta_{0(10)}^{*}}{\Delta_{(10)}^{*}}\right]$ |
| $\alpha$ | $\eta$ | $\alpha_1$ | $\alpha_2$ | $\widehat{t}_d^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | $MSE_{\min}(\widehat{t}_d^{**})=\overline{Y}^2\left[1-\frac{H_6^{*2}}{H_0^{*}}\right]$ |
| $\alpha$ | $\eta$ | 0 | 0 | $\widehat{t}_{1d}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}$ | $MSE_{\min}(\widehat{t}_{1d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(1)}^{*2}}{H_{0(1)}^{*}}\right]$ |
| 0 | 0 | $\alpha_1$ | $\alpha_2$ | $\widehat{t}_{2d}^{**}=w_0\widehat{\bar{y}}^{**}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | $MSE_{\min}(\widehat{t}_{2d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(2)}^{*2}}{H_{0(2)}^{*}}\right]$ |
| $\alpha$ | 0 | 0 | $\alpha_2$ | $\widehat{t}_{3d}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | $MSE_{\min}(\widehat{t}_{3d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(3)}^{*2}}{H_{0(3)}^{*}}\right]$ |
| $\alpha$ | $\eta$ | 0 | 0 | $\widehat{t}_{4d}^{**}=w_0\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}$ | $MSE_{\min}(\widehat{t}_{4d}^{**})=\overline{Y}^2\left[1-\frac{H_{6(4)}^{*2}}{H_{0(4)}^{*}}\right]$ |
| $\alpha$ | $\eta$ ($w_0=1$) | $\alpha_1$ | $\alpha_2$ | $\widehat{t}_{ds}^{**}=\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | |
| $\alpha$ | $\eta$ ($w_0=1$) | 0 | 0 | $\widehat{t}_{1ds}^{**}=\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}$ | |
| 0 | 0 ($w_0=1$) | $\alpha_1$ | $\alpha_2$ | $\widehat{t}_{2ds}^{**}=\widehat{\bar{y}}^{**}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | $MSE_{\min}(\widehat{t}_j^{**})=\left[A+\kappa-\frac{(C^*D^{*2}-2D^*E^*F^*+B^*F^2)}{(B^*C^*-E^{*2})}\right]$, j=ds, 1ds, 2ds, 3ds and 4ds |
| $\alpha$ | 0 ($w_0=1$) | 0 | $\alpha_2$ | $\widehat{t}_{3ds}^{**}=\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_1'}{\bar{x}_1^{**}}\right)^{\alpha}\exp\left\{\frac{\alpha_2(\bar{x}_2'-\bar{x}_2^{**})}{\bar{x}_2'+\bar{x}_2^{**}}\right\}$ | |
| $\alpha$ | $\eta$ ($w_0=1$) | 0 | 0 | $\widehat{t}_{4ds}^{**}=\widehat{\bar{y}}^{**}\left(\frac{\bar{x}_2^{**}}{\bar{x}_2'}\right)^{\eta}\exp\left\{\frac{\alpha_1(\bar{x}_1'-\bar{x}_1^{**})}{\bar{x}_1'+\bar{x}_1^{**}}\right\}$ | |

# 4. Efficiency Comparisons

To compare the mean squared error of the proposed estimator $\widehat{T}_d^{**}$ to the mean squared error of the class of proposed estimator and Kumar & Zhang (2023) estimator, we establish the efficiency requirements as follows:

(i) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(1d)}^{**})$,

$$if \quad \Delta_{(1)}^* H_6^* \Delta_0^* - \Delta^* H_{6(1)}^* \Delta_{0(1)}^* < 0,$$

(ii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(2d)}^{**})$,

$$if \quad \Delta_{(2)}^* H_6^* \Delta_0^* - \Delta^* H_{6(2)}^* \Delta_{0(2)}^* < 0,$$

(iii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(3d)}^{**})$,

$$if \quad \Delta_{(3)}^* H_6^* \Delta_0^* - \Delta^* H_{6(3)}^* \Delta_{0(3)}^* < 0,$$

(iv) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(4d)}^{**})$,

$$if \quad \Delta_{(4)}^* H_6^* \Delta_0^* - \Delta^* H_{6(4)}^* \Delta_{0(4)}^* < 0,$$

(v) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(5d)}^{**})$,

$$if \quad \Delta_{(5)}^* H_6^* \Delta_0^* - \Delta^* H_{6(5)}^* \Delta_{0(5)}^* < 0,$$

(vi) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(6d)}^{**})$,

$$if \quad \overline{Y}^2 \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right] - \left[(A+K) - \frac{(C^* D^{*2} - 2D^* E^* F^* + B^* F^{*2})}{(B^* C^* - E^{*2})}\right] < 0,$$

(vii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(7d)}^{**})$,

$$if \quad \overline{Y}^2 \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right] - \left[(A+K) - \frac{D^*}{B^*}\right] < 0,$$

(viii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(8d)}^{**})$,

$$if \quad \overline{Y}^2 \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right] - \left[(A+K) - \frac{F^{*2}}{C^*}\right] < 0,$$

(ix) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(9d)}^{**})$,

$$if \quad \left[1 - \frac{H_6^* \Delta_0^*}{\Delta^*}\right] - \frac{\left[(A+K)B^* - D^{*2}\right]}{B^* \overline{Y}^2 + \{(A+K)B^* - D^{*2}\}} < 0,$$

(x) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(10d)}^{**})$,

$if \quad \Delta_{(10)}^* H_6^* \Delta_0^* - \Delta^* H_{6(10)}^* \Delta_{0(10)}^* < 0$,

(xi) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_d^{**})$,

$if \quad \Delta_0^* H_0^* - \Delta^* H_6^* < 0$,

(xii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_{(1d)}^{**})$,

$if \quad \Delta_{0(1)}^* H_0^* - \Delta^* H_{6(1)}^* < 0$,

(xiii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_{(2d)}^{**})$,

$if \quad \Delta_{0(2)}^* H_0^* - \Delta^* H_{6(2)}^* < 0$,

(xiv) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_{(3d)}^{**})$,

$if \quad \Delta_{0(3)}^* H_0^* - \Delta^* H_{6(3)}^* < 0$,

(xv) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_{(4d)}^{**})$,

$if \quad \Delta_{0(4)}^* H_0^* - \Delta^* H_{6(4)}^* < 0$,

(xvi) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{t}_{(dj)}^{**}), j = 0, 1, 2, 3, 4$,

$$if \quad \overline{Y}^2\left[1 - \frac{H_6^{*2}}{H_0^*}\right] - \left[A + k - \frac{C^* D^{*2} - 2D^* E^* F^* + B^* F^{*2}}{(B^* C^* - E^{*2})}\right] < 0,$$

(xvii) $MSE_{\min}(\widehat{T}_d^{**}) < MSE_{\min}(\widehat{T}_{(sszd)}^{**})$

$$\begin{aligned} if \quad & \overline{Y}^2\left[1 - \frac{H_6^{*2}}{H_0^*}\right] - \overline{Y}^2(4\widehat{\alpha}_{dopt}^{**} - 1)^2 + 16\widehat{\alpha}_{dopt}^{**2} A + 16\beta_{yx_1}^2 B + 16\beta_{yx_2}^2 C - \\ & 16\lambda' \beta_{yx_1}^2 S_{x_1}^2 - 16\lambda' \beta_{yx_2}^2 S_{x_2}^2 - 32\widehat{\alpha}_{dopt}^{**} \beta_{yx_1} D + 32\lambda' \widehat{\alpha}_{dopt}^{**} \beta_{yx_1} \rho_{yx_1} S_y S_{x_1} + \\ & 32\lambda' \widehat{\alpha}_{dopt}^{**} \beta_{yx_2} \rho_{yx_2} S_y S_{x_2} + 32\beta_{yx_1} \beta_{yx_2} E + 32\lambda' \beta_{yx_1} \beta_{yx_2} S_{x_1} S_{x_2} - \\ & 32\widehat{\alpha}_{dopt}^{**} \beta_{yx_2} F + 16\widehat{\alpha}_{dopt}^{**2} \kappa < 0. \end{aligned}$$

If the above relations from (i) to (xvii) holds true then it is clear that the proposed estimator is more efficient than the other class of proposed estimator(s) and Kumar & Zhang (2023) estimator that are used for comparison purpose.

In the next section, we conducted a simulation study by using `R` software to verify the above results.

# 5. Simulation Study

To evaluate the performance of proposed estimator, we conduct a simulation study to compare the mean squared error (MSE) of the proposed class of estimator by using R software. We assume three finite populations of size $N = 10,000$ generated from a normal distribution and from $N$ we picked a large sample of size $n' = 7000$ and a small sample of sizes $n = 3000, 4000, 5000$ is taken from $n'$. In the first phase, only 1600 ($n_1$) provide a response to the survey question and $n_2 = n - n_1$ i.e. 2400 of them do not respond. In the second phase, we take another sample $\left(n_s = \frac{n_2}{k}\right)$ from the non-respondent group by using different values of $k = 2, 3, 4, 5$. First a variable $X_1 \sim N(0.2, 1)$, $X_2 \sim N(0.2, 1)$ and variable $Y$ which is defined as $Y = aX_1 + aX_2 + N(0, 1)$ generated from a normal distribution where $a = 0.02$. And second a variable $Y$ which is defined as $Y = bX_1 + bX_2 + N(0, 1)$ generated from a normal distribution where $b = 0.05$. The scrambling variable(s) $S_1$ is taken from a normal distribution with a mean of 1 and a variance of 0.5, whereas the scrambling variable $S_2$ is drawn from a normal distribution with a mean of 0 and a variance of 1, respectively. The MSE's of the considered estimators for different levels of $W$ (i.e. 0.2 to 1) and $k$ (i.e. $2, 3, 4, 5$) are shown in Table 2 to 7.

Next, we calculate PRE's of the proposed and existing estimator for different values of $n, k$ and $W$.

Thus, the percent relative efficiency of the proposed estimator $(T_d^{**})$ among the different classes of proposed estimator i.e. $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_{d}^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$, unbiased estimator $(\widehat{y}_d^{**})$ and Kumar & Zhang (2023) estimator $(\widehat{T}_{sszd}^{**})$, respectively, are as

$$PRE(\iota) = \left[\frac{MSE(\widehat{y}_d^{**})}{MSE(\iota)}\right] \times 100 \tag{8}$$

where, $\iota = T_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_{d}^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$ and $\widehat{T}_{sszd}^{**}$.

Tables 2, 3 and 4 represents the comparison of percent relative efficiency (PRE) of the proposed estimator over different class of estimator(s), unbiased estimator $(\widehat{y}_d^{**})$ and Kumar & Zhang (2023) estimator for different values of '$n$', '$k$' and '$W$' in double sampling when $Y = aX_1 + aX_2 + N(0, 1)$ where $a = 0.02$.

1. It is clear from Table 2, 3 and 4 that if we increase the sample size from 3000 to 5000, the percent relative efficiency of all the considered estimator(s) decreases.

Table 2: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 3000$ and model $Y = aX_1 + aX_2 + N(0,1)$, where $a = 0.02$.

| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{y}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 328.14 | 420.50 | 518.09 | 604.42 | 470.91 | 598.90 | 708.54 | 814.96 | 718.32 | 973.33 | 1186.42 | 1478.51 |
| $\widehat{T}_{(1d)}^{**}$ | 323.94 | 412.01 | 505.92 | 584.72 | 472.04 | 598.34 | 707.69 | 811.91 | 721.92 | 975.02 | 1186.71 | 1474.00 |
| $\widehat{T}_{(2d)}^{**}$ | 302.38 | 374.24 | 439.28 | 500.50 | 459.13 | 579.20 | 677.83 | 777.29 | 712.52 | 963.03 | 1167.73 | 1450.62 |
| $\widehat{T}_{(3d)}^{**}$ | 311.40 | 389.98 | 460.49 | 526.80 | 469.27 | 594.20 | 697.18 | 800.48 | 723.46 | 978.94 | 1188.00 | 1475.83 |
| $\widehat{T}_{(4d)}^{**}$ | 292.85 | 361.50 | 424.29 | 483.51 | 452.02 | 570.09 | 667.13 | 765.26 | 706.49 | 955.18 | 1158.33 | 1439.61 |
| $\widehat{T}_{(5d)}^{**}$ | 292.71 | 361.13 | 423.86 | 482.87 | 451.43 | 569.23 | 666.07 | 763.95 | 705.62 | 953.90 | 1156.71 | 1437.69 |
| $\widehat{T}_{(6d)}^{**}$ | 101.99 | 103.92 | 105.31 | 106.66 | 101.09 | 102.15 | 102.88 | 103.62 | 100.56 | 101.37 | 102.11 | 103.27 |
| $\widehat{T}_{(7d)}^{**}$ | 100.92 | 101.71 | 102.03 | 102.46 | 100.51 | 100.93 | 101.11 | 101.36 | 100.29 | 100.67 | 100.87 | 101.23 |
| $\widehat{T}_{(8d)}^{**}$ | 101.05 | 102.18 | 103.17 | 104.05 | 100.58 | 101.22 | 101.74 | 102.23 | 100.28 | 100.70 | 101.21 | 102.01 |
| $\widehat{T}_{(9d)}^{**}$ | 291.65 | 358.92 | 420.58 | 478.67 | 450.86 | 568.01 | 664.30 | 761.68 | 705.35 | 953.20 | 1155.55 | 1435.50 |
| $\widehat{T}_{(10d)}^{**}$ | 291.78 | 359.39 | 421.71 | 480.25 | 450.92 | 568.29 | 664.93 | 762.56 | 705.34 | 953.23 | 1155.86 | 1436.30 |
| $\widehat{t}_d^{**}$ | 295.03 | 362.60 | 425.80 | 484.70 | 454.93 | 572.78 | 670.84 | 769.22 | 709.76 | 957.77 | 1162.10 | 1443.51 |
| $\widehat{t}_{(1d)}^{**}$ | 296.93 | 364.51 | 427.84 | 486.17 | 458.27 | 575.62 | 673.71 | 771.36 | 714.15 | 960.64 | 1163.90 | 1442.32 |
| $\widehat{t}_{(2d)}^{**}$ | 297.41 | 366.09 | 428.72 | 487.72 | 456.49 | 574.89 | 672.28 | 770.61 | 711.10 | 960.43 | 1163.90 | 1445.20 |
| $\widehat{t}_{(3d)}^{**}$ | 306.15 | 380.72 | 447.94 | 511.00 | 466.41 | 589.31 | 690.66 | 792.39 | 721.91 | 975.95 | 1183.51 | 1469.20 |
| $\widehat{t}_{(4d)}^{**}$ | 288.48 | 354.64 | 415.63 | 473.25 | 449.60 | 566.24 | 662.28 | 759.50 | 705.17 | 952.81 | 1154.95 | 1434.90 |
| $\widehat{t}_{d_j}^{**}$ | 101.99 | 103.92 | 105.31 | 106.66 | 101.09 | 102.15 | 102.88 | 103.62 | 100.56 | 101.37 | 102.11 | 103.27 |
| $\widehat{T}_{sszd}^{**}$ | 290.36 | 358.85 | 420.94 | 479.28 | 449.02 | 566.35 | 662.15 | 758.91 | 703.20 | 950.60 | 1152.10 | 1431.00 |

TABLE 3: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 4000$ and model $Y = aX_1 + aX_2 + N(0,1)$, where $a = 0.02$.

| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{\bar{y}}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 275.59 | 387.18 | 511.71 | 629.86 | 354.83 | 470.33 | 569.72 | 666.74 | 530.99 | 712.10 | 908.35 | 1109.62 |
| $\widehat{T}_{(1d)}^{**}$ | 269.01 | 372.52 | 491.39 | 594.52 | 353.71 | 466.87 | 566.03 | 660.29 | 532.21 | 712.45 | 907.91 | 1107.75 |
| $\widehat{T}_{(2d)}^{**}$ | 241.75 | 305.69 | 358.19 | 410.53 | 340.45 | 442.56 | 528.24 | 618.42 | 524.17 | 699.82 | 891.27 | 1087.24 |
| $\widehat{T}_{(3d)}^{**}$ | 245.64 | 316.87 | 375.51 | 433.26 | 345.43 | 452.25 | 542.32 | 635.92 | 529.52 | 708.55 | 902.84 | 1102.12 |
| $\widehat{T}_{(4d)}^{**}$ | 232.99 | 292.90 | 342.47 | 392.53 | 334.49 | 434.37 | 518.23 | 607.20 | 519.80 | 694.09 | 884.62 | 1079.41 |
| $\widehat{T}_{(5d)}^{**}$ | 233.15 | 292.79 | 341.96 | 391.64 | 334.27 | 433.85 | 517.33 | 605.98 | 519.43 | 693.48 | 883.77 | 1078.26 |
| $\widehat{T}_{(6d)}^{**}$ | 102.76 | 106.09 | 107.85 | 109.47 | 101.44 | 103.17 | 104.09 | 104.87 | 100.67 | 101.44 | 101.97 | 102.60 |
| $\widehat{T}_{(7d)}^{**}$ | 101.33 | 102.87 | 103.72 | 104.70 | 100.69 | 101.53 | 102.02 | 102.53 | 100.29 | 100.59 | 100.77 | 100.99 |
| $\widehat{T}_{(8d)}^{**}$ | 101.40 | 103.07 | 103.78 | 104.30 | 100.74 | 101.60 | 101.96 | 102.21 | 100.38 | 100.84 | 101.18 | 101.57 |
| $\widehat{T}_{(9d)}^{**}$ | 231.72 | 289.56 | 337.83 | 386.86 | 333.53 | 432.21 | 515.27 | 603.64 | 519.04 | 692.63 | 882.57 | 1076.62 |
| $\widehat{T}_{(10d)}^{**}$ | 231.79 | 289.76 | 337.89 | 386.47 | 333.57 | 432.28 | 515.20 | 603.32 | 519.14 | 692.88 | 882.98 | 1077.21 |
| $\widehat{t}_d^{**}$ | 232.75 | 289.55 | 337.50 | 385.42 | 335.39 | 433.53 | 516.38 | 603.95 | 521.72 | 695.85 | 886.64 | 1081.45 |
| $\widehat{t}_{(1d)}^{**}$ | 232.82 | 289.41 | 336.97 | 383.87 | 336.55 | 433.86 | 516.17 | 602.5 | 523.78 | 697.30 | 887.41 | 1081.18 |
| $\widehat{t}_{(2d)}^{**}$ | 235.8 | 294.61 | 344.15 | 393.78 | 337.38 | 437.01 | 521.30 | 610.34 | 522.66 | 697.21 | 887.94 | 1083.00 |
| $\widehat{t}_{(3d)}^{**}$ | 239.63 | 304.61 | 359.06 | 412.72 | 342.26 | 446.19 | 534.41 | 626.42 | 527.95 | 705.74 | 899.15 | 1097.30 |
| $\widehat{t}_{(4d)}^{**}$ | 227.77 | 283.68 | 331.19 | 379.40 | 331.67 | 429.41 | 512.18 | 600.25 | 518.37 | 691.68 | 881.56 | 1075.60 |
| $\widehat{t}_{d_j}^{**}$ | 102.76 | 106.09 | 107.85 | 109.47 | 101.44 | 103.17 | 104.09 | 104.87 | 100.67 | 101.44 | 101.97 | 102.60 |
| $\widehat{T}_{sszd}^{**}$ | 232.40 | 292.06 | 341.14 | 390.71 | 333.56 | 433.16 | 516.41 | 604.78 | 518.56 | 692.30 | 881.97 | 1075.60 |

TABLE 4: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 5000$ and model $Y = aX_1 + aX_2 + N(0, 1)$, where $a = 0.02$.

| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{\bar{y}}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 251.23 | 362.79 | 510.94 | 683.75 | 301.83 | 396.74 | 493.49 | 583.82 | 422.12 | 574.66 | 720.72 | 848.61 |
| $\widehat{T}_{(1d)}^{**}$ | 243.05 | 340.38 | 467.12 | 612.79 | 299.50 | 390.98 | 485.27 | 573.84 | 422.10 | 573.03 | 717.33 | 845.74 |
| $\widehat{T}_{(2d)}^{**}$ | 208.37 | 260.13 | 309.82 | 355.87 | 284.78 | 363.92 | 444.36 | 522.90 | 414.81 | 561.55 | 702.32 | 823.48 |
| $\widehat{T}_{(3d)}^{**}$ | 209.00 | 267.48 | 322.82 | 373.52 | 286.85 | 370.14 | 454.09 | 535.71 | 417.35 | 566.99 | 710.25 | 833.72 |
| $\widehat{T}_{(4d)}^{**}$ | 199.98 | 247.59 | 294.41 | 338.50 | 279.48 | 356.41 | 435.32 | 512.71 | 411.15 | 556.63 | 696.41 | 816.83 |
| $\widehat{T}_{(5d)}^{**}$ | 200.31 | 247.93 | 294.53 | 338.28 | 279.43 | 356.26 | 434.95 | 512.01 | 410.93 | 556.27 | 695.88 | 816.13 |
| $\widehat{T}_{(6d)}^{**}$ | 103.85 | 107.84 | 110.43 | 111.82 | 102.01 | 104.09 | 105.43 | 106.17 | 100.89 | 101.90 | 102.74 | 103.00 |
| $\widehat{T}_{(7d)}^{**}$ | 101.80 | 103.43 | 104.56 | 105.35 | 100.95 | 101.83 | 102.43 | 102.94 | 100.42 | 100.83 | 101.12 | 101.12 |
| $\widehat{T}_{(8d)}^{**}$ | 101.99 | 104.22 | 105.44 | 105.85 | 101.04 | 102.23 | 102.90 | 103.06 | 100.46 | 101.07 | 101.61 | 101.83 |
| $\widehat{T}_{(9d)}^{**}$ | 198.26 | 243.52 | 288.66 | 331.82 | 278.38 | 354.00 | 431.95 | 508.78 | 410.47 | 555.20 | 694.26 | 814.25 |
| $\widehat{T}_{(10d)}^{**}$ | 198.45 | 244.31 | 289.54 | 332.31 | 278.47 | 354.40 | 432.42 | 508.9 | 410.50 | 555.44 | 694.75 | 814.96 |
| $\widehat{t}_d^{**}$ | 197.85 | 242.02 | 286.46 | 329.01 | 278.89 | 354.12 | 431.84 | 508.07 | 411.61 | 556.53 | 695.97 | 817.16 |
| $\widehat{t}_{(1d)}^{**}$ | 197.39 | 241.52 | 285.39 | 327.19 | 279.07 | 353.75 | 430.68 | 505.93 | 412.34 | 556.58 | 695.16 | 816.26 |
| $\widehat{t}_{(2d)}^{**}$ | 201.16 | 246.82 | 292.43 | 336.15 | 281.12 | 357.29 | 435.82 | 513.30 | 413.12 | 558.46 | 698.11 | 818.84 |
| $\widehat{t}_{(3d)}^{**}$ | 201.94 | 253.23 | 303.07 | 350.17 | 283.17 | 363.11 | 444.69 | 524.82 | 415.63 | 563.74 | 705.71 | 828.61 |
| $\widehat{t}_{(4d)}^{**}$ | 193.63 | 236.50 | 280.39 | 322.92 | 276.09 | 350.45 | 427.80 | 504.37 | 409.54 | 553.73 | 692.51 | 812.58 |
| $\widehat{t}_{d_j}^{**}$ | 103.85 | 107.84 | 110.43 | 111.82 | 102.01 | 104.09 | 105.43 | 106.17 | 100.89 | 101.90 | 102.74 | 103.00 |
| $\widehat{T}_{sszd}^{**}$ | 199.57 | 246.46 | 292.74 | 336.59 | 278.99 | 355.55 | 434.15 | 511.25 | 410.56 | 555.69 | 695.03 | 814.92 |

TABLE 5: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 3000$ and model $Y = bX_1 + bX_2 + N(0,1)$ where $b = 0.05$.

| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{\bar{y}}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 248.39 | 318.52 | 417.77 | 497.85 | 284.34 | 349.68 | 410.79 | 464.71 | 382.60 | 499.95 | 601.47 | 737.43 |
| $\widehat{T}_{(1d)}^{**}$ | 225.96 | 284.32 | 362.77 | 418.08 | 279.82 | 343.71 | 404.43 | 456.25 | 384.26 | 501.57 | 603.22 | 736.53 |
| $\widehat{T}_{(2d)}^{**}$ | 197.67 | 233.37 | 264.40 | 293.82 | 263.10 | 317.93 | 362.61 | 407.59 | 371.60 | 482.71 | 573.35 | 698.55 |
| $\widehat{T}_{(3d)}^{**}$ | 199.66 | 243.83 | 281.34 | 317.09 | 269.55 | 330.37 | 379.91 | 429.42 | 380.39 | 497.50 | 593.05 | 724.08 |
| $\widehat{T}_{(4d)}^{**}$ | 184.89 | 216.14 | 244.22 | 270.94 | 254.27 | 306.49 | 349.22 | 392.54 | 364.62 | 473.44 | 562.22 | 685.35 |
| $\widehat{T}_{(5d)}^{**}$ | 185.97 | 217.05 | 245.29 | 271.89 | 254.35 | 306.37 | 349.04 | 392.16 | 364.14 | 472.58 | 561.20 | 683.98 |
| $\widehat{T}_{(6d)}^{**}$ | 102.33 | 104.29 | 105.66 | 107.01 | 101.27 | 102.35 | 103.07 | 103.80 | 100.66 | 101.48 | 102.22 | 103.38 |
| $\widehat{T}_{(7d)}^{**}$ | 101.09 | 101.88 | 102.18 | 102.61 | 100.60 | 101.02 | 101.20 | 101.44 | 100.34 | 100.72 | 100.92 | 101.28 |
| $\widehat{T}_{(8d)}^{**}$ | 101.23 | 102.37 | 103.35 | 104.23 | 100.67 | 101.32 | 101.84 | 102.33 | 100.32 | 100.75 | 101.27 | 102.07 |
| $\widehat{T}_{(9d)}^{**}$ | 184.73 | 214.64 | 241.81 | 267.49 | 253.68 | 305.05 | 347.17 | 389.8 | 363.82 | 471.82 | 559.91 | 681.87 |
| $\widehat{T}_{(10d)}^{**}$ | 184.87 | 215.13 | 242.98 | 269.11 | 253.75 | 305.34 | 347.81 | 390.69 | 363.81 | 471.85 | 560.25 | 682.66 |
| $\widehat{t}_d^{**}$ | 187.03 | 217.57 | 246.00 | 272.54 | 257.34 | 309.60 | 353.26 | 396.93 | 368.24 | 476.97 | 566.86 | 690.27 |
| $\widehat{t}_{(1d)}^{**}$ | 185.00 | 217.35 | 247.05 | 274.36 | 258.95 | 312.28 | 356.94 | 401.09 | 372.04 | 481.26 | 571.59 | 694.36 |
| $\widehat{t}_{(2d)}^{**}$ | 189.92 | 221.79 | 249.95 | 276.76 | 259.01 | 311.91 | 355.12 | 398.82 | 369.39 | 479.15 | 568.44 | 691.86 |
| $\widehat{t}_{(3d)}^{**}$ | 192.37 | 231.73 | 265.29 | 297.06 | 265.41 | 323.83 | 371.43 | 419.08 | 378.07 | 493.5 | 587.31 | 715.87 |
| $\widehat{t}_{(4d)}^{**}$ | 178.45 | 207.06 | 233.29 | 258.38 | 250.64 | 301.32 | 342.96 | 385.33 | 362.59 | 470.26 | 557.96 | 679.66 |
| $\widehat{t}_{d_j}^{**}$ | 102.33 | 104.29 | 105.66 | 107.01 | 101.27 | 102.35 | 103.07 | 103.80 | 100.66 | 101.48 | 102.22 | 103.38 |
| $\widehat{T}_{sszd}^{**}$ | 178.88 | 210.24 | 237.49 | 263.08 | 247.33 | 298.31 | 338.91 | 379.93 | 356.05 | 461.76 | 546.91 | 665.12 |

TABLE 6: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 4000$ and model $Y = bX_1+bX_2+N(0,1)$ where $b = 0.05$.

| | b | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{y}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 236.23 | 364.52 | 631.76 | 986.23 | 235.66 | 302.47 | 366.11 | 418.22 | 299.82 | 384.87 | 475.24 | 568.66 |
| $\widehat{T}_{(1d)}^{**}$ | 209.78 | 305.99 | 484.97 | 650.64 | 228.86 | 293.11 | 356.98 | 406.86 | 299.09 | 384.31 | 475.15 | 568.49 |
| $\widehat{T}_{(2d)}^{**}$ | 171.26 | 205.55 | 232.49 | 258.94 | 210.69 | 258.55 | 298.18 | 339.25 | 288.33 | 366.38 | 450.73 | 537.29 |
| $\widehat{T}_{(3d)}^{**}$ | 167.33 | 211.44 | 245.95 | 279.97 | 211.45 | 265.66 | 310.49 | 356.03 | 291.31 | 373.56 | 461.34 | 551.71 |
| $\widehat{T}_{(4d)}^{**}$ | 159.04 | 187.31 | 210.16 | 233.24 | 202.96 | 247.70 | 284.99 | 324.46 | 283.10 | 359.42 | 442.65 | 527.84 |
| $\widehat{T}_{(5d)}^{**}$ | 160.28 | 188.32 | 210.81 | 233.46 | 203.35 | 247.80 | 284.73 | 323.85 | 283.10 | 359.22 | 442.26 | 527.18 |
| $\widehat{T}_{(6d)}^{**}$ | 103.10 | 106.48 | 108.20 | 109.81 | 101.62 | 103.37 | 104.27 | 105.04 | 100.76 | 101.53 | 102.05 | 102.69 |
| $\widehat{T}_{(7d)}^{**}$ | 101.49 | 103.05 | 103.88 | 104.86 | 100.78 | 101.63 | 102.11 | 102.61 | 100.33 | 100.63 | 100.81 | 101.02 |
| $\widehat{T}_{(8d)}^{**}$ | 101.56 | 103.25 | 103.94 | 104.45 | 100.82 | 101.70 | 102.04 | 102.28 | 100.43 | 100.89 | 101.22 | 101.62 |
| $\widehat{T}_{(9d)}^{**}$ | 158.67 | 184.89 | 206.49 | 228.51 | 202.52 | 246.06 | 282.57 | 321.42 | 282.67 | 358.33 | 441.01 | 525.52 |
| $\widehat{T}_{(10d)}^{**}$ | 158.75 | 185.09 | 206.56 | 228.11 | 202.56 | 246.13 | 282.50 | 321.09 | 282.77 | 358.58 | 441.43 | 526.11 |
| $\widehat{t}_d^{**}$ | 158.78 | 184.56 | 206.41 | 227.95 | 203.98 | 247.50 | 284.39 | 323.03 | 285.07 | 361.45 | 445.05 | 530.48 |
| $\widehat{t}_{(1d)}^{**}$ | 155.46 | 183.00 | 205.47 | 227.42 | 203.48 | 247.85 | 285.24 | 323.92 | 286.43 | 363.42 | 447.41 | 532.99 |
| $\widehat{t}_{(2d)}^{**}$ | 162.48 | 190.60 | 213.91 | 237.05 | 206.18 | 251.19 | 289.21 | 328.99 | 286.08 | 362.90 | 446.46 | 532.07 |
| $\widehat{t}_{(3d)}^{**}$ | 159.58 | 196.21 | 225.52 | 254.12 | 207.13 | 257.98 | 300.61 | 344.20 | 289.07 | 369.91 | 456.69 | 545.82 |
| $\widehat{t}_{(4d)}^{**}$ | 151.77 | 175.87 | 196.61 | 217.79 | 198.93 | 241.41 | 277.58 | 316.12 | 281.01 | 356.26 | 438.83 | 523.23 |
| $\widehat{t}_{d_j}^{**}$ | 103.10 | 106.48 | 108.20 | 109.81 | 101.62 | 103.37 | 104.27 | 105.04 | 100.76 | 101.53 | 102.05 | 102.69 |
| $\widehat{T}_{sszd}^{**}$ | 156.70 | 185.78 | 208.04 | 230.39 | 200.11 | 244.82 | 280.94 | 319.10 | 279.46 | 354.66 | 435.98 | 518.62 |

TABLE 7: Percent Relative Efficiency (PRE) of estimator(s) at varying values of $k$ and $W$ when $n = 5000$ and model $Y = bX_1 + bX_2 + N(0,1)$ where $b = 0.05$.

| Estimator(s) | $W = 0.2$ | | | | $W = 0.6$ | | | | $W = 1$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k \rightarrow$ | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |
| $\widehat{\bar{y}}_d^{**}$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $\widehat{T}_d^{**}$ | 233.00 | 373.17 | 691.57 | 1636.50 | 213.21 | 271.63 | 333.26 | 389.13 | 252.25 | 324.45 | 393.12 | 456.01 |
| $\widehat{T}_{(1d)}^{**}$ | 204.68 | 295.68 | 462.85 | 764.78 | 205.89 | 258.98 | 316.99 | 371.30 | 250.68 | 321.68 | 389.15 | 453.69 |
| $\widehat{T}_{(2d)}^{**}$ | 157.11 | 186.30 | 212.56 | 235.68 | 186.28 | 224.14 | 261.49 | 297.40 | 240.39 | 305.93 | 368.5 | 421.87 |
| $\widehat{T}_{(3d)}^{**}$ | 149.05 | 187.33 | 221.39 | 251.02 | 183.86 | 227.39 | 269.36 | 309.29 | 240.33 | 309.77 | 375.44 | 431.34 |
| $\widehat{T}_{(4d)}^{**}$ | 145.17 | 168.10 | 190.09 | 210.34 | 179.21 | 213.93 | 249.18 | 283.56 | 235.88 | 299.73 | 361.02 | 413.55 |
| $\widehat{T}_{(5d)}^{**}$ | 146.46 | 169.58 | 191.42 | 211.33 | 179.68 | 214.39 | 249.46 | 283.49 | 235.95 | 299.72 | 360.9 | 413.33 |
| $\widehat{T}_{(6d)}^{**}$ | 104.17 | 108.17 | 110.74 | 112.09 | 102.17 | 104.26 | 105.58 | 106.30 | 100.97 | 101.98 | 102.82 | 103.07 |
| $\widehat{T}_{(7d)}^{**}$ | 101.95 | 103.58 | 104.7 | 105.47 | 101.03 | 101.91 | 102.50 | 103.01 | 100.46 | 100.87 | 101.16 | 101.15 |
| $\widehat{T}_{(8d)}^{**}$ | 102.14 | 104.38 | 105.59 | 105.97 | 101.12 | 102.31 | 102.97 | 103.12 | 100.50 | 101.11 | 101.65 | 101.87 |
| $\widehat{T}_{(9d)}^{**}$ | 144.25 | 164.98 | 185.38 | 204.71 | 178.54 | 212.04 | 246.39 | 280.20 | 235.45 | 298.61 | 359.25 | 411.42 |
| $\widehat{T}_{(10d)}^{**}$ | 144.44 | 165.79 | 186.27 | 205.21 | 178.63 | 212.44 | 246.86 | 280.31 | 235.48 | 298.85 | 359.74 | 412.13 |
| $\widehat{t}_d^{**}$ | 143.12 | 163.01 | 183.05 | 202.22 | 178.78 | 212.09 | 246.49 | 280.21 | 236.51 | 299.97 | 361.07 | 414.34 |
| $\widehat{t}_{(1d)}^{**}$ | 139.96 | 161.26 | 181.88 | 201.31 | 177.78 | 211.72 | 246.36 | 280.11 | 236.73 | 300.57 | 361.75 | 415.46 |
| $\widehat{t}_{(2d)}^{**}$ | 147.14 | 169.02 | 190.37 | 210.63 | 181.27 | 215.71 | 250.89 | 285.59 | 238.02 | 301.99 | 363.32 | 416.21 |
| $\widehat{t}_{(3d)}^{**}$ | 140.66 | 170.54 | 197.75 | 222.59 | 179.23 | 218.89 | 258.08 | 296.17 | 238.05 | 305.75 | 369.96 | 425.21 |
| $\widehat{t}_{(4d)}^{**}$ | 136.91 | 154.87 | 173.89 | 192.58 | 174.72 | 206.68 | 240.32 | 273.87 | 233.66 | 296.12 | 356.35 | 408.51 |
| $\widehat{t}_{d_j}^{**}$ | 104.17 | 108.17 | 110.74 | 112.09 | 102.17 | 104.26 | 105.58 | 106.30 | 100.97 | 101.98 | 102.82 | 103.07 |
| $\widehat{T}_{sszd}^{**}$ | 144.41 | 167.99 | 189.82 | 209.64 | 178.08 | 212.99 | 247.87 | 281.49 | 234.19 | 297.72 | 358.30 | 409.53 |

2. From Table 2, when the value of $k$ increases from 2 to 5 and $W$ increases from 0.2 to 1, the percent relative efficiency of all considered estimator(s) increases i.e. unbiased estimator ($\widehat{y}_d^{**}$), $\widehat{T}_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_d^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$, $\widehat{T}_{sszd}^{**}$ also increases.

3. It is also illustrated from Table 2, 3 and 4 that the percent relative efficiency of proposed estimator $\widehat{T}_d^{**}$ is highest among all other class of estimator(s) i.e. $\widehat{y}_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_d^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$ and Kumar & Zhang (2023) estimator ($\widehat{T}_{sszd}^{**}$). In the end, we obtain that the proposed estimator is more efficient than the other different classes of proposed estimator and Kumar & Zhang (2023) estimator in terms of having highest percent relative efficiency.

Tables 5, 6 and 7 represents the comparison of percent relative efficiency (PRE) of the proposed estimator over different class of estimator(s) and Kumar & Zhang (2023) estimator for different values of '$n$', '$k$' and '$W$' in double sampling when $Y = bX_1 + bX_2 + N(0, 1)$ where $b = 0.05$.

1. It is visualize from Table 5, 6 and 7 that if we increase the sample size, the percent relative efficiency of all the considered estimator(s) decreases.

2. From Table 5, 6 and 7 when the value of $k$ increases from 2 to 5 and $W$ increases from 0.2 to 1, the percent relative efficiency of all considered estimator(s) increases i.e. unbiased estimator ($\widehat{y}_d^{**}$), $\widehat{T}_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_d^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$, $\widehat{T}_{sszd}^{**}$ also increases.

3. It is also illustrated from Table 5, 6 and 7 that the percent relative efficiency of proposed estimator $\widehat{T}_d^{**}$ is highest among all other class of estimator(s) i.e. $\widehat{y}_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_d^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$ and Kumar & Zhang (2023) estimator ($\widehat{T}_{sszd}^{**}$). In the end, we obtain that the proposed estimator is more efficient than the other different classes of proposed estimator and Kumar & Zhang (2023) estimator in terms of having highest percent relative efficiency.

## 6. Conclusion

The brief summary of this paper has addressed the challenging task of estimating the population mean of a sensitive variable under non-response and measurement error using ORRT models under double sampling. The efficiency of the proposed estimator and class of estimator are evaluated up to the first order of approximation and the conditions are also determined in comparison to other examined estimators and Kumar & Zhang (2023) estimator. We have conducted two model based simulation study i.e., $Y = aX_1 + aX_2 + N(0, 1)$ and

$Y = bX_1 + bX_2 + N(0,1)$ with varying values of sample size $n$, non response rate $k$ and sensitive question probability $W$. Through the simulation study, the effectiveness and performance of the proposed class of estimator is evaluated and found that the proposed estimator $\widehat{T}_d^{**}$ obtain the highest percent relative efficiency among the different classes of proposed estimator i.e. $\widehat{y}_d^{**}$, $\widehat{T}_{(1d)}^{**}$, $\widehat{T}_{(2d)}^{**}$, $\widehat{T}_{(3d)}^{**}$, $\widehat{T}_{(4d)}^{**}$, $\widehat{T}_{(5d)}^{**}$, $\widehat{T}_{(6d)}^{**}$, $\widehat{T}_{(7d)}^{**}$, $\widehat{T}_{(8d)}^{**}$, $\widehat{T}_{(9d)}^{**}$, $\widehat{T}_{(10d)}^{**}$, $\widehat{t}_d^{**}$, $\widehat{t}_{(1d)}^{**}$, $\widehat{t}_{(2d)}^{**}$, $\widehat{t}_{(3d)}^{**}$, $\widehat{t}_{(4d)}^{**}$, $\widehat{t}_{(jd)}^{**}$ and Kumar & Zhang (2023) estimator ($\widehat{T}_{sszd}^{**}$). As a result, we favour the proposed estimator $\widehat{T}_d^{**}$ for future studies by the researchers in practice.

[Received: February 2024 — Accepted: June 2024]

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