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A Novel Global Probabilistic Fuzzy System for Occupational Risk Assessment (GPFSORA)

Un novedoso sistema probabilístico difuso global para la evaluación de riesgos laborales (GPFSORA)

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ABSTRACT

Occupational risk assessment is the process of estimating the magnitude of risks that cannot be avoided. Then, the corresponding assessment is carried out, using comparative tables with different evaluation methods. Current risk assessment techniques enable the individual assessment of each potential risk, but there is no method to globally assess potential risks in an organization. The motivation of this research was to develop an objective and quantitative risk assessment system through a diffuse probabilistic model integrating stochastic and non-stochastic uncertainty. To this effect, an empirical collective record was used, whose attribute of interest was the occurrence of different accident types over a period of 52 weeks. Here, each of the collectives represented a linguistic input variable. In the probabilistic fuzzification stage, the frequentist probability of the occurrence of accidents was determined. One of our most important contributions to probabilistic fuzzy systems lies in our classification of language labels based on the linguistic projection of frequentist probabilities via a projection membership function determined by experts. The use of the total probability theorem in the implication stage is also proposed. The output of the system determines the type of risk, its evaluation, and the probability of its occurrence, vital factors to be considered in prevention work. The system's stages are explicitly described and applied to real data corresponding to construction materials distribution company. One of the relevant conclusions of this research is that the integration of stochastic and imprecise uncertainty allows for a more reliable risk assessment system.

Keywords: probabilistic fuzzy system, frequentist probability, total probability theorem

RESUMEN

La evaluación de riesgos laborales es el proceso de estimar la magnitud de los riesgos que no se pueden evitar. Luego, se lleva a cabo la evaluación correspondiente, utilizando tablas comparativas con diferentes métodos de evaluación. Las técnicas actuales de evaluación de riesgos permiten la evaluación individual de cada riesgo potencial, pero no hay un método para evaluar globalmente los riesgos potenciales en una organización. La motivación de esta investigación fue desarrollar un sistema objetivo y cuantitativo de evaluación de riesgos a través de un modelo probabilístico difuso que integrara la incertidumbre estocástica y no estocástica. Para ello, se utilizó un registro colectivo empírico, cuyo atributo de interés fue la ocurrencia de diferentes tipos de accidentes durante un período de 52 semanas. Aquí, cada uno de los colectivos representaba una variable de entrada lingüística. En la etapa de difusión probabilística, se determinó la probabilidad frecuentista de la ocurrencia de accidentes. Una de nuestras contribuciones más importantes a los sistemas difusos probabilísticos radica en la clasificación de etiquetas de lenguaje con base en la proyección lingüística de probabilidades frecuentistas a través de una función de membresía de proyección determinada por expertos. También se propone el uso del teorema de probabilidad total en la etapa de implicación. La salida del sistema determina el tipo de riesgo, su evaluación y la probabilidad de su ocurrencia, factores vitales a tener en cuenta en el trabajo de prevención. Las etapas del sistema se describen explícitamente y se aplican a datos reales de una empresa de distribución de materiales de construcción. Una de las conclusiones relevantes de esta investigación es que integrar incertidumbre estocástica e imprecisa permite un sistema de evaluación de riesgos más confiable.

Palabras clave: sistema probabilístico difuso, probabilidad frecuentista, teorema de probabilidad total

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Introduction

The fuzzy logic system cannot handle various uncertainties in practical applications and is notoriously not the best option to process stochastic uncertainties, since it can control, classify, or evaluate the process in two-dimensional natures (Sozhamadevi *et al*., 2012). Moreover, their linguistic expression cannot handle stochastic uncertainty, and the Mamdani and Takagi-Sugeno-type inference engine may not be suitable for working under a stochastic environment with incomplete dynamics (Li and Liu, 2009). Due to the complexity and diversity of big data, it is difficult to deal with uncertain and fuzzy information (Xue *et al*., 2020). Fuzzy systems emulate human reasoning. However, decision-making is currently based on all the information available from systems, including stochastic uncertainty conditions. This represents an area of opportunity to strengthen the

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inference results of fuzzy systems, which constitutes the motivation of our research. To date, researchers have paid more attention to multi-attribute decision-making problems in imprecise and uncertain environments (De *et al*., 2019). Fuzzy entropy provides a measure of fuzziness (ambiguity). Therefore, it is important in decision-making applications with imprecise (diffuse) values (Aggarwal, 2021). In multi-criteria group decision-making, those in charge often do not share the same concepts (Farhadinia *et al*., 2020). The fuzzy probabilistic set is designed to handle uncertainties that are blurred and stochastic in nature, so the its logic system can handle more complex uncertainties (Huang *et al*., 2019). Additionally, a fuzzy probability inference system provides a measure of dispersion and seems to be essential to designing systems that incorporate linguistic and/or numerical uncertainties, which are transformed into uncertainty rules, analogous to how variance relates to the mean (Sozhamadevi and Sathiyamoorty, 2015). The approach proposed in this research integrates non-stochastic uncertainty conditions, emulating the human cognitive process, and stochastic uncertainty conditions, using the empirical collectives of the number of accidents associated with different types of occupational risks.

In general, occupational health and safety (OSH) risk assessment models are established upon the basis of four basic steps: hazard identification, hazard characterization, exposure assessment, and risk characterization. Currently, there is little guidance for choosing the most suitable model for a given application, which is based on the individual judgment of the expert and can therefore lead to highly variable, experience-dependent results (Tian *et al*., 2018). One of the most important stages of a risk assessment system is the categorization of risk levels for the operations under study, *i.e.*, a characterization of risk levels, which are subjectively assigned according to the heuristic knowledge of the experts based on specific criteria. Then, the corresponding evaluations are conducted upon the bases of comparative tables. Boyaci (2022) described OSH risk assessment techniques in two categories: qualitative and quantitative. In qualitative techniques, a numerical value is assigned to each parameter, such as the probability and severity of the hazard. These values can be processed using mathematical and logical methods and a calculated risk value.

Gül (2020) stated that some of the most common methods for risk assessment are based on the subjective estimation of two or three criteria. The matrix method has two parameters called *probability* and *gravity*, and, in the Fine-Kinney method, there are three parameters: *gravity*, *probability*, and *frequency*. Moreover, in the failure modes and effects analysis (FMEA) method there are three parameters: *severity*, *probability*, and *detectability*.

These assessment methods are used to individually analyze and assess each potential risk in an organization, based on the knowledge and experience of experts. However, they are not useful for making a comprehensive assessment of all potential risks in the organization or for predicting their probability of occurrence. A comparison is often made in a table showing possible combinations, and the reference criteria are qualitatively described to carry out a subjective evaluation based on the interpretation of the experts, which may be imprecise and vague. Nevertheless, in each organization, there are registers of different types of risks and accidents, which can be used to design a global prediction, classification, and evaluation system according to specific needs and priorities, rather than an individual and subjective approach. This constitutes the motivation of our research, which develops a probabilistic system based on fuzzy logic that objectively integrates the calculated probabilities of each type of accident, analyzed by adding stochastic uncertainty. Fuzzy logic is used to integrate non-stochastic uncertainty and obtain the results in a language that is simple and clear to categorize, assess, and estimate. Labor-, material-, and site condition-related accident records are analyzed as an empirical collective. Frequentist probability is determined, which is projected on language labels through weighted percentages determined by experts according to their cognitive process. For the implication stage, we propose using the total probability theorem, where each of the linguistic variables has an important weighting factor. Finally, with the aggregation stage, there is an estimate of the occurrence of the three types of accidents. The novelty of this study lies in four points:

1. The proposal of a comprehensive method for assessing occupational risks in an organization

2. The integration of stochastic and non-stochastic uncertainty into a robust prediction, categorization, and evaluation system

3. The linguistic projection of frequentist probability as a mechanism to integrate the imprecise uncertainty of cognitive processes and emulate human reasoning, proposing new membership functions for projection on language labels

4. The use of the total probability theorem as the trigger level for each of the fuzzy rules in the implication stage

This manuscript is organized as follows. The introduction presents the limitations of diffuse systems and the area of opportunity to integrate stochastic uncertainty conditions to diffuse systems, aiming for a more robust system. A literature review is presented in the second section. The methodology is presented thereafter, and its application to a case study related to the construction industry is presented at the end.

Literature review

Below are the results of a comprehensive state of the art review regarding qualitative and quantitative risk assessment methods in various industries, which are aimed at analyzing, planning for, and preventing accidents.

Gül (2020) proposed a quantitative assessment of occupational risk via a technique based on classification for order performance by similarity to ideal solutions, in order to manage risks in the aluminum extrusion industry. This quantitative method, like qualitative and subjective methods, makes it possible to assess each of the risks individually and does not present an alternative for assessing risks jointly or globally in order to predict, classify, and evaluate the risks in an organization.

Xu *et al*. (2020) explored the quantitative differences between common occupational health risk assesment (OHRA) models. Risk ratios in five typical industries (leather manufacturing, wood furniture, printing and dyeing, and printing and clothing) were analyzed by using six OHRA models. The consistency, correlation, and reliability of quantitative differences between the models were evaluated. Additionally, Tian *et al*. (2018) identified the essential models of the Environmental Protection Agency, Singapore, Australia, Romania, the International Council of Mining and Metals, and Health Hazardous Substances Control as the six most common based on a literature review. Each OHRA qualitative comparison model had its own strengths and limitations and exhibited a diverse distribution at different levels regarding each evaluation indicator. Note that risk ratios are defined as the relationship between the risk level of a given risk factor (obtained through the given model) and the maximum risk level of that model. Both studies, which compared the most commonly used methodologies for risk assessment, highlighted the absence of a global method for assessing potential risks in an organization.

Fuzzy systems have been developed in several productive sectors to create stronger systems in risk assessment, as shown below.

Boyaci (2022) introduced a new approach to OSH risk assessment, combining the Fine-Kinney method and multi-criteria group decision-making to identify hazards and assess, prioritize, and mitigate risks. The proposed approach was applied to the operating theater of a public hospital in Turkey. The analysis involved three experts with occupational health and safety responsibilities in the unit.

Tang *et al*. (2021) developed a hybrid risk prioritization approach to the Fine-Kinney method through the widespread use of interactive and multi-criteria, better-worse decision-making and the diffuse type II set. First, language terms and type II fuzzy range numbers were employed to handle the problem of expressing uncertain evaluation information on team members. Second, in order to obtain the objective weights of the risk parameters, the best-worst method was proposed for determining their degrees of materiality. Third, blurred range II numbers were incorporated into the generalized method for determining hazard risk priority orders, which can simulate the limited rational behavior of experts under uncertain environment.

Fattahi *et al*. (2020) proposed a novel fuzzy multi-criteria decision-making model based on FMEA to more accurately assess the risks of different failure modes. Fattahi *et al*. (2020) used a method called *Modified-SIRA* (enhanced security risk assessment). The criteria and alternatives were prioritized based on the risk priority number and the methodology called *order preference technique by similarity to the ideal solution*, respectively. The results obtained regarding the risk priority number show that most accidents in the Pakistani construction industry are due to the deficiency of PPEs, followed by electrocution, the improper use of the available PPE, and falling from elevated platforms.

Gül and Celik (2018) proposed a combination of the Fine-Kinney method and a fuzzy rule-based expert system. This approach captures nonlinear causal relationships between Fine-Kinney parameters. Since there is a high level of vagueness involved in the OHS risk assessment data, the rule-based expert system was developed for probability, exposure, and consequence in evaluating risk scores.

Each of the studies proposing the integration of diffuse systems use the same approach as traditional systems, only aiming to assess each individual hazard and not predict, evaluate, and categorize the whole. None of the studies with diffuse logic integrate the stochastic uncertainty condition. So far, no research has been found in the literature which uses traditional models and modifies them with diffuse logic to assess the risks of a company collectively or integrates stochastic and non-stochastic conditions, which constitutes the motivation of this research.

Some recent studies integrating stochastic and non-stochastic conditions in applications different from those involving a risk assessment system are mentioned below.

Wang *et al*. (2021) proposed a probabilistic fuzzy inference method to improve the accuracy of evaluation results by considering the uncertainty of indicators. They elaborated a prediction model based on the Gaussian distribution function. The outputs of the model represent the inputs to the system, and the fuzzy membership functions are made probabilistic by multiplying their value by their corresponding probability.

Jiang and Liao (2021) proposed a network consensus analysis of probabilistic linguistic preference relations based on a new measure of probabilistic distance by Kolmogorov-Smirnov, introducing the cumulative probability distribution of sets of probabilistic linguistic terms. They used the quantitative terms of a probability label and the qualitative terms of a language label.

Modares and Desch (2021) discussed and compared various probabilistic and possibilistic methods based on finite elements, presenting the results of case studies on structures using static and dynamic uncertainty. These results suggest that incorporating uncertainty into the analysis procedure provides a higher level of confidence.

De Ridder *et al*. (2020) presented a novel approach based on machine learning for uncertainty quantification problems involving random and epistemic variables. They employed Bayesian optimization to efficiently propagate this hybrid uncertainty throughout the performance of the system under study.

Liang *et al*. (2020) proposed a multi-attribute group decision-making method under an uncertain and diffuse environment that considers the psychological state of decision makers. A weight determination model was constructed, incorporating objective information on WED decision-making and subjective preferences based on a scoring function and the principle of minimal relative entropy.

Gupta *et al*. (2020) extracted reference values using a controller based on probabilistic and diffuse set theory, and Liang *et al*. (2020) solved group decision-making problems containing inconsistent probabilistic preferred language relations and unknown expert weights.

Each of the above-presented studies integrates stochastic and non-stochastic uncertainty conditions, making decision-making more robust. The use of this approach is aimed at the design and development of diffuse probabilistic risk assessment systems by integrating the stochastic uncertainty state, using the frequency probabilities of different types of occupational risks as input variables, as well as their disaggregation based on the interpretation and knowledge of experts of the vagueness in their values, categorizing them into language labels.

Compared to existing diffuse probabilistic approaches, the novelty and contributions of our proposal can be summarized as follows:

1. An effective method based on probabilistic fuzzy inference is proposed, which can obtain better evaluation results under uncertain conditions.

2. Frequentist probability is broken down into diffuse labels based on the knowledge of the experts, who determine reliability percentages.

3. Based on the total probability theorem, we propose determining the stage of implication of the diffuse probabilistic system.

Fuzzy systems use fuzzy operators such as *maximum*, *minimum*, and *product* to define the compositional rules of inference in the stages of implication and aggregation, employing fuzzified values in the rules of knowledge. In this research, probabilistic values are used. We propose integrating the probabilistic values of each input variable through the total probability theorem in the stage of implication, based on the fact that each of the input variables is an independent event and the conditional probability for each language label is known through the projection of its relative frequencies, *i.e.*, the system's inference mechanism. This, instead of using a compositional rule with diffuse operators.

Methodology

The GPFSORA methodology consists of seven stages (Figure 1). The first stage of the methodology is to identify accident records as empirical groups, where the attribute to be analyzed is the number of accidents per week in a construction company over a period of 52 weeks. In the second stage, the frequentist probability of each group is determined. The third stage identifies the categories of importance or relevance according to the frequency of the number of accidents [a; b], and the frequentist probability is projected on three language labels based on the knowledge of the experts, as well as determined through the product of the frequentist probability of the type of accident and the proposed weighted projection membership function (top middle of Figure 1). In the fourth stage, diffuse rules are established, creating the knowledge base. One of the main contributions of this research lies in the fifth stage of implication, where the use of the total probability theorem is proposed. Finally, in the aggregation phase, the maximum operator is employed, and the probability, evaluation, and classification of accidents is estimated. The corresponding assessment is performed using the central limit theorem.

This research was validated with real data in the southern region of the state of Guanajuato, Mexico. It is worth mentioning that fuzzy systems are based on the fuzzification of input variables in an estimated domain, analyzing the different possible combinations in the established domains. In our research proposal, real data on accidents are used, which determine the domain of the system. They are based on evidence from historical records of accidents, as opposed to defining a domain estimate based on accidents that can happen. In summary, the definition of domains in the input variables is based on accidents that actually occurred, rather than estimating those that might occur.

Empirical collectives

The first stage consists of analyzing the records of the main types of accidents as empirical groups based on objective evidence of the reality of their occurrence over a specific period (Table 1).

Table 1. Accidents records

Week	Type of accident 1	Type of accident 2	....	Type of accident m
1	X11	X21		Xm1
2	X12	X22		Xm2
.				
.				
n	X1n	X2n		Xmn

Where: m = type of accident, n = week number
Source: Authors

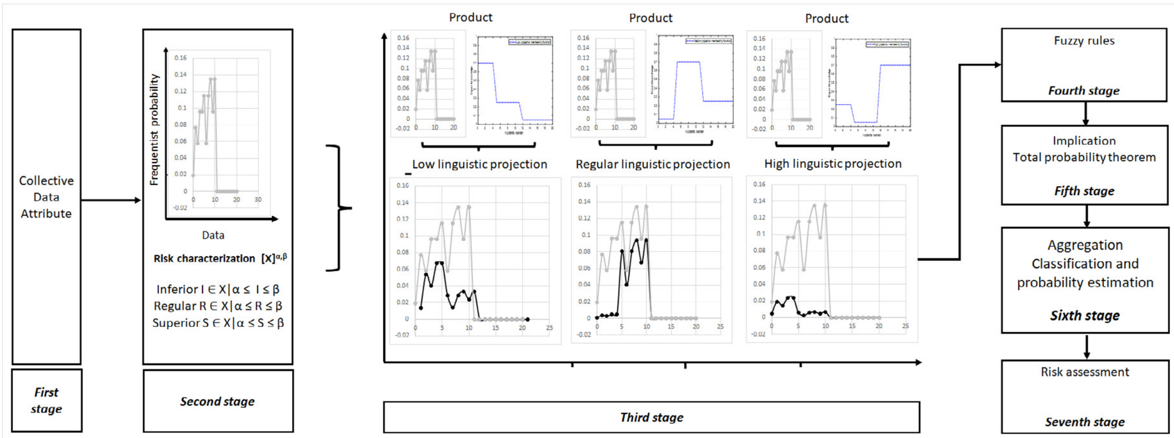


Figure 1. Research methodology
Source: Authors

Frequentist probability and risk characterization

The second stage consists of determining the frequentist probability or relative frequency, as expressed in Equation (1).

$$Rf = \frac{Absolute\ frecuency}{Total} \tag{1}$$

Based on records of the main types of accidents as empirical groups of the organization, integrating stochastic uncertainty into the system as the probability of the associated event occurring, the random variable X is the number of accidents per week in the organization, which can only take integer values between 0 and 20, based on historical records (Table 2).

Table 2. Frequentist probability records

Number of accidents X	Absolute frequency of number of accidents occurred per week	Relative frequency (Rf) of number of accidents	Accumulated relative frequency
0	1	1/52	1/52
1	4	4/52	5/52
2	3	3/52	8/52
3	5	5/52	13/52
4	5	5/52	18/52
5	6	6/52	24/52
6	3	3/52	27/52
7	6	6/52	33/52
8	7	7/52	40/52
9	5	5/52	45/52
10	7	7/52	52/52
11	0	0/52	52/52
12	0	0/52	52/52
13	0	0/52	52/52
14	0	0/52	52/52
15	0	0/52	52/52
16	0	0/52	52/52
17	0	0/52	52/52
18	0	0/52	52/52
19	0	0/52	52/52
20	0	0/52	52/52
Total of weeks	52	52/52	

Source: Authors

Experts determine an α-level set of crisp sets of X to classify them as inferior, regular, or superior for each type of accident in a clear and simple language based on subjective interpretation. In an α-level set of a crisp set I, R and S of X are a crisp set denoted by $[I]^{\alpha,\beta}$, $[R]^{\alpha,\beta}$, and $[S]^{\alpha,\beta}$ (Table 3). The characterization of the α-level sets is obtained by analyzing the history of accident records at work meetings by three plant industrial safety staff experts and two company managers who reached a consensus.

Table 3. Risk characterization

Risk characterization	$[X]^{\alpha,\beta}$	α, β
Inferior (I)	$I \in X \mid \alpha \leq I \leq \beta$	0.4
Regular (R)	$R \in X \mid \alpha \leq R \leq \beta$	5.11
Superior (S)	$S \in X \mid \alpha \leq S \leq \beta$	12.20

Source: Authors

Linguistic projection

Each of the frequentist probabilities is projected on three language labels as a mechanism to integrate the knowledge of experts to the interpretation of relative frequencies. A robust evaluation system is established by integrating stochastic uncertainty, using frequentist probability as input variable as well as vague uncertainty by projecting frequentist probability on labels based on expert knowledge.

One of the most important contributions of our research is the proposal of weighted projection membership functions based on risk categorization level sets to interpret and project the relative frequency of accident types into three categories or language tags, which are shown below. Each of these functions must be multiplied by the frequentist probability of each type of accident (Table 4).

The relative frequency value is projected on three membership functions. In each membership function projection, weightings of 70, 25, and 5% are given to each risk category. 70% of the value is assigned to the similar risk category with its projection.

Table 4. Weighted membership functions of linguistic projection

Low $[Rf]^{\lambda Rf}$	Medium $[Rf]^{\lambda Rf}$	High $[Rf]^{\lambda Rf}$
I ∈ Rf| I =0.7 Rf	I ∈ Rf| I =0.05 Rf	I ∈ Rf| I =0.25 Rf
R ∈ Rf| R =0.25 Rf	R ∈ Rf| R =0.7 Rf	R ∈ Rf| R =0.05 Rf
S ∈ Rf| S =0.05 Rf	S ∈ Rf| S =0.25 Rf	S ∈ Rf| S =0.7 Rf

Source: Authors

The relative frequency value is projected on three membership functions. In each membership function projection, weightings of 70, 25, and 5% are given to each risk category. 70% of the value is assigned to the similar risk category with its projection.

For a low linguistic projection, the weighting 70% is assigned to the inferior risk category (Figure 2). For the medium linguistic projection, the relevant weight 70% is assigned to the regular risk category (Figure 3). Finally, for the high linguistic projection, the weighting 70% is assigned to the superior risk category (Figure 4).

In this vein, the frequentist probability of each type of accident is projected to a language label called *low*, with weights of 70, 25, and 5%. The value of the relative frequency within the lower risk category of zero to four accidents is propagated while maintaining 70% of its value for a low linguistic projection (Figure 2).

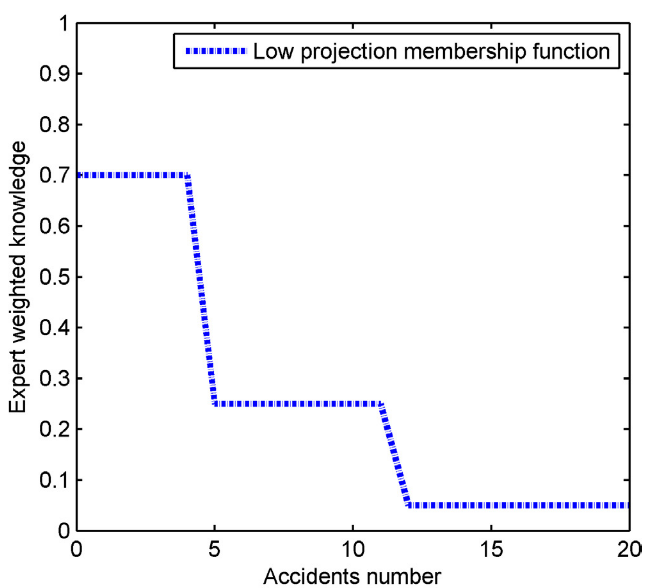


Figure 2. Low membership function projection
Source: Authors

The frequentist probability of each type of accident is projected to a language label called *medium*, with weights of 5, 70, and 25%. The value of the relative frequency within the regular risk category of five to 11 accidents is propagated while maintaining 70% of its value for medium linguistic projection (Figure 3).

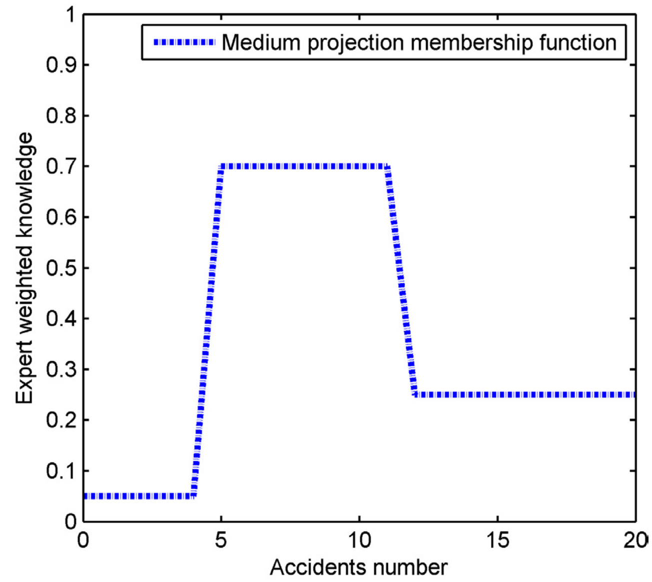


Figure 3. Medium membership function projection
Source: Authors

The frequentist probability of each type of accident is projected to a language label called *high*, with weights of 25, 5, and 70%. The value of the relative frequency within the superior risk category of 12 to 20 accidents is propagated while maintaining 70% of its value for the high linguistic projection (Figure 4).

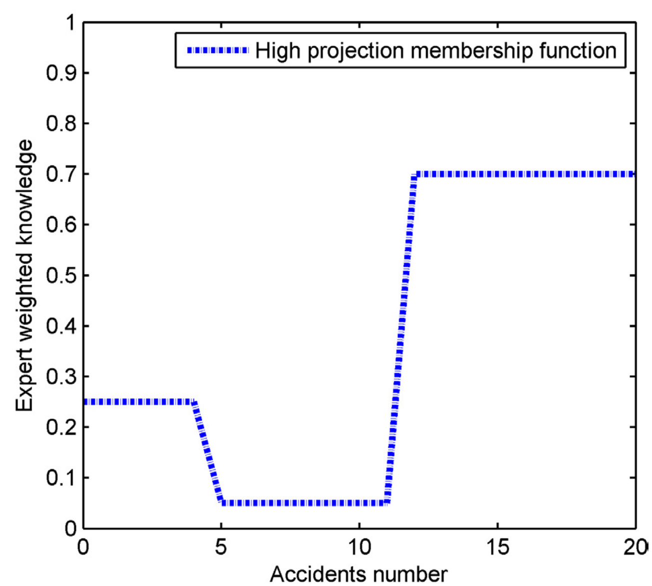


Figure 4. High membership function projection
Source: Authors

These percentages are based on the experts' interpretation of relative frequencies, which are projected on language labels as a mechanism to emulate the experts' reasoning when analyzing and categorizing the relative frequencies of accidents. If the category is lower, a higher percentage is given to the lower risk level. If the category is regular, a higher percentage is given to the average risk level, and, finally, if the category is higher, a higher percentage is given to the higher risk level. The percentage supplements are distributed in the other two categories to meet the weighted projection (Table 5).

Table 5. Projection of frequentist probability

Risk category	Number of accidents of type 1	Relative frequency	Frequentist probability projection		
			Low	Medium	High
Inferior	0	1/52	70%	5%	25%
	1	4/52	70%	5%	25%
	2	3/52	70%	5%	25%
	3	5/52	70%	5%	25%
	4	5/52	70%	5%	25%
Regular	5	6/52	25%	70%	5%
	6	3/52	25%	70%	5%
	7	6/52	25%	70%	5%
	8	7/52	25%	70%	5%
	9	5/52	25%	70%	5%
	10	7/52	25%	70%	5%
	11	0/52	25%	70%	5%
Superior	12	0/52	5%	25%	70%
	13	0/52	5%	25%	70%
	14	0/52	5%	25%	70%
	15	0/52	5%	25%	70%
	16	0/52	5%	25%	70%
	17	0/52	5%	25%	70%
	18	0/52	5%	25%	70%
	19	0/52	5%	25%	70%
	20	0/52	5%	25%	70%
	Total	**52/52**			

Source: Authors

The proposed membership functions project a percentage of the actual value of the relative frequency. For the low membership function, 70% of the actual value of the relative frequency corresponds to the inferior risk category (zero to four accidents), 25% of the actual relative frequency value is for the regular risk category (5-11 accidents), and 5% of the actual relative frequency value corresponds to the higher risk category (12-20 accidents).

Example

$$\mathrm{Re}\textit{lative frequency of } 0 \textit{ accidents} = Rf(0) = (1/52) = 0.019231 \tag{2}$$

$$\textit{Low linguistic projection of } 0 \textit{ accidents} = Low(0) = (0.7)*(1/52) = 0.013462 \tag{3}$$

$$\mathrm{Re}\textit{lative frequency of } 5 \textit{ accidents} = Rf(5) = 6/52 = 0.11538 \tag{4}$$

$$\textit{Low linguistic projection of } 5 \textit{ accidents} = Low(5) = (0.25)*(6/52) = 0.02886 \tag{5}$$

$$\mathrm{Re}\textit{lative frequency of } 12 \textit{ accidents} = Rf(12) = 0/52 = 0.00000 \tag{6}$$

$$\textit{Low linguistic projection of } 12 \textit{ accidents} = Low(12) = (0.05)*(0/52) = 0.00000 \tag{7}$$

Fuzzy rules

The proposed system analyzes three types of accidents and is classified into three language labels, for which 21 of the 27 feasible fuzzy rules are determined as the basis of knowledge. This is presented in Table 10.

Implication: total probability theorem

In the research proposal, each linguistic variable has a priority weight and is classified as an independent variable.

$$A_1 = \mathrm{Pr}\textit{iority weight of the labor - related } \mathrm{var}\textit{iable} \tag{8}$$

$$A_2 = \mathrm{Pr}\textit{iority weight of the material - related } \mathrm{var}\textit{iable} \tag{9}$$

$$A_3 = \mathrm{Pr}\textit{iority weight of the site condition - related } \mathrm{var}\textit{iable} \tag{10}$$

where

$$P(A_1) = 0.2; P(A_2) = 0.3; P(A_3) = 0.5$$

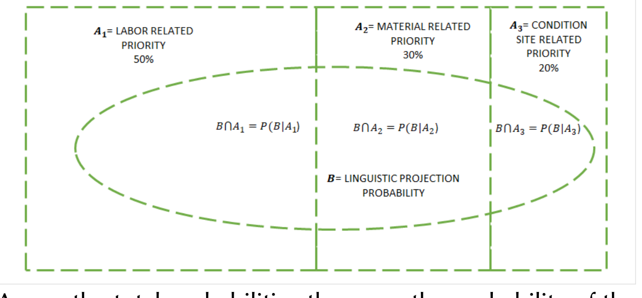
Note that the events A_1, A_2 *and* A_3 are incompatible and their probabilities add up to 1, so they satisfy the hypotheses of the total probability theorem (Figure 5).

In each proposed fuzzy rule, there is a probabilistic fuzzy value, given that it belongs to a specific linguistic projection represented as $P(B \mid A_i)$.

$$P(B \mid A_1) = \textit{Probability of the linguistic projection, given that it belongs to the labor} - \textit{related } \mathrm{var}\textit{iable} \tag{11}$$

$$P(B \mid A_2) = \textit{Probability of the linguistic projection, given that it belongs to the material - related } \mathrm{var}\textit{iable} \tag{12}$$

$$P(B \mid A_3) = \textit{Probability of the linguistic projection, given that it belongs to the site condition - related } \mathrm{var}\textit{iable} \tag{13}$$



As per the total probabilities theorem, the probability of the linguistic projection $P(B)$ is described using Equation (14).

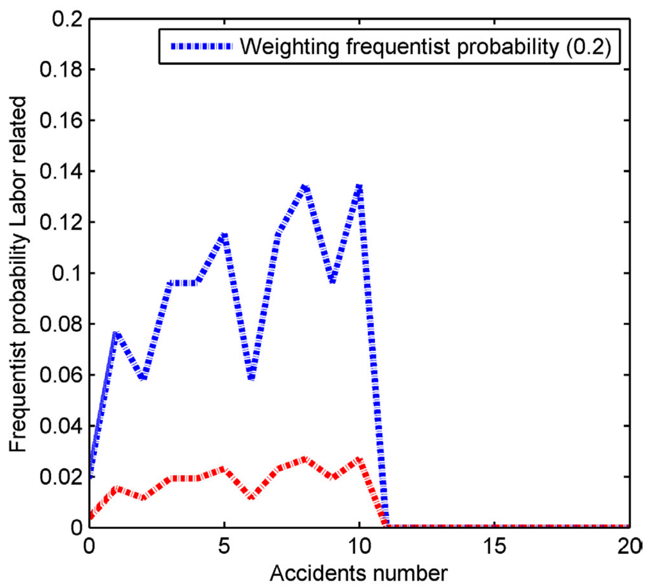
$$P(B) = P(B \mid A_1)P(A_1) + P(B \mid A_2)P(A_2) + P(B \mid A_3)P(A_3) \tag{14}$$

Figure 5 shows the representative Venn diagram of the total probability theorem, where the probability of linguistic projection $P(B)$ can be determined, since the conditional probabilities of the linguistic labels are known, given the factor to which they belong, $P(B|A_i)$.

The total probability theorem is represented by the product of the weighted probability of the input variable $P(A_i)$ and the known conditional probability $P(B \mid A_i)$. Visually, the product $P(A_i) * P(B \mid A_i)$ is represented by a projection of the known conditional probability. This projection represents the contraction of the conditional probability behavior set at a weighted percentage.

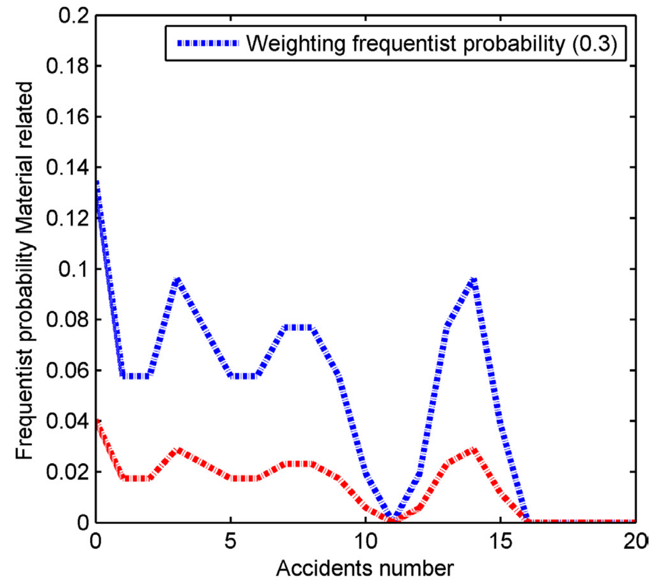
The projections of the weighted frequency probabilities $P(A_i)$ for each of the known conditional probabilistic $P(B|A_i)$ are shown below in Figures 6, 7, and 8.

The behavior of the blue trajectory represents the conditional probability $P(B|A_i)$, and the behavior of the red trajectory represents the projection or contraction of the conditional probability, which represents the product $P(A_i) * P(B \mid A_i)$ as a term to determine the total probability proposed in this research. The projection of the weighting behavior of the first type of labor-related accident with 20% of the relative frequency can be seen in Figure 6.



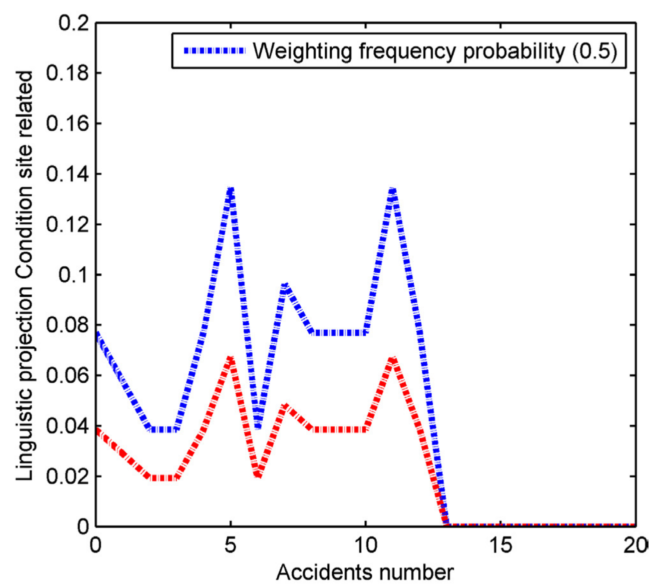
Source: Authors

The blue trajectory represents the conditional probability $P(B \mid A_2)$, and the red one represents the projection or contraction of the conditional probability, which represents the product of the probabilities $P(A_2) * P(B \mid A_2)$ as a term to determine the total probability. The projection of the weighting behavior of the material-related accidents, with 30% of the relative frequency, can be seen in Figure 7.

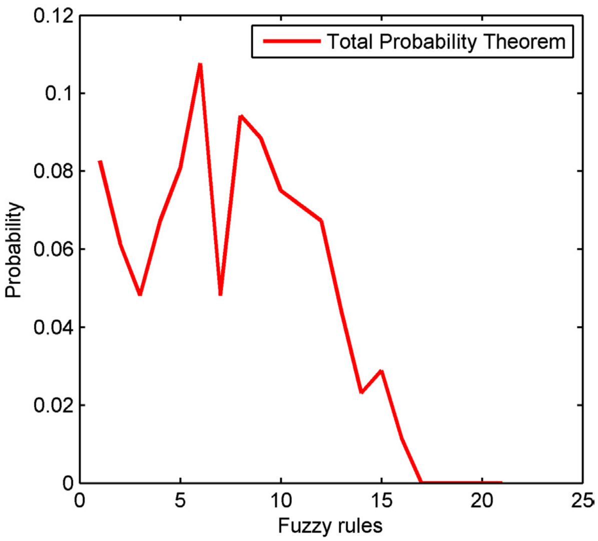


Source: Authors

The blue trajectory represents the conditional probability $P(B|A_3)$ and the red one represents the projection or contraction of the conditional probability, which represents the product of the probabilities $P(A_3) * P(B \mid A_3)$. The projection of the weighting behavior of site condition-related accidents, with 50% of the relative frequency, can be seen in Figure 8.



Finally, the sum of weighted frequency probabilities is added as the proposed implication mechanism by using the total probability theorem (Figure 9).



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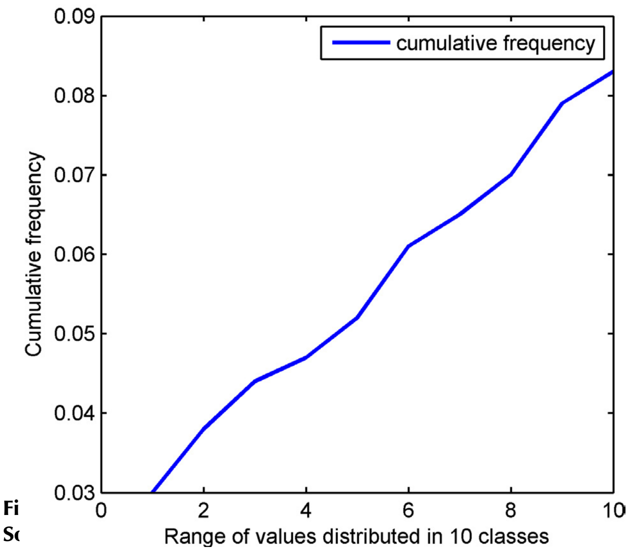

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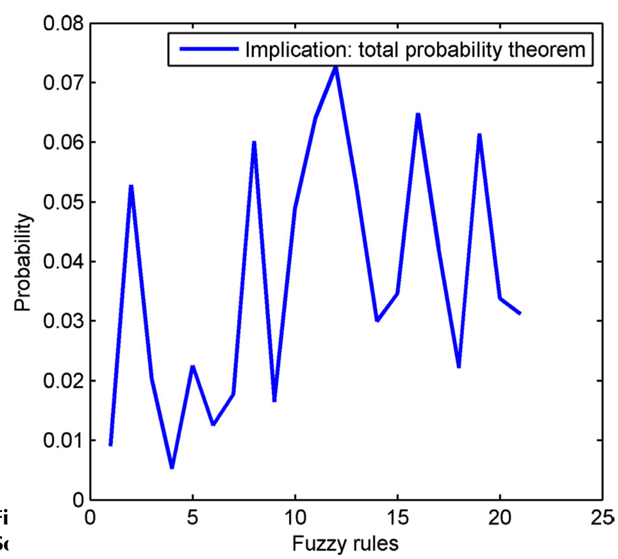
Aggregation

At the aggregation stage, the maximum fuzzy operator is used to find the maximum value of the total probability for each output linguistic label, as well as to identify the linguistic classification of the output variable to which it belongs.

Risk assessment

Below is another scientific contribution of this research, which is related to the assessment phase. The results of 26 training points in the implication phase range from 0 to 0.09. Figure 10 shows the results of the implication stage for a training point oscillating between the established range of values.



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A cumulative probability function is determined in that range of values, distributed in ten classes (Figure 11). Several zones are proposed based on the output risk category.

In each zone, the central limit theorem is used for the corresponding evaluations, along with its main mean and standard deviation parameters (Table 6). In summary, when the maximum value is obtained at the aggregation stage, the risk classification is identified, and its normal distribution function in the relevant area is used to assess the company's risk.

Classification and assessment zones

Table 6 shows the classification and assessment zones based on the behavior of the total probability of the system's training points, as well as the parameters of the mean and the standard deviation. The central limit theorem is used as a decoding mechanism, as described using Equation (15).

$$z = \frac{x-\mu}{\sigma} \tag{15}$$

The value of x in the central limit theorem is cleared in Equation (16).

$$x = z\sigma + \mu \tag{16}$$

Table 6. Classification and assessment zones

Output classification	Zones	Probability	Mean	Standard deviation
Very Serious	1	0-0.09	9	1
		0.065-0.09	4	2
		0.061-0.065	7	2
		0.052-0.061	3	2
Moderate	5	0.044-0.052	7	2
		0.000-0.044	9	2
		0.065-0.09	3	0.75
		0.047-0.065	4	0.75
		0.038-0.047	2.5	0.75
Very Slight	5	0.030-0.038	4	0.75
		0.000-0030	1.9	0.75

Source: Authors

Results

The top five causes of accidents are ignorance of safety issues, lack of personal protective equipment, lack of safety measures, unfit equipment, and lack of knowledge and training on equipment. The main types of accidents in the company under study, are related to daily work, material handling, and site conditions.

The company keeps weekly records of the number of accidents. These records were analyzed as empirical collectives to integrate stochastic uncertainty into the proposed system.

The firm specializes in the sale of construction materials, specifically grey, white, and red sand, crushed gravel, *tezontle*, *tepetate*, *confitillo*, and fathom. These materials are delivered to the address indicated by the customer. The main current issues of the company include the lack of control of internal processes and the lack of training for employees regarding material handling and the use of the machinery used. These factors cause various types of accidents and damages to machines and trucks. The owner must incur a maintenance expense, which can result in a large cash outlay, especially with regard to machinery.

Empirical collectives

Each of the accident records related to daily labor, material, and site conditions have been classified as empirical collectives. The value ranges are 0-10 for labor-related accidents, 0-15 for material-related ones, and 0-12 for site condition-related ones (Table 7).

Frequentist probability and risk characterization

The attribute to be analyzed is the weekly relative frequency of the number of accidents of each type over a period of 52 weeks. Table 8 and Figure 12 show the frequentist probability of the number of accidents of each type. A risk characterization based on a defined range of accidents in each category is observed. The range from zero to four represents a low category, the range of five to 11 accidents represents a regular category, and the range of 12 to 20 represents a high-risk category.

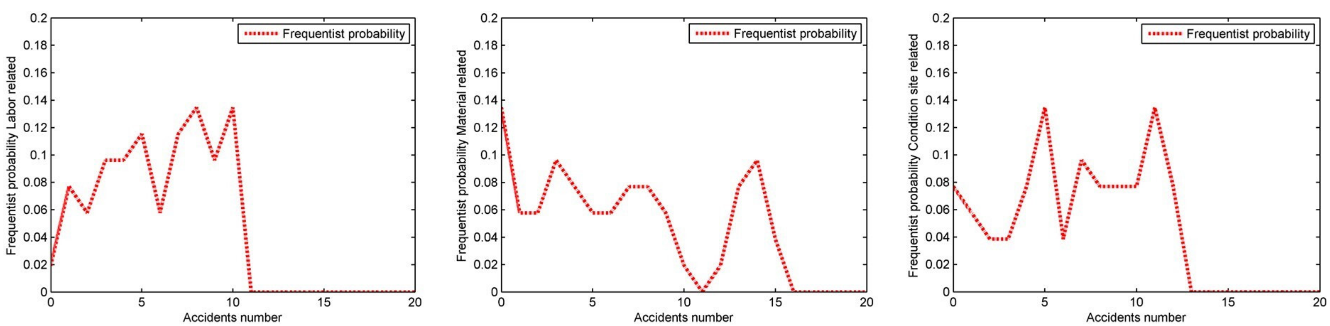


Figure 12. Frequentist probabilities
Source: Authors

Table 7. Accident records

Week	Labor-related	Material-related	Site condition-related	Week	Labor-related	Material-related	Site condition-related	Week	Labor-related	Material-related	Site condition-related
1	1	5	8	**19**	10	9	1	**37**	10	3	9
2	1	9	6	**20**	3	3	12	**38**	10	4	7
3	8	0	8	**21**	7	0	6	**39**	2	6	0
4	9	7	0	**22**	5	13	0	**40**	8	7	3
5	10	14	0	**23**	5	10	11	**41**	3	14	5
6	2	3	2	**24**	8	8	8	**42**	8	0	1
7	6	1	9	**25**	4	7	10	**43**	9	14	11
8	7	2	11	**26**	10	8	12	**44**	9	0	4
9	8	3	5	**27**	9	15	1	**45**	7	8	11
10	5	13	5	**28**	9	12	5	**46**	4	6	11
11	5	2	7	**29**	8	4	7	**47**	4	9	12
12	0	15	7	**30**	5	1	9	**48**	1	5	10
13	10	8	4	**31**	8	4	8	**49**	5	2	11
14	7	4	3	**32**	6	1	5	**50**	10	7	4
15	3	3	2	**33**	4	6	10	**51**	7	13	4
16	7	13	5	**34**	3	5	7	**52**	2	14	10
17	1	0	11	**35**	6	0	9				
18	3	0	5	**36**	4	14					

Source: Authors

Table 8. Frequentist probability

Risk characterization	Accident number	Absolute frequency, labor-related	Relative frequency, labor-related	Absolute frequency, material-related	Relative frequency, material-related	Absolute frequency, site condition-related	Relative frequency, site condition-related
	0	1	0.019230769	7	0.134615385	4	0.07692308
	1	4	0.076923077	3	0.057692308	3	0.05769231
Inferior	2	3	0.057692308	3	0.057692308	2	0.03846154
	3	5	0.096153846	5	0.096153846	2	0.03846154
	4	5	0.096153846	4	0.076923077	4	0.07692308
	5	6	0.115384615	3	0.057692308	7	0.13461538
	6	3	0.057692308	3	0.057692308	2	0.03846154
	7	6	0.115384615	4	0.076923077	5	0.09615385
Regular	8	7	0.134615385	4	0.076923077	4	0.07692308
	9	5	0.096153846	3	0.057692308	4	0.07692308
	10	7	0.134615385	1	0.019230769	4	0.07692308
	11	0	0	0	0	7	0.13461538
	12	0	0	1	0.019230769	4	0.07692308
	13	0	0	4	0.076923077	0	0
	14	0	0	5	0.096153846	0	0
	15	0	0	2	0.038461538	0	0
Superior	16	0	0	0	0	0	0
	17	0	0	0	0	0	0
	18	0	0	0	0	0	0
	19	0	0	0	0	0	0
	20	0	0	0	0	0	0
	Total	**52**		**52**		**52**	

Source: Authors

Linguistic projection

Subsequently, the frequentist probability is projected to three language labels: low, medium, and high. The projection is based on the membership functions proposed in the research. For each risk characterization associated with each language label, a spread is performed based on percentages determined by the experts (Figure 13). Table 9 shows the linguistical projection of labor-related accidents.

$\mathbf{[x]^{\alpha,\beta}, [Rf]^{\lambda Rf}}$

Low$=[I]^{0,4}, [Rf]^{0.7Rf}; [R]^{5,11}, [Rf]^{0.25Rf}; [S]^{12,20}, [Rf]^{0.05Rf}$

Medium$=[I]^{0,4}, [Rf]^{0.05Rf}; [R]^{5,11}, [Rf]^{0.7Rf}; [S]^{12,20}, [Rf]^{0.25Rf}$

High$=[I]^{0,4}, [Rf]^{0.25Rf}; [R]^{5,11}, [Rf]^{0.05Rf}; [S]^{12,20}, [Rf]^{0.70Rf}$

Fuzzy rules

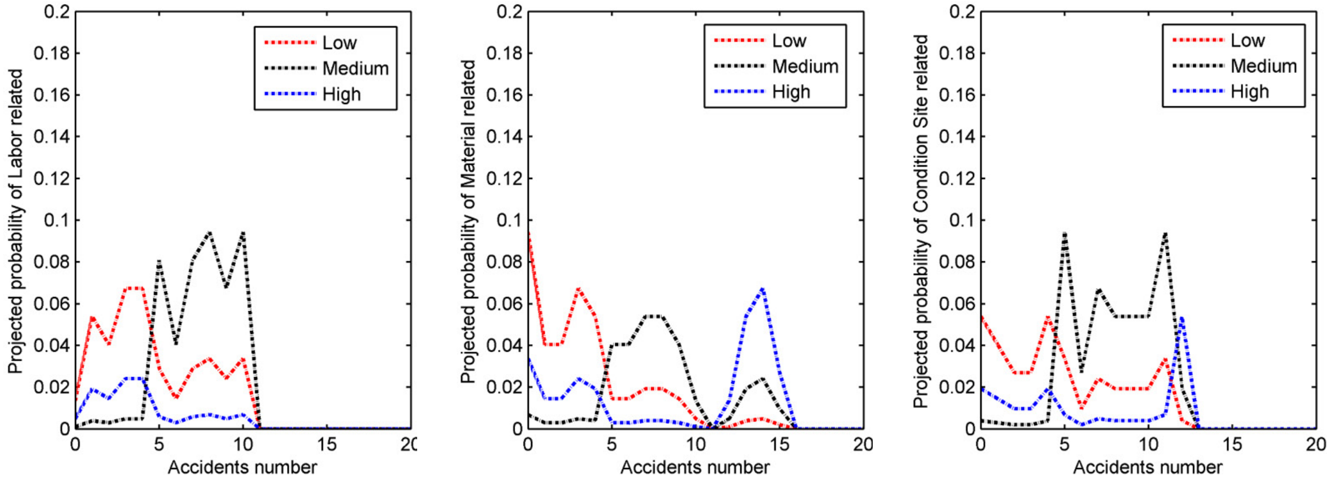
Table 10 shows the 21 fuzzy rules that make up the knowledge base of the proposed system. The output of the system is the classification of a risk as very slight, moderate, and very serious. Seven rules were determined for each label type of the output variable.

Table 10. Knowledge base.

Labor related	Material related	Condition site related	Output classification
HIGH	HIGH	HIGH	VERY SERIOUS
HIGH	HIGH	MEDIUM	VERY SERIOUS
HIGH	MEDIUM	HIGH	VERY SERIOUS
MEDIUM	HIGH	HIGH	VERY SERIOUS
HIGH	HIGH	LOW	VERY SERIOUS
HIGH	LOW	HIGH	VERY SERIOUS
LOW	HIGH	HIGH	VERY SERIOUS
MEDIUM	MEDIUM	MEDIUM	MODERATE
MEDIUM	MEDIUM	HIGH	MODERATE
MEDIUM	HIGH	MEDIUM	MODERATE
HIGH	MEDIUM	MEDIUM	MODERATE
LOW	MEDIUM	MEDIUM	MODERATE
MEDIUM	LOW	MEDIUM	MODERATE
MEDIUM	MEDIUM	LOW	MODERATE
LOW	LOW	LOW	VERY SLIGHT
LOW	LOW	MEDIUM	VERY SLIGHT
LOW	MEDIUM	LOW	VERY SLIGHT
MEDIUM	LOW	LOW	VERY SLIGHT
LOW	LOW	HIGH	VERY SLIGHT
LOW	HIGH	LOW	VERY SLIGHT
HIGH	LOW	LOW	VERY SLIGHT

Source: Authors

Figure 13. Linguistic projection of frequentist probabilities



Source: Authors

Table 9. Linguistic projection of labor related frequentist probability

Test number	Labor-related	Material-related	Site condition-related	Weighted average	System probability	Absolute error	Weighted average	System probability	Efficiency
1	0	1	4	2.3	2.09	0.21	VERY SLIGHT	VERY SLIGHT	1
2	2	4	6	4.6	4.59	0.01	VERY SLIGHT	VERY SLIGHT	1
3	6	5	8	6.7	6.68	0.02	MODERATE	MODERATE	1
4	4	7	12	8.9	8.86	0.04	MODERATE	SERIOUS	0
5	8	6	4	5.4	5.04	0.36	MODERATE	MODERATE	1
6	10	4	2	4.2	4.59	0.39	VERY SLIGHT	VERY SLIGHT	1
7	5	7	1	3.6	3.64	0.04	MODERATE	MODERATE	1
8	5	5	5	5	5.04	0.04	MODERATE	MODERATE	1
9	10	7	5	6.6	6.7	0.1	MODERATE	MODERATE	1
10	12	11	5	8.2	9	0.8	VERY SERIOUS	VERY SERIOUS	1
11	9	10	7	8.3	8.53	0.23	MODERATE	MODERATE	1
12	6	11	10	9.5	9.36	0.14	MODERATE	MODERATE	1
13	5	8	7	6.9	6.56	0.34	MODERATE	MODERATE	1
14	2	4	6	4.6	4.59	0.01	VERY SLIGHT	VERY SLIGHT	1
15	5	7	7	6.6	6.56	0.04	MODERATE	MODERATE	1
16	12	8	9	9.3	9.38	0.08	MODERATE	MODERATE	1
17	8	7	10	8.7	8.07	0.63	VERY SERIOUS	VERY SERIOUS	1
18	10	10	6	8	7.9	0.1	MODERATE	MODERATE	1
19	2	7	4	4.5	4.56	0.06	VERY SLIGHT	VERY SLIGHT	1
20	8	8	8	8	8.07	0.07	MODERATE	MODERATE	1
21	2	3	5	3.8	3.59	0.21	VERY SLIGHT	VERY SLIGHT	1
22	0	2	3	2.1	1.57	0.53	VERY SLIGHT	VERY SLIGHT	1
23	0	1	3	1.8	1.57	0.23	VERY SLIGHT	VERY SLIGHT	1
24	1	3	3	2.6	2.16	0.44	VERY SLIGHT	VERY SLIGHT	1
25	2	6	4	4.2	454	0.34	VERY SLIGHT	VERY SLIGHT	1
26	3	6	5	4.9	5.06	0.16	MODERATE	MODERATE	1

Source: Authors

Implication: Total probability theorem

Test point

Below is a test point of the system's training. The records to be evaluated for each type of accident are as follows: labor-related: 18, material-related: 1, site condition-related: 12. Figure 14 shows the inference values using the total probability theorem for each of the 21 fuzzy rules.

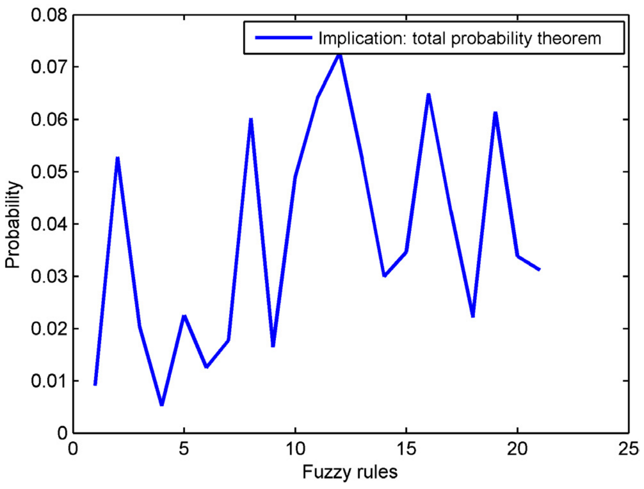


Figure 14. Individual evaluation, inference system
Source: Authors

Aggregation

The maximum value in the implication phase is 0,039, corresponding to fuzzy rule 6, with a *very serious* output label. This value represents the possibility of occurrence for the training point (Figure 14).

Classification: very serious, probability: 3.9%.

Risk assessment

Finally, in the assessment stage, the central limit theorem is used to obtain the corresponding evaluation. The maximum value is scaled with a factor of 10, to be used in Equation (17), since the classification is very serious and there is only one assessment zone with average parameters equal to 9 and a standard deviation 1.

$$x = (z * \sigma) + \mu = (0.39 * 1) + 9 = 9.39 \tag{17}$$

Figure 15 shows the result of the training point. Evaluation: 9.39.

Comparison of results

Below are 26 system training points. Our comparison with the traditional method is based on a weighted average. The results are reliable, and the efficiency of the system is 96.15%, classifying 25 of 26 training patterns correctly and constitute a robust, reliable, and simple system (Table 11).

The classification made by experts based on the weighted average is flexible with regard to the interpretation of the output classification. Although the weighted average value is 8.9, it was assigned a moderate classification, considering that the first two types of accidents have low or moderate values, preserving subjectivity in their output classification. The proposed model eliminates subjectivity and is based on the integration of stochastic uncertainty by using the relative frequencies of the accidents that occurred, as well as on that of linguistic uncertainty when projecting frequencies based on risk categorization. This was the only pattern that showed variations in the proposed probabilistic risk assessment system, with a reliability of 96,15%.

Figure 16 shows the results of the implication stage for the 26 test points.

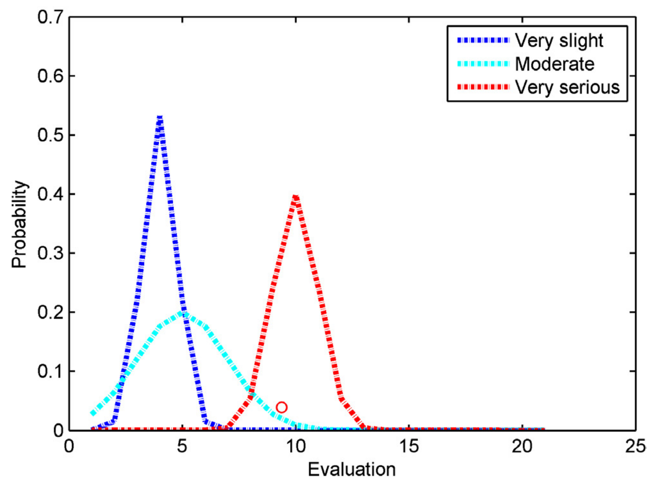


Figure 15. System output
Source: Authors

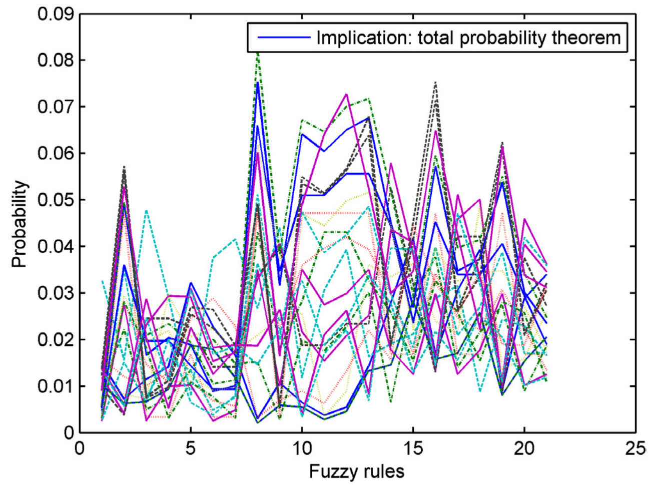


Figure 16. Total evaluation, inference system.
Source: Authors

Figure 17 shows the inference of the proposed system for the 26 test points.

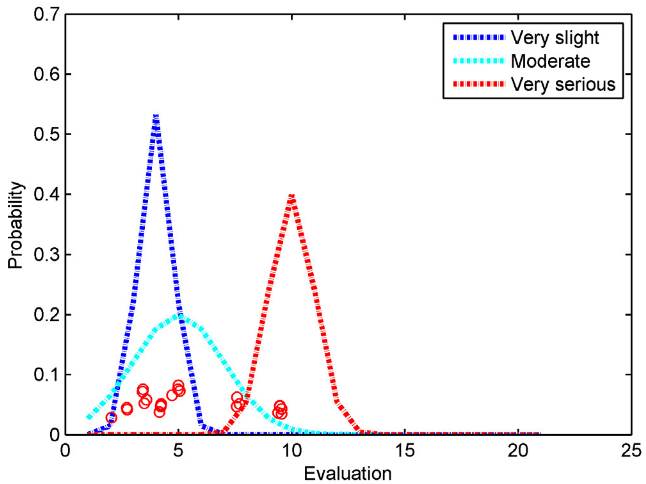


Figure 17. System output
Source: Authors

The results of the proposed system and the weighted average benchmark are compared in Figure 18.

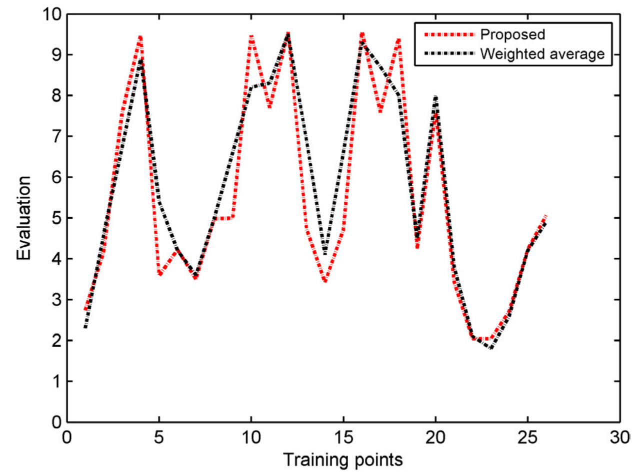


Figure 18. Comparative results
Source: Authors

Table 11. Comparison of results

Test number	Labour related	Material related	Site condition related	Weighted average	Probability system	Absolute error	Weighted average	Probability system	Efficiency
1	0	1	4	2.3	2.09	0.21	VERY SLIGHT	VERY SLIGHT	1
2	2	4	6	4.6	4.59	0.01	VERY SLIGHT	VERY SLIGHT	1
3	6	5	8	6.7	6.68	0.02	MODERATE	MODERATE	1
4	4	7	12	8.9	8.86	0.04	MODERATE	SERIOUS	0
5	8	6	4	5.4	5.04	0.36	MODERATE	MODERATE	1
6	10	4	2	4.2	4.59	0.39	VERY SLIGHT	VERY SLIGHT	1
7	5	7	1	3.6	3.64	0.04	MODERATE	MODERATE	1
8	5	5	5	5	5.04	0.04	MODERATE	MODERATE	1
9	10	7	5	6.6	6.7	0.1	MODERATE	MODERATE	1
10	12	11	5	8.2	9	0.8	VERY SERIOUS	VERY SERIOUS	1
11	9	10	7	8.3	8.53	0.23	MODERATE	MODERATE	1
12	6	11	10	9.5	9.36	0.14	MODERATE	MODERATE	1
13	5	8	7	6.9	6.56	0.34	MODERATE	MODERATE	1
14	2	4	6	4.6	4.59	0.01	VERY SLIGHT	VERY SLIGHT	1
15	5	7	7	6.6	6.56	0.04	MODERATE	MODERATE	1
16	12	8	9	9.3	9.38	0.08	MODERATE	MODERATE	1
17	8	7	10	8.7	8.07	0.63	VERY SERIOUS	VERY SERIOUS	1
18	10	10	6	8	7.9	0.1	MODERATE	MODERATE	1
19	2	7	4	4.5	4.56	0.06	VERY SLIGHT	VERY SLIGHT	1
20	8	8	8	8	8.07	0.07	MODERATE	MODERATE	1
21	2	3	5	3.8	3.59	0.21	VERY SLIGHT	VERY SLIGHT	1
22	0	2	3	2.1	1.57	0.53	VERY SLIGHT	VERY SLIGHT	1
23	0	1	3	1.8	1.57	0.23	VERY SLIGHT	VERY SLIGHT	1
24	1	3	3	2.6	2.16	0.44	VERY SLIGHT	VERY SLIGHT	1
25	2	6	4	4.2	4.54	0.34	VERY SLIGHT	VERY SLIGHT	1
26	3	6	5	4.9	5.06	0.16	MODERATE	MODERATE	1

Source: Authors

Conclusions

Our research proposed a novel global probabilistic fuzzy system for occupational risk assessment. Stochastic uncertainty conditions were integrated, using the frequentist probability of the number of accidents of each type as input variables. Non-stochastic uncertainty was incorporated by classifying the input variable via language labels in a clear and simple language (*i.e.*, high, medium, and low) based on expert interpretation.

The accidents assessed were labor-related, material-related, and site condition-related in a construction company dedicated to the sale of materials.

The conceptualization was based on the analysis of accident records as empirical collectives, where the frequentist probability could be determined for different numbers of identified accidents. A crucial phase in the methodology was to identify the risk category based on parameters corresponding to expert interpretation.

One of the scientific contributions to diffuse probabilistic systems is the projection of frequentist probabilities in three language categories, which are based on the definition of percentages related to the heuristic knowledge of experts and constitute three weighted membership functions. 21 fuzzy rules were defined to infer the result. Another important contribution is the use of the total probability theorem in the implication stage as the inference mechanism of the output variable. In the aggregation stage, the maximum fuzzy operator was used to estimate the probability of occurrence of each type of accident and its classification. The central limit theorem was used in the risk assessment stage. Different zones were proposed for each classification of the output variable. For the *very serious* classification, there is only one zone, and, for the *moderate* and *very slight* classifications, there are five zones. The main parameters of the mean and the standard deviation were determined for each zone. Finally, a benchmarking comparison was performed via a simple weighted average technique used and interpreted by experts. The results show the robustness and reliability of the proposed system.

It should be noted that the heuristic knowledge of experts was used to determine percentages in the frequentist probability breakdown. It was also used in determining the ranges of each risk category.

The proposal for a diffuse probabilistic system can be replicated in any production or service sector, as well as in organizations of any size.

As future work, a novel alternative will be presented for the involvement stage, aiming to group the output categories objectively and not subjectively.

CRedit author statement

All authors have participated in a) conception and design or data analysis and interpretation; (b) drafting the article or revising it critically for important intellectual content; and (c) approving of the final version of the manuscript.

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