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Corrigendum to "Descriptive Measures of Poisson-Lomax Distribution"

Corrección de errores de "Medidas descriptivas de la distribución Poisson-Lomax"

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Abstract

This corrigendum focuses on the correction of numerical results derived from Poisson-Lomax Distribution (PLD) originally proposed by Al-Zahrani & Sagor (2014). Though the mathematical properties and derivations by Al-Zahrani & Sagor (2014) were immaculate but during the execution of the R codes using Monte Carlo simulation some anomalies occurred in the calculation of the mean values. The same anomalies are addressed in the present corrigendum. The outcome of the corrigendum will provide basic guidelines for the academia and reviewers of various journals to match the numerical results with the shape of the probability distribution under study. The results will also emphasize the fact that code writing is a cumbersome process and due diligence be exercised in executing the codes using any programming language. Relevant R codes are appended in Appendix 'A'.

***Key words*:** Monte Carlo method; Poisson-Lomax distribution; probability distribution.

Resumen

Esta corrección se centra en la corrección de los resultados numéricos derivados de la Distribución Poisson-Lomax (PLD) propuesta originalmente por Al-Zahrani & Sagor (2014). Aunque las propiedades matemáticas y las derivaciones de Al-Zahrani & Sagor (2014) son correctas, durante la ejecución de los códigos R utilizando la simulación de Montecarlo se produjeron algunas anomalías en el cálculo de los valores medios. La corrección proporciona directrices para que académicos y revisores hagan coincidir los resultados numéricos con la forma de la distribución de probabilidad objeto de estudio. Los resultados también ponen de relieve el hecho de que la escritura y ejecución de los códigos, en cualquier lenguaje de programación, es un proceso engorroso que requiere precaución. Los códigos R relevantes se adjuntan en el Apéndice 'A'.

***Palabras clave*:** Distribución Poisson-Lomax; distribución de probabilidad; método de Montecarlo.

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1. Raison d'être for the corrigendum

For positively skewed distributions usually $Mode < Median < Mean$ which means that Mean is the largest value followed by Median and then Mode. Normally in a positively skewed distribution the value of Median is usually less than the value of the Mean, for illustrative purposes the same has been exhibited in Table 1 and four panels of Figure 1 using different values of $\lambda(0.5, 1, 2, 3)$ for the following equation.

$$y = e^{-\lambda x} \tag{1}$$

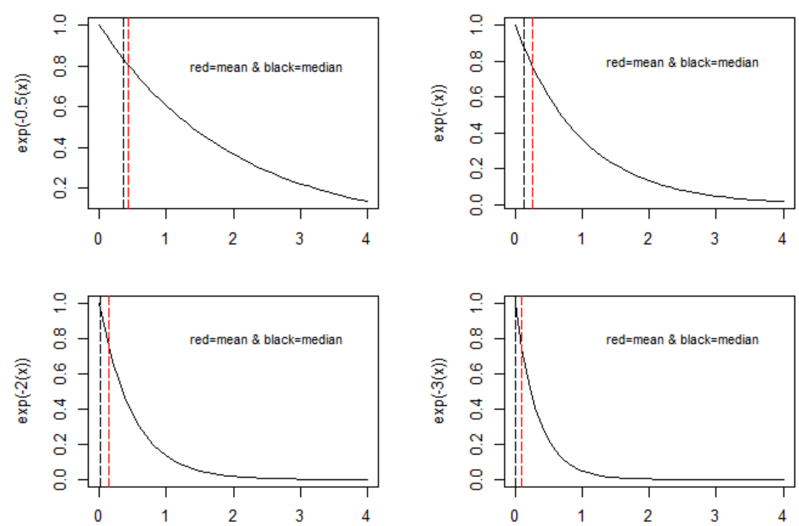

Figure 1: Means and Medians for Positively Skewed Distribution with different values of the parameter λ.

Since PLD is also a positively skewed distribution so the professed relationship between Mean and Median should hold to some extent. But in by merely looking at the values of the Means and Medians (Q_2) from Tables 1 and 3 in the reference paper, it is seen that the values of Mean are less than the values of Median (Q_2) which is an anomaly and needs to be investigated and corrected. If the values of Means are incorrect than this will have a domino effect on the values of the other measures associated with the Mean.

Table 1: Values of Means and Medians for different values of λ.

Values of λ	Mean	Median
0.5	0.43572	0.36787
1.0	0.25205	0.13533
2.0	0.13451	0.01831
3.0	0.09410	0.00247

2. Objective

The main objective of this corrigendum is to rectify the anomalies found in executing the programming codes in Al-Zahrani & Sagor (2014) with a view to correct the scientific record for the potential readership.

3. Methodology

Following methodology will be adopted in order to address the anomalies found in Al-Zahrani & Sagor (2014):

a. Comparison of the empirical and theoretical results of PLD with the reference paper.

b. Calculation of the corrected Skewness and Kurtosis results.

c. Producing probability density plots for different values of the parameters.

4. Results

4.1. Comparison of Empirical and Theoretical Results

All calculations in the present corrigendum are carried out using equation (10), (17) and (22) of the reference paper Al-Zahrani & Sagor (2014). All relevant calculations are shown in Table 2 to Table 5 for different combinations of the parameters α, β and λ. The last two columns in Table 2-4 are the values from the reference paper Al-Zahrani & Sagor (2014). Values in columns 3-4 in Table 2-4 are values of empirical means ($\mu(E)$) and empirical medians (Q_2) whereas, in column 6 values of theoretical means ($\mu(T)$) are shown. Both empirical and theoretical mean values are larger than the medians or Q_2. Mean values from the reference paper Al-Zahrani & Sagor (2014) as shown in the last two columns are smaller than median (Q_2) barring few exceptions. Since PLD being essentially a positively skewed distribution shall have mean values greater than the median. This points to the fact that the means values in the reference paper were incorrect. Similarly, the values of the variances from the reference paper are more inflated as compared to empirical and theoretical variances of the current corrigendum.

4.2. Calculation of the corrected Skewness and Kurtosis Results

Both the skewness and kurtosis are decreasing functions of α and both are increasing of λ as posited by Al-Zahrani & Sagor (2014). But according to the corrected values of skewness and kurtosis as depicted in Table 5 the former part of the aforementioned posited statement stands correct but as far as the latter part of the posited statement is concerned it is seen from Table 5 that with an increase in the value of λ skewness somewhat decreases whereas the kurtosis increases.

Table 2: Values of the Empirical and Theoretical means and variances of PLD for $\beta = 0.5$.

	$\beta = 0.5$						values from ref paper	
	2	3	4	5	6	7	8	9
	α	$\mu(E)$	Q_2	$\sigma^2(E)$	$\mu(T)$	$\sigma^2(T)$	$\mu(ref)$	$\sigma^2(ref)$
$\lambda = 0.5$	4.0	0.7542	0.4600	0.8813	0.7652	1.0009	0.1184	1.6233
	4.5	0.6461	0.4040	0.5977	0.6545	0.6908	0.1013	1.1089
	5.0	0.5651	0.3602	0.4303	0.5717	0.4872	0.0885	0.8062
	5.5	0.5021	0.3250	0.3238	0.5075	0.3614	0.0785	0.6128
	6.0	0.4517	0.2960	0.2521	0.4562	0.2786	0.0706	0.4816
	6.5	0.4104	0.2718	0.2016	0.4144	0.2210		
$\lambda = 1.5$	4.0	0.9639	0.6453	1.1769	0.9760	1.3899	0.5890	1.9955
	4.5	0.8225	0.5644	0.7865	0.8315	0.9006	0.5018	1.3402
	5.0	0.7170	0.5014	0.5598	0.7241	0.6285	0.4369	0.9618
	5.5	0.6354	0.4510	0.4174	0.6412	0.4623	0.3869	0.7237
	6.0	0.5705	0.4098	0.3226	0.5752	0.3538	0.3471	0.5641
	6.5	0.5175	0.3756	0.2564	0.5216	0.2791		
$\lambda = 2.0$	4.0	1.0715	0.7442	1.3227	1.0840	1.5570	0.8104	2.0752
	4.5	0.9126	0.6494	0.8778	0.9218	1.0023	0.6892	1.377
	5.0	0.7944	0.5760	0.6214	0.8017	0.6957	0.5993	0.9791
	5.5	0.7032	0.5174	0.4613	0.7091	0.5096	0.5301	0.7313
	6.0	0.6307	0.4696	0.3552	0.6356	0.3886	0.4752	0.5668
	6.5	0.5717	0.4298	0.2815	0.5759	0.3057		
$\lambda = 4.0$	4.0	1.4811	1.1233	1.8466	1.4952	2.1602	1.4409	2.3195
	4.5	1.2536	0.9724	1.1948	1.2639	1.3566	1.2179	1.4705
	5.0	1.0861	0.8569	0.8291	1.0939	0.9237	1.0542	1.0089
	5.5	0.9577	0.7658	0.6056	0.9639	0.6661	0.9289	0.7322
	6.0	0.8562	0.6921	0.4602	0.8614	0.5013	0.8301	0.5542
	6.5	0.7741	0.6312	0.3606	0.7784	0.3901		

4.3. Probability Density Plots for different combinations of the Parameters

Pdf for the PLD is given in equation (10) of Al-Zahrani & Sagor (2014), relevant graphs of the pdf for different combinations of the parameters α, β and λ are shown in Figure 2 for illustrative purposes which does match the refpaper Al-Zahrani & Sagor (2014).

TABLE 3: Values of the Empirical and Theoretical means and variances of PLD for $\beta = 1.0$.

	$\beta = 1.0$						values from ref paper	
	2	3	4	5	6	7	8	9
	α	$\mu(E)$	Q_2	$\sigma^2(E)$	$\mu(T)$	$\sigma^2(T)$	$\mu(ref)$	$\sigma^2(ref)$
$\lambda = 0.5$	4.0	0.3771	0.2300	0.2203	0.3826	0.2630	0.0592	0.4058
	4.5	0.3231	0.2020	0.1494	0.3272	0.1727	0.0506	0.2772
	5.0	0.2825	0.1801	0.1076	0.2858	0.1218	0.0442	0.2015
	5.5	0.2510	0.1625	0.0809	0.2537	0.0903	0.0392	0.1532
	6.0	0.2258	0.1480	0.0630	0.2281	0.0697	0.0353	0.1204
	6.5	0.2052	0.1358	0.0504	0.2072	0.0553		
$\lambda = 1.5$	4.0	0.4820	0.3226	0.2942	0.4880	0.3475	0.2945	0.4988
	4.5	0.4112	0.2822	0.1966	0.4157	0.2252	0.2509	0.3350
	5.0	0.3585	0.2507	0.1400	0.3620	0.1572	0.2184	0.2404
	5.5	0.3177	0.2256	0.1044	0.3206	0.1156	0.1934	0.1809
	6.0	0.2852	0.2050	0.0806	0.2876	0.0885	0.1735	0.1410
	6.5	0.2587	0.1878	0.0641	0.2608	0.0698		
$\lambda = 2.0$	4.0	0.5357	0.3720	0.3307	0.5420	0.3892	0.4052	0.5188
	4.5	0.4563	0.3247	0.2195	0.4609	0.2506	0.3446	0.3442
	5.0	0.3972	0.2880	0.1553	0.4008	0.1740	0.2996	0.2447
	5.5	0.3516	0.2586	0.1153	0.3545	0.1274	0.2650	0.1828
	6.0	0.3154	0.2348	0.0888	0.3178	0.0972	0.2376	0.1417
	6.5	0.2859	0.2149	0.0704	0.2879	0.0764		
$\lambda = 4.0$	4.0	0.7405	0.5616	0.4616	0.7476	0.5401	0.7204	0.5798
	4.5	0.6268	0.4862	0.2987	0.6319	0.3392	0.6089	0.3676
	5.0	0.5430	0.4284	0.2073	0.5470	0.2309	0.5271	0.2522
	5.5	0.4789	0.3829	0.1514	0.4820	0.1665	0.4644	0.1830
	6.0	0.4281	0.3460	0.1150	0.4307	0.1253	0.4150	0.1385
	6.5	0.3871	0.3156	0.0902	0.3892	0.0975		

TABLE 4: Values of the Empirical and Theoretical means and variances of PLD for $\beta = 2.0$.

	$\beta = 2.0$						values from ref paper	
	2	3	4	5	6	7	8	9
	α	$\mu(E)$	Q_2	$\sigma^2(E)$	$\mu(T)$	$\sigma^2(T)$	$\mu(ref)$	$\sigma^2(ref)$
$\lambda = 0.5$	4.0	0.1885	0.1150	0.0551	0.1913	0.0657	0.0296	0.1014
	4.5	0.1615	0.1010	0.0374	0.1636	0.0432	0.0253	0.0693
	5.0	0.1413	0.0900	0.0269	0.1429	0.0305	0.0221	0.0503
	5.5	0.1255	0.0812	0.0202	0.1269	0.0226	0.0196	0.0383
	6.0	0.1129	0.0740	0.0158	0.1141	0.0174	0.0176	0.0301
	6.5	0.1026	0.0679	0.0126	0.1036	0.0138		
$\lambda = 1.5$	4.0	0.2410	0.1614	0.0736	0.2440	0.0869	0.1472	0.1247
	4.5	0.2056	0.1411	0.0492	0.2079	0.0563	0.1254	0.0837
	5.0	0.1793	0.1254	0.0350	0.1810	0.0392	0.1092	0.0601
	5.5	0.1589	0.1128	0.0261	0.1603	0.0289	0.0967	0.0452
	6.0	0.1426	0.1025	0.0202	0.1438	0.0221	0.0867	0.0352
	6.5	0.1294	0.0939	0.0160	0.1304	0.0174		
$\lambda = 2.0$	4.0	0.2679	0.1860	0.0827	0.2710	0.0974	0.2026	0.1297
	4.5	0.2281	0.1624	0.0549	0.2305	0.0627	0.1723	0.0860
	5.0	0.1986	0.1440	0.0388	0.2004	0.0434	0.1498	0.0611
	5.5	0.1758	0.1294	0.0288	0.1773	0.0319	0.1325	0.0457
	6.0	0.1577	0.1174	0.0222	0.1589	0.0243	0.1188	0.0354
	6.5	0.1429	0.1074	0.0176	0.1440	0.0191		
$\lambda = 4.0$	4.0	0.3703	0.2808	0.1154	0.3738	0.1350	0.3602	0.1449
	4.5	0.3134	0.2431	0.0747	0.3160	0.0847	0.3044	0.0919
	5.0	0.2715	0.2142	0.0518	0.2735	0.0577	0.2635	0.0630
	5.5	0.2394	0.1914	0.0379	0.2410	0.0416	0.2322	0.0457
	6.0	0.2141	0.1730	0.0288	0.2153	0.0313	0.2075	0.0346
	6.5	0.1935	0.1578	0.0225	0.1946	0.0244		

TABLE 5: Corrected Skewness and Kurtosis.

	α	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$
			Skewness			Kurtosis	
$\lambda = 0.5$	4.5	5.1667	5.1649	5.1504	93.4524	23.3597	5.8342
	5.0	4.3818	4.3796	4.3815	32.4803	8.1191	2.0283
	5.5	3.9010	3.9046	3.9020	16.3027	4.0768	1.0192
	6.0	3.5745	3.5703	3.6068	9.6956	2.4234	0.6041
	6.5	3.3418	3.3364	3.3487	6.3838	1.5957	0.4036
$\lambda = 1.5$	4.5	4.7424	4.7408	4.7424	105.453	26.3592	6.5921
	5.0	3.9997	3.9982	4.0093	36.1124	9.0266	2.2593
	5.5	3.5460	3.5421	3.5370	17.9163	4.4774	1.1188
	6.0	3.2354	3.2318	3.2470	10.5531	2.6387	0.6592
	6.5	3.0121	3.0141	3.0411	6.8908	1.7215	0.4309
$\lambda = 2.0$	4.5	4.5993	4.5996	4.5968	111.749	27.9373	6.9812
	5.0	3.8705	3.8694	3.8808	38.0045	9.4990	2.3770
	5.5	3.4238	3.4234	3.4243	18.7475	4.6849	1.1702
	6.0	3.1195	3.1159	3.1319	10.9934	2.7474	0.6874
	6.5	2.8983	2.8991	2.9171	7.1492	1.7890	0.4498
$\lambda = 4.0$	4.5	4.3214	4.3201	4.3255	137.6450	34.4054	8.6068
	5.0	3.6125	3.6129	3.6150	45.6889	11.4233	2.8580
	5.5	3.1767	3.1785	3.1814	22.1036	5.5270	1.3812
	6.0	2.8809	2.8817	2.8783	12.7593	3.1912	0.7968
	6.5	2.6654	2.6654	2.6365	8.1880	2.0471	0.5107

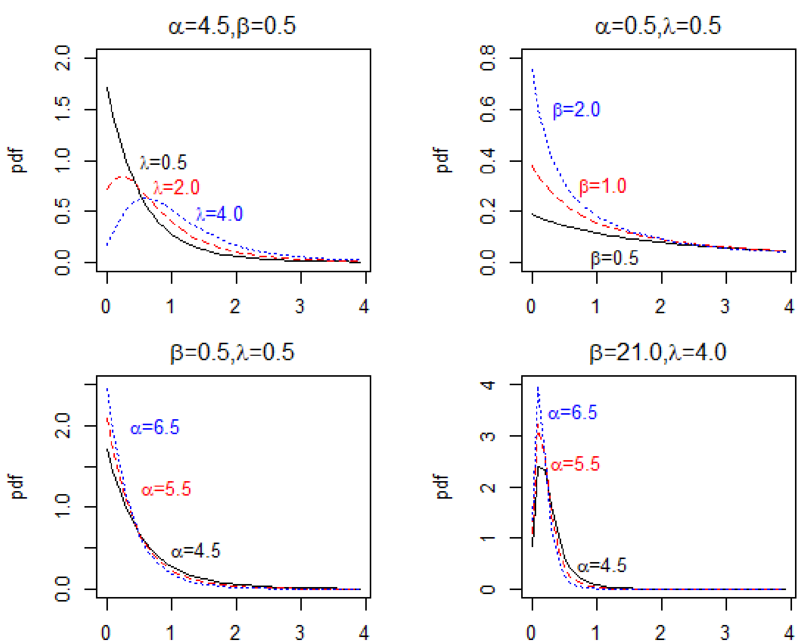

FIGURE 2: Probability distribution function for different values of the parameters α, β and λ.

5. Conclusion

Shape of the distribution defines the relationship between three measures of central tendency (mean, median or Q_2 and mode). While simulating the values of the said three measures using any programming language one must keep in mind the basic relationship which exists between the measures from antiquity. Also, while implementing the codes to extract numerical values the researcher should keep in mind the shape of the probability distribution under study. Current corrigendum were undertaken to address a minor anomaly in calculating the values of the means and variances with a view of enhancing the efforts of the reference authors.

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Appendix 'A'

R Codes (Empirical Results).

```
library(moments)
set.seed(1234)
nsim=10000;lem=0.5;b=0.5;a=4.0;
u=runif(nsim);
x=1-exp(-lem);
y=1/b;
z=round(y*((((-log((u*x)+exp(-lem)))/lem)$\mathrm{\wedge}$(-1/a))-1),4);
round(cbind(mean(z),median(z),var(z),4)

#R Codes (Theoretical Results)-means
m=sapply(c(4.0,4.5,5.0,5.5,6.0,6.5),function(a,x=seq(0,50,1),l=0.5)
,(1/(1-exp(-l)))*sum(((-l)$\mathrm{\wedge}$(k+1))/(factorial(k+1)*(1-((k+1)*a)))));
d=round(sapply(c(1/0.5,1/1.0,1/2),function(b)(m*b)),4);d
t=as.matrix(d);t

#R Codes (Theoretical Results)-variances
m=sapply(c(4.0,4.5,5.0,5.5,6.0,6.5),function(a,x=seq(1,50,1),l=0.5),
(1/(1-exp(-l)))*sum(((-l)$\mathrm{\wedge}$(k+1))/(factorial(k+1)*(1-((k+1)*a)))));
d=round(sapply(c(1/0.5,1/1.0,1/2),
function(b)(m*b)),4);t=as.matrix(d);msq=round(t$\mathrm{\wedge}$2,4);msq
v=sapply(c(4.0,4.5,5.0,5.5,6.0,6.5),function(a,x=seq(1,50,1),l=0.5),
(-2/(1-exp(-l)))*sum(((-l)$\mathrm{\wedge}$(k+1))/(factorial(k+1)*(1-((k+1)*a)),
,,,,,,,,,,,,,,,,*(2-((k+1)*a)))));
d=round(sapply(c(1/0.5,1/1.0,1/2),function(b)(v*(b$\mathrm{\wedge}$2))),4);
z=as.matrix(d);  z;msq;z-msq
```

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References

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