Research Article/Mechanical, Mechatronics, and Materials Engineering

http://doi.org/10.15446/ing.investig.111284

Analysis, Modeling, and Simulation Solution of Induced-Draft Fan Rotor with Excessive Vibration: A Case Study

Solución de análisis, modelado y simulación de rotor de ventilador de tiro inducido con vibración excesiva: un caso de estudio

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ABSTRACT

In the modern industry, computer modeling and simulation tools have become fundamental to estimating the behavior of rotodynamic systems. These computational tools allow analyzing possible modifications as well as alternative solutions to changes in design, with the aim of improving performance. Nowadays, rotodynamic systems, present in various industrial applications, require greater efficiency and reliability. Although there are deep learning methodologies for monitoring and diagnosing failures which improve these standards, the main challenge is the lack of databases for learning, a problem that can be addressed through experimental monitoring and computer analysis. This work analyzes the vibrations of two induced-draft fans with excess vibration in a thermoelectric plant in Mexico. A vibration analysis was carried out through the instrumentation and monitoring of accelerometers located at crucial points in the fans. The results of this experimental analysis were validated by computer simulation based on FEM. The results show that the operating speed of the induced-draft fans is very close to their natural frequency, causing considerable stress and potential failures due to excessive vibration. Finally, this work presents a practical solution to modify the natural frequency of induced-draft fans, so that they can function correctly at the required operating speed, thus mitigating excessive vibration issues.

Keywords: Monitoring and data analysis, fault diagnosis, computer simulation, mitigate mechanical vibrations.

RESUMEN

En la industria moderna, las herramientas de modelado y simulación computacional se han vuelto fundamentales para estimar el comportamiento de los sistemas rotodinámicos. Estas herramientas computacionales permiten analizar posibles modificaciones y soluciones alternativas a cambios en el diseño, con el objetivo de mejorar el rendimiento. Hoy en día, los sistemas rotodinámicos, presentes en diversas aplicaciones industriales, requieren mayor eficiencia y fiabilidad. Aunque existen metodologías de aprendizaje profundo para el monitoreo y diagnóstico de fallas que mejoran estos estándares, el principal desafío es la falta de bases de datos para el aprendizaje. Este problema puede ser abordado a través del monitoreo experimental y el análisis computacional. Este trabajo analiza las vibraciones de dos ventiladores de tiro inducido con exceso de vibración en una planta termoeléctrica en México. Se realizó un análisis de vibración a través de la instrumentación y el monitoreo de acelerómetros ubicados en puntos cruciales de los ventiladores. Los resultados de este análisis experimental fueron validados por simulación computacional basada en el método de elementos finitos. Los resultados muestran que la velocidad de operación de los ventiladores de tiro inducido está muy cerca de su frecuencia natural, causando un estrés considerable y posibles fallas debido a la vibración excesiva. Finalmente, este trabajo presenta una solución práctica para modificar la frecuencia natural de los ventiladores de tiro inducido, de modo que puedan funcionar correctamente a la velocidad de operación requerida, mitigando así los problemas de vibración excesiva.

Palabras clave: Monitoreo y análisis de datos, diagnóstico de fallas, simulación por computadora, mitigar vibraciones mecánicas.

Received: September 25th 2023
Accepted: February 27th 2024

Introduction

Vibration issues in rotating machinery depend on multiple factors, including misalignment, critical speeds or resonances, system deterioration due to continuous use, bearings defects, and imbalance. It is well known in the field of vibrations that critical velocities or resonances are physical phenomena that arise when the natural frequency of the system matches the operating frequency. These occurrences can be mitigated by altering the system design and adjusting its natural or operating frequency (Xiangyang *et al.*, 2023). Simultaneously, imbalances can manifest as either mechanical or electrical in nature. Mechanical imbalance arises when the principal axis of inertia does not align with the geometric axis of the system. According to (Blanco-Ortega *et al.*, 2010), several active and passive methods or devices have been developed trough vibration analysis to mitigate this mechanical phenomenon. Electrical imbalance is generated by voltage variations or harmonic distortion in the voltage, which, in the typical case of induction motors, affects their dynamic behavior and vibrations (Donolo *et al.*, 2016; Ren *et al.*, 2023). For steam turbines, the vibration in the rotor is divided into two categories (Kaneko *et al.*, 2022): forced vibration and self-excited vibration. Forced vibration is caused by an external

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force and is divided as follows: a) by rotor imbalance, b) by mechanical imbalance in gears and connections or couplings, c) by electric excitation from the motor or generator, d) by fluid excitation, and e) by uncommon factors. Self-excited vibration is mainly caused by oil whip and steam whirl.

Long-term vibration defects in the machinery significantly impact both the equipment's service life and the regular, stable operation of the unit (Li *et al.*, 2020). According to studies on different types of turbomachines, the leading causes of vibrations are cavitation and resonance. In addition, excess speed and long periods of operation contribute to this effect (Doshi *et al.*, 2021). Although vibrations are unwanted in systems, they also serve as support to identify defective designs, poor installation, and wear or deterioration. Over time, improper installation, manufacturing defects, waste accumulation, and the gradual degradation of machines can lead to rotor eccentricity. This eccentricity adversely impacts their performance, causing high noise and vibration levels, elevated losses, and the risk of overheating (Chapagain and Silwal, 2023), (Yu-Ling *et al.*, 2023).

Induced-draft fans are used to evacuate air from a space or to create a negative air pressure in a system. They are one of the crucial elements of a thermal power plant's auxiliary equipment. Vibration studies on industrial fans focus on the main static and dynamic parts, such as the rotor and bearings (Zenglin and Gordon, 2003; Shabaneh and Zu, 2003; Trebuna *et al.*, 2014; Jagtap *et al.*, 2020), the blades (Pingchao *et al.*, 2018; Wang *et al.*, 2020), the casing and structure (Niko *et al.*, 2011), and the ducts. At times, vibration issues arise from the intricate interaction between various components of the fan-duct-foundation system.

Rotating machinery can be analyzed using physical models (Shabaneh and Zu, 2003; Zenglin and Gordon, 2003), experimental tests through instrumentation (Jerzy *et al.*, 2014), simulation via mechanical/acoustic and computational methods (Manish *et al.*, 2015; Kalmár-Nagy *et al.*, 2015), or a combination of approaches (Pingchao *et al.*, 2018; Novotný *et al.*, 2019; Xie *et al.*, 2023). In the case of rotating machinery (*e.g.*, fans), vibration measurement has become a fundamental tool for monitoring operating conditions (Guo *et al.*, 2021; Dhamande *et al.*, 2023). In some studies, comprehensive analyses are conducted to identify fan failure. The methods employed encompass vibration analysis, noise measurement, wear debris analysis, and ultrasonic monitoring (Jagtap *et al.*, 2020).

In recent years, various modern techniques have emerged from analyzing, determining, and predicting failures in rotating machinery based on a) artificial intelligence (Liu *et al.*, 2018) through data acquisition, feature extraction, and fault recognition; b) neural networks (Benrahmoune *et al.*, 2018), by analyzing and comparing monitoring data; and c) machine learning models (Benchekroun *et al.*, 2023) to accurately predict the vibration problem in fans. Considering the above, and bolstered by preventive maintenance to reduce downtime and enhance efficiency, operators can proactively identify potential issues before they reach critical levels. (Di *et al.*, 2022) propose a novel method for detecting anomalies in rotating machinery, leveraging vibration vectors inspired by the Polar plot. These vectors contain amplitude and phase values from various characteristic frequencies. By converting the original Polar vectors to Cartesian plots using fast Fourier transform-based order analysis (FFT-OA), calculations are simplified and visualization is improved. Wei *et al.* (2022) introduce an innovative condition monitoring approach for induced-draft systems. They combine a genetic algorithm with a long-short-term memory network (GA-LSTM) to establish dynamic and static thresholds for anomaly detection. Applying this method to a coal-fired power plant enables early fault diagnosis and detection. Experimental results demonstrate the effectiveness of this approach, successfully detecting minor anomalies in advance and contributing to improved system reliability and proactive maintenance.

Physical models allow estimating a system's real behavior to better perform and interpret computational and simulation analyses (Xie *et al.*, 2023; Novotný *et al.*, 2019). The study by Zenglin and Gordon (2003) is based on the Jeffcott rotor model with external damping for vertical and horizontal directions. The Lund Stability method is applied for the horizontal direction, whereas an analytical and numerical analysis is conducted for the vertical direction to study the threshold speeds. The authors also consider improving the stability characteristics of the system by adjusting external damping values. Shabaneh and Zu (2003) perform a dynamic analysis of a single-rotor shaft system with nonlinear elastic bearings at the ends mounted on viscoelastic suspension. The Timoshenko shaft model represents the flexibility of the shaft, and the viscoelastic supports are modeled using the Kelvin-Voigt model. The authors also find that the primary resonance peak is modified for higher frequencies based on increased nonlinear elastic bearing characteristics.

Bearings play a critical role in rotating machines. The inevitability of degradation and bearing failures arises from prolonged and continuous operation, encompassing factors such as poor maintenance practices, inadequate care, and dynamic stress induced by the dynamic loads of rotating parts. Recent research endeavors employ innovative analysis techniques for the vibration analysis of bearings, recognizing their inherent complexity (Mohamad *et al.*, 2023; Lu *et al.*, 2023). Furthermore, probabilistic techniques such as the remaining useful life (RUL) as well as those based on deep learning have become very important. To predict the failure of bearings through vibration analysis or estimate their remaining service life, modern techniques are complemented with the Vold-Kalman parametric filter method (Cui *et al.*, 2019). This method, based on the state-space framework, has recently found applications in the navigation and control of vehicles and spacecraft. In the context of techniques rooted in deep learning, such as the innovative PIResNet (Ni *et al.*, 2023), their necessity arises from the inherent variations in speed and load experienced by bearings during the operation of machinery. PIResNet stands out by offering a consistent physical solution even in the presence of imperfect data. Given the critical significance of bearings, their behavior has undergone evaluation in both previous unsuccessful actions and our ongoing research project. In both instances, they have been ruled out as the causes of excessive vibrations.

The main contribution of this work lies in the comprehensive study and identification of the cause of excessive vibration in two induced-draft fans of a thermoelectric plant in Mexico.

In addition, through computer simulation, a solution is proposed. Previous attempts to resolve the vibration issue had been unsuccessful, prompting the need for a more rigorous and systematic approach. The hypothesis of this work assumes that a practical and theoretical analysis through simulation will shed light on the leading causes of excessive vibration, thus allowing to propose solutions. This hypothesis employed a methodology involving the instrumentation of rotor bearings with accelerometers, fan modeling, simulation, and data analysis.

The research objectives of this work are as follows: (i) to study and identify the cause of excessive vibration in an induced-draft fan at a Mexican thermoelectric power plant; (ii) to develop a methodology that includes instrumentation, fan modeling, simulation, and data analysis to evaluate vibrations and determine the natural frequencies and vibration modes of the fan's operation; (iii) to assess the hydraulic bearings' stiffness and damping constants for a more accurate computer simulation; (iv) to emphasize the importance of vibration measurement and monitoring for the proper operation and maintenance of fans; (v) to provide valuable insights and recommendations for mitigating vibration issues and improving the performance and reliability of induced-draft fans in thermoelectric power plants.

This document is organized into sections focusing on analyzing, modeling, and simulating an induced-draft fan model with excessive vibration. It starts with an *Introduction* that provides an overview of vibration issues in rotating machines and a case study conducted in a thermoelectric plant in Mexico, in addition to stressing the need for analyzing fan vibration issues. Figure 1 depicts the proposed approach while considering previous actions and presents an overview of the methodology. The *Problem definition* section provides information on the system's design, vibration limits, and the need for mitigating excessive vibrations. It also describes previous actions to address the vibration issues. The *Metodology* section outlines the analysis, covering both experimental instrumentation and computer simulation. It explains the approach to vibration analysis, which involved the placement of accelerometers in the fan bearings to validate the natural frequencies. The document further details the process of modeling and simulating the fan using the finite element method (FEM) in the ANSYS software. The *Results* section presents the data obtained from static and dynamic tests, encompassing the modeling and meshing of individual fan components as well as the complete simulation. Subsequently, the *Discussion* and *Conclusion* sections draw upon relevant studies, offering valuable insights and recommendations to enhance the performance and reliability of a particular induced-draft fan in a thermoelectric power plant.

Problem definition

Unpredictable rotor vibrations have been observed in two fans at a thermoelectric plant in Mexico, with each occurrence displaying distinctive characteristics. In order to mitigate random vibrations, it red was necessary to balance the shaft-compressor system of the rotor. This process has been frequently performed over the past decade whenever the system experiences vibration. However, this issue has resulted in significant economic losses due to the inadequate operation of fan motors or insufficient maintenance times to achieve balance.

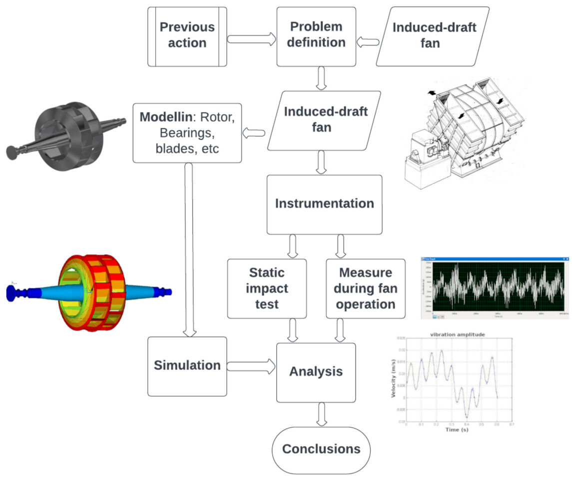


Figure 1. Proposed approach
Source: Authors

Table 1. Fan information

Description	Quantity
Volumetric flow	2000 $\frac{m^3}{hr}$
Static pressure	0.602 m water column
Temperature	417.5 K
Type	Double suction blade or vane control, constant speed

Source: Authors

The system comprises two 2500 kW motors, each integrated into a rotor, and each duct is supplied by one of these motors (Figure 2). Table 1 presents information on the system. Airflow is supplied to the ducts of these motors. The air inflow is in the vertical direction from the outside in, and the air outflow is in the horizontal direction towards the entrance of the ducts (Figures 3a and 3b).

The mechanical vibration issues of the system have persisted since it was put into operation two decades ago, with the bearings reaching unbalanced speeds between 10 and

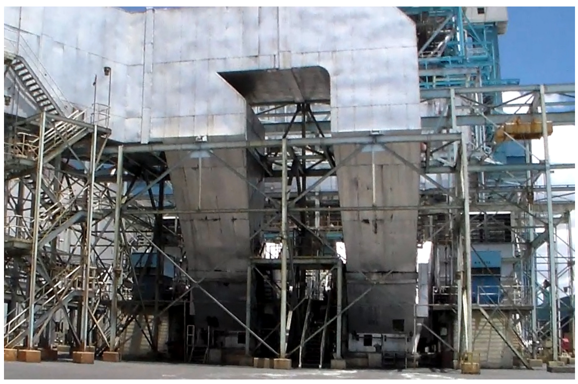
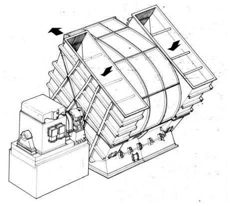


Figure 2. Lateral section of the ducts and actuation engines
Source: Authors



(a) Manufacturer design

(b) Induced-draft motor

Figure 3. Fan motor
Source: Authors

Table 2. Typical band boundaries regarding the effective vibration velocity of machines in class III according to STN ISO 10816-1 (Trebuna *et al.*, 2014)

Band	v_{eff} ($\frac{mm}{s}$)	Characteristics
A	0.28-1.80	Typical vibration of new machines
B	2.28-4.50	Vibration of machines assigned for non-stop long-term operation
C	7.10-11.2	Vibration of machines that are unsuitable for long-term non-stop operation
D	18.0-45.0	No vibration allowed in the machines

Source: Authors

$11 mm/s$, exceeding the limit established by the regulations (Table 2). While induced-draft fan 1A initially experienced minimal issues, it has required frequent balancing since 2002. Achieving motor stability as it passes through the system's natural frequencies is crucial, and the balancing process varies for each of the two 2500 kW motors. Sometimes, up to 20 motor starts are necessary for correct balancing without complications.

Previous actions

Earlier, the internal staff of the thermoelectric, as well as private companies, used to analyze vibration issues in the system. In this regard, previous actions included the following:

- In 1996, a new rotor and shaft design was implemented for induced draft-fan 1A, significantly reducing the need for more continuous balancing. The revised design specifically involved the switch to a more rigid steel material.
- Ash residues, with quantities of approximately 0.300 kg, were detected on the fan blades. In response, a maintenance and cleaning program with increased frequency was proposed. While this initiative has contributed to a partial mitigation of excessive vibration issues, it remains a necessary measure.
- In some instances, repairing the edges of the fan blades has been necessary.
- To achieve system stability, the structure of the ducts was modified, and gates were added to the fan suction. These actions were also unsuccessful.
- The bearings were also tested to determine whether they were the cause of excessive vibration, and the foundation of the fans was modified. Both modifications were unsuccessful.

In light of the above, the following hypotheses were proposed:

1. The operating frequency of the engine is very close to one of the system's natural frequencies (simulation, hammer test and instrumentation sections will be used to test these hypotheses).
2. Improper balancing, whereby an additional mass is added to a subsequent imbalance, may be the cause of excessive vibration.
3. The accumulation of external elements (*i.e.*, ash, additives, moisture) or blade wear causes the rotor to be unbalanced.

In light of the above, a methodology was structured, with the aim of examining the system's excessive vibration issues. This methodology comprises two main parts: experimental instrumentation and computer simulation.

Methodology

Our approach began with the system's instrumentation for static and dynamic testing. We modeled and meshed the fan parts. In this model, master nodes connect the meshed parts. A vibration analysis of the complete system was carried out through a computer simulation of the fan's operation.

Instrumentation of the induced-draft fan

Vibration curves and natural frequencies were obtained by conducting dynamic and static tests with accelerometers placed on two fans. One fan was operational for the dynamic case, while the other was turned off for the static case.

Dynamic testing

Through the instrumentation process, the operation of the induced draft-fan was evaluated in order to estimate the system's natural frequencies. To properly take measurements of these machines' bearings, the first step involved adjusting the position of the accelerometers. One accelerometer was used to measure vibrations in the vertical direction of the bearing, and the other one was employed to measure vibrations in the longitudinal or horizontal direction of the bearing (Figure 4).

Static impact test

An impact test was performed on induced-draft fan 1B, taking advantage of the fact that the unit was out of service due to major maintenance (*i.e.*, joint expansion replacement). The impact test is a vibration measurement using accelerometers, wherein the system is excited using a high-intensity mechanical impulse, forcing it to vibrate in its natural modes. If the system is operating, the damping factor of the system can also be obtained. Nevertheless, the only information that could be obtained from this test were the natural frequencies and the first vibration modes.

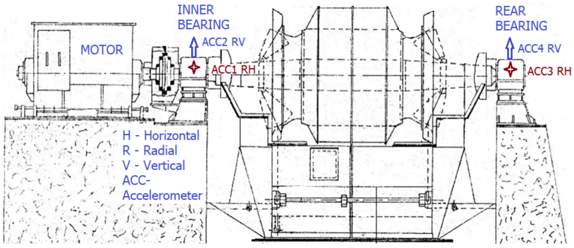


Figure 4. Placement of the accelerometers and directions of response measurements in the induced-draft fan
Source: Authors

(a) Real model of the fan rotor

(b) CAD model of the fan rotor

Figure 5. Rotor and CAD model
Source: Authors

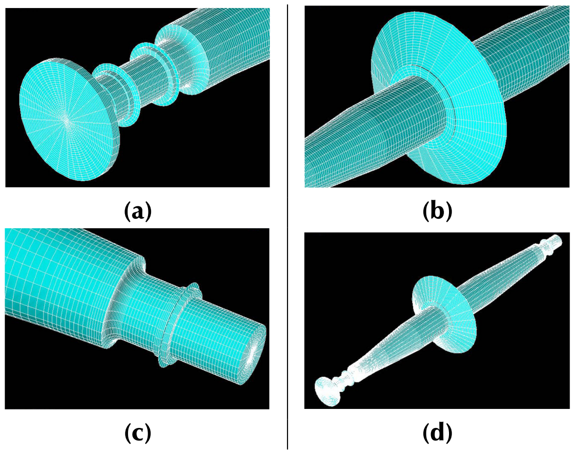


Figure 6. Sectional meshing: a) connection to the motor of the rotor shaft component, b) middle section of the rotor shaft component, c) final section of the rotor shaft component, d) complete rotor shaft mesh
Source: Authors

Modeling the induced-draft fans

The goal of this vibration analysis was to provide a solution to the vibration issues, which cause mechanical damage and result in downtimes, leading to economic losses for the company. The analytical solution aimed to extend the service life of the fans and ensure optimal performance without frequent maintenance.

This work focuses on three main aspects:

1. the material's mechanical properties, using specialized techniques to understand its behavior under mechanical stress;
2. the root causes of vibration failure in the system;
3. proposing a series of solutions to mitigate vibration issues in the system.

Rotor

In rotating machinery, vibration can stem from various sources, with imbalance being a major cause. While rotors ideally consist of homogeneous material with a uniform mass distribution along their axis and perfect symmetry, the presence of small randomly distributed mass concentrations unfortunately leads to imbalance. This imbalance can be caused by factors such as corrosion and dirt. Balancing a rotor involves ensuring that its principal axis of inertia aligns with the center of gravity, coinciding with the axis of rotation, nearly eliminating vibration.

Rotating machinery may experience instability due to various factors. Factors that can be attributed to incorrect rotor assemblies include inadequate shaft alignment and incorrect bearing placement. In many cases, rotors operate above their natural frequency, causing resonance to occur briefly when the operating speed (or impeller speed) passes through this frequency. These occurrences can cause damage to machinery components, such as seals and bearings, leading to instability. In addition, high-speed operation and friction in the bearings can lead to wear and temperature increases, modifying the dynamic characteristics of lubricants and resulting in poor lubrication and further contributing to instability in the bearings.

There are various models available for dynamic rotor analysis, ranging from very simple to extremely complex. The complexity of these models is based on the number of degrees of freedom (Zenglin and Gordon, 2003; Shabaneh and Zu, 2003; Yang *et al.*, 2024). While some models may be unrealistic and inadequate, others may need to be more complex to be practically useful, making their mathematical representation and solution nearly impossible. Figure 5a shows the fan rotor. The model was compared against design models based on manufacturer and field measurements to ensure accuracy. The 3D CAD model of the fan rotor is shown in Figure 5b.

After creating the geometric model, the rotor was meshed to generate a sectional model. This involved selecting certain parts or components of the rotor and assembling them into a single model, as illustrated in Figure 6.

The mesh corresponding to the complete rotor shaft shell has a complex geometry due to the shape of the blades, as shown in Figure 7, which depicts the appropriate assembly of elements.

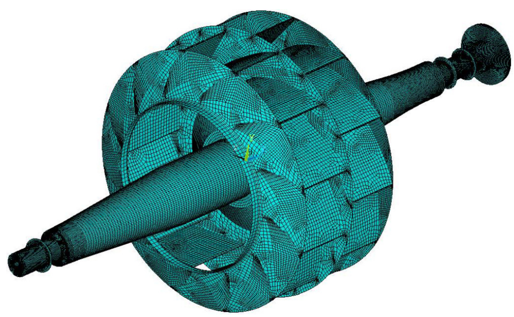


Figure 7. Total mesh of the rotor shaft
Source: Authors

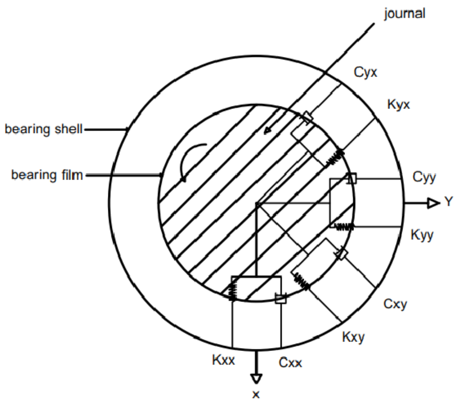


Figure 8. Schematic representation of a hydraulic bearing
Source: Authors

Table 3. Stiffness and damping coefficients for the simulated hydraulic bearing

Constant	Value
K_{xx}	$190.673496665 \times 10^6[N/m]$
K_{yy}	$52.599585288 \times 10^6[N/m]$
C_{xx}	$4.389542179 \times 10^6[(N-s)/m]$
C_{yy}	$0.6270774542 \times 10^6[(N-s)/m]$
K_{xy}	$203.823392957 \times 10^6[N/m]$
K_{yx}	$85.474326105 \times 10^6[N/m]$
C_{xy}	$0.2429925136 \times 10^6[(N-s)/m]$
C_{yx}	$0.219477109 \times 10^6[(N-s)/m]$

Bearings

The bearings used in these fans are hydrodynamic and can be represented by four stiffness and four damping coefficients. These coefficients are usually modeled as four spring-damper systems placed at a separation of 45^o (Jorgen *et al.*, 1965; Čorović and Miljavec, 2020; Xie *et al.*, 2023), as shown in Figure 8.

Based on Sommerfeld numbers, and assuming that the system is in resonance when the natural and the operating frequencies through the angular velocity are similar, the eight stiffness and damping constants to be introduced into the software for simulation are defined in Table 3. The following is also considered:

- Each bearing carries 9394.849 kg due to the weight of the rotor.
- The radial clearance of the bearing is c = 0.0014 m.
- The critical angular velocity of the system is also considered.

Based on a Sommerfeld number of 0.9, and assuming that the first resonance is the most pronounced, the critical speed is 0.9 (890 rpm) = 801 rpm (83.88 rad/s). The value of 890 rpm is the system's operating speed and corresponds to a frequency of 14.8333 Hz. The value of 0,9 was chosen instead of 1 to prevent the system's natural and operating frequencies from being equal, thus only approaching resonance.

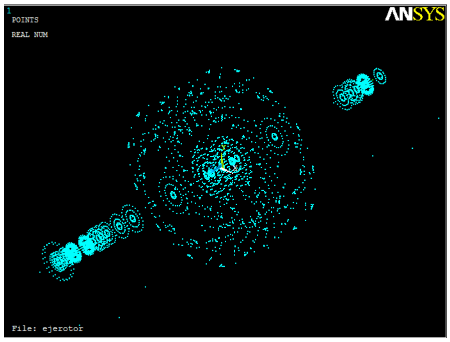


Figure 9. Key points to be connected with the master node
Source: Authors

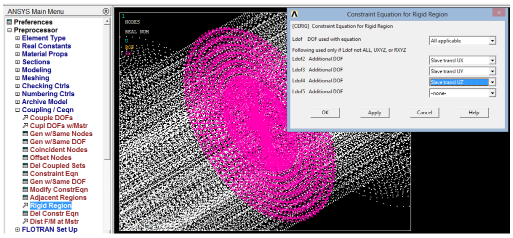


Figure 10. Association of slave nodes with the master node
Source: Authors

Preparing the performance simulation

The simulation was carried out in the ANSYS software. The analysis was performed based on several factors, *i.e.*, for two materials, considering increases in diameter and the accuracy of bearings. The materials defined for the rotor shaft were 1026 steel (E=205GPa) and 4340 steel (E=215GPa), both with a Young modulus of 0.29. The MASS21 element assigned mass values to key points and consequently provided the values of the loads applied to each bearing. These key points were used as master nodes. For the real constant of the MASS21 element, the approximate value of the bearing weight was introduced.

To simulate the spring-damper pairs, the COMBIN14 element must be assigned to assume the values in Table 3 for the eight different values of the stiffness and damping constants.

Key points were created in the part of the fan where the bearings are mounted. These key points were then meshed as a mass element and used as a master node with which all slave nodes were associated. Attributes were assigned to the previously created key points, the element type (MASS21), and its real constant. This yielded a mesh, which implies a mass element. The key points were created to be connected with lines to the mass element (master node) that simulates the springs and dampers through the COMBIN14 element. This process can be observed in Figure 9. Here, the key points are outside the rotor.

Afterwards, the master node was associated with the slave nodes. The master node was the MASS21 element, and the slave nodes were all those around it, as shown in Figure 10.

As depicted in Figure 11, lines were created to represent the spring-damper system. Each of the lines starts from the master node (MASS21 element) and goes towards the different key points that were created earlier.

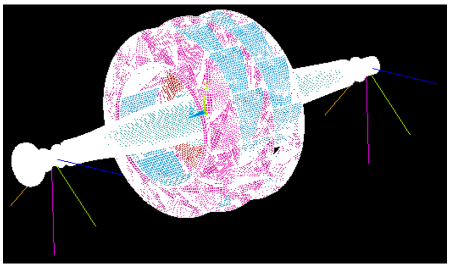


Figure 11. Lines to simulate each spring-damper assembly
Source: Authors

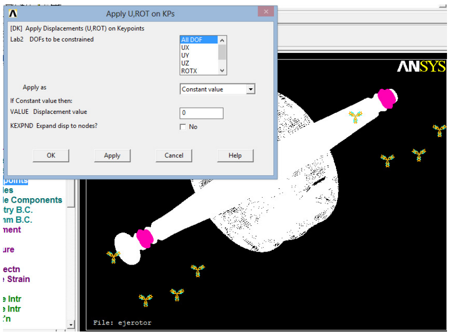


Figure 12. Restriction of displacements in the spring-damper system
Source: Authors

To assign attributes to the previously created lines created, these steps were followed:

1. The element type (COMBIN14) was given.
2. The corresponding real constant for each line was assigned.
3. Meshing was performed on the lines.

These steps aimed to simulate the operation of the spring-damper system. To avoid unwanted displacement, the following actions were performed (Figure 12):

1. The key points connected to the elements of COMBIN14 were restricted in all directions.
2. The master node was restricted in the direction of z.

The finished model can be observed in Figure 13, which is sufficient for the modal analysis. This analysis is very important in the design and development of a fan to prevent premature failure (Manish *et al.*, 2015; Čorović and Miljavec, 2020), as it helps to obtain the vibration modes of the system.

Results

The results presented herein start with the data obtained from static and dynamic experimental tests using instrumentation. Following this, the modeling and meshing of the fan parts are detailed, leading to the ultimate stage of computer simulation.

Dynamic test

Measurements were obtained for the induced-draft fan 1A. It was possible to measure the vibration in both bearings for

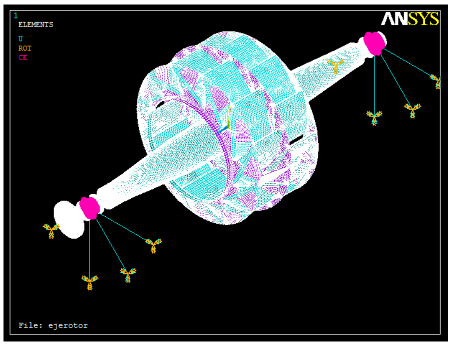


Figure 13. Completed model prior to modal analysis
Source: Authors

the connection closest to the motor (inner bearing) and the opposite or furthest from the motor (rear bearing) (Figure 4). Figures 14 and 15 show the vibration corresponding to the horizontal and vertical directions. Despite the presence of some noise, both figures exhibit a harmonic behavior. This consistent behavior allowed eliminating any possibility of damage to the bearings.

Figures 16 and 17 show the integral of the acceleration curves, which clearly shows the harmonic behavior of the cycles, eliminating the noise as well.

Figures 18 and 19 show the Fourier spectrum and the integration of the spectrum of the rotating behavior of the bearing in the horizontal and vertical directions. The maximum peak occurs at approximately 15 Hz for both cases.

The vibration values in the horizontal direction exceed those obtained in the vertical direction. This discrepancy arises from the lower rigidity in the horizontal direction, in contrast to the vertical direction, due to the presence of vertical supports. This behavior has been experimentally confirmed.

Static impact test

Figure 20 shows the results obtained from the impact test conducted on the out-of-operation fan 1B. The top part shows the vibration amplitude, and the bottom part shows the Fourier spectrum indicating the natural frequencies. As seen in this test, values of 15 and 20 Hz were obtained for this particular configuration. It is important to emphasize that the unit was out of service: the bearings were not operating, and there was no mechanical connection between the electric motor and the fan rotor. The results show that the natural frequencies of the rotor are very close to its operating frequency, as already validated with the accelerometers mounted during the operation of the other fan.

Simulation without bearings

Through simulation, it is possible to determine the different vibration modes of a fan. These modes are fundamental to determining primary conditions such as angular velocity or system design, which are essential for a correct operation. The simulation of the system is based on the real operating conditions of the fan, meaning that the model is close to reality. Figure 21 shows the frequencies of the first vibration mode of the rotor for two different materials.

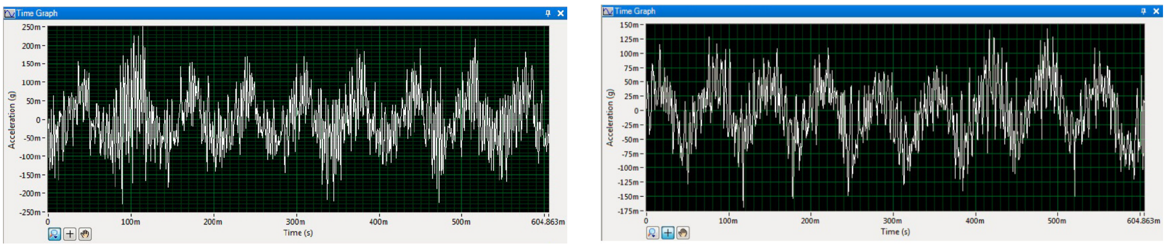


Figure 14. Vibration obtained in the horizontal direction of the inner bearing, left; and rear bearing, right. Induced-draft fan 1A.

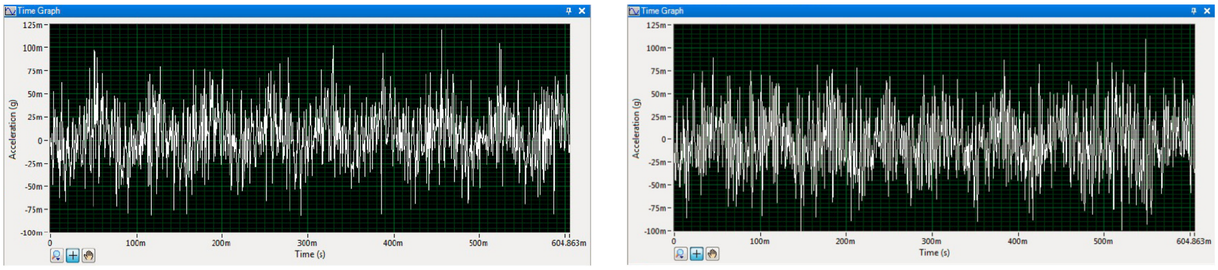


Figure 15. Vibration obtained in the vertical direction of the inner bearing, left; and rear bearing, right. Induced-draft fan 1A.
Source: Authors

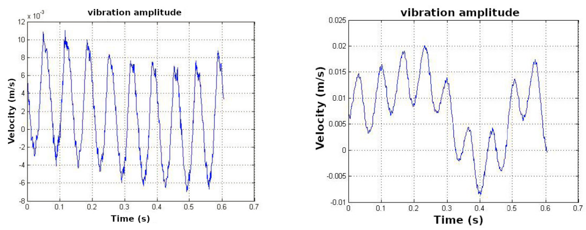


Figure 16. Integral of the vibration obtained in the horizontal direction of the inner bearing, left; and rear bearing, right. Induced-draft fan 1A.
Source: Authors

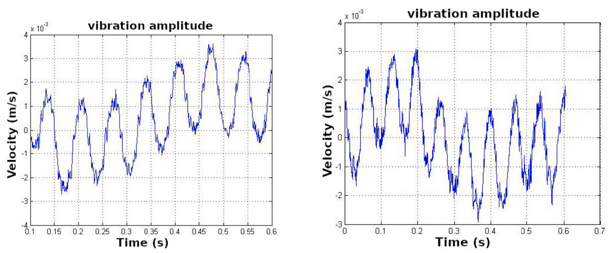


Figure 17. Integral of the vibration obtained in the vertical direction of the inner bearing, left; and rear bearing, right. Induced-draft fan 1A.
Source: Authors

According to the results shown in Figure 21, the system's natural frequency is sensitive to the material. Another factor that we consider fundamental is the rotor diameter. In this vein, to increase the cross-section, analyses were conducted in order to evaluate the impact on the natural frequency. The results are shown in Table 4.

According to the results regarding the change in the radius of the inner shaft, by increasing the stiffness of the shaft, it is possible to increase the natural frequencies of the system's vibration modes. It should be noted that the longitudinal dimension cannot be modified. It was also necessary to analyze the system's vibration modes. Figures 21 and 22 show the first to the fourth mode. These modes were included to rule out their similarity to the functional frequency of the system.

Table 4. Natural frequency of the undamped system according to the radial increment of the shaft and the two materials used

Radial increment [m]	Material	Natural frequency [Hz]
0.01		19,302
0.02		20.009
0.03	1026 steel	20.834
0.04		21.343
0.05		25.254
0.01		19.671
0.02		20.392
0.03	4340 steel	21.111
0.04		21.923
0.05		25.945

Table 5. Results of the two main simulations

First mode of vibration without bearings	First mode of vibration with bearings
18.559 Hz	15.3622 Hz

Simulation with bearings

Figure 23 shows the vibration mode obtained with the original characteristics of the system, including the bearings. It can be observed that the natural frequency (15.3622 Hz) of the system is very close to its excitation or operating frequency (890 rpm, 14.8333 Hz), which likely indicates the reason for excessive vibration in the system.

Table 5 compares the results obtained from the two initial simulations of the system in its first mode of vibration, represented in Figures 21 and 23.

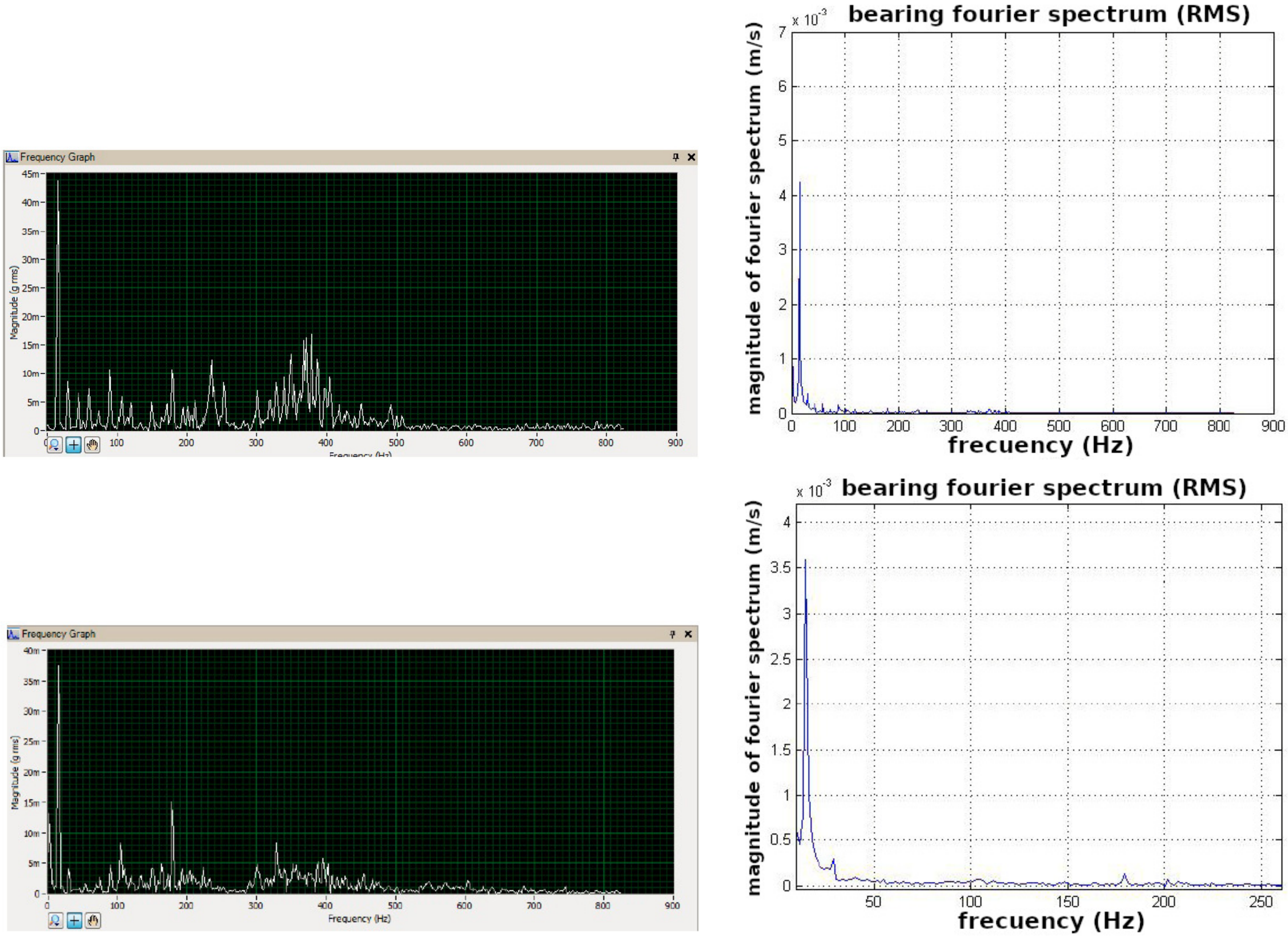


Figure 18. Fourier spectrum and integral of the Fourier effect in the horizontal direction of the inner bearing, top; rear bearing, below. Induced-draft fan 1A.
Source: Authors

Discussion

The first natural frequency is 15.3622 Hz. This frequency was obtained from experimental tests, and it was validated using computer simulation (Table 5). The operating frequency (14.833 Hz), obtained based on the impeller speed (890 rpm), is close to the system's natural frequency. Excessive vibration occurs due to the convergence of these values toward mechanical resonance.

Modifying the rotation speed of the fan rotor in order to move it away from its natural frequency values, with the purpose of preventing resonance, was not possible due to two main factors: a) the airflow and the efficiency of the system are altered and no longer meet the functionality requirements; and b), according to Qingjie *et al.* (2020), it is not possible to eliminate the torsional vibration generated in this type of systems when adding a variable frequency drive. Inertial factors generate this torsional vibration due to speed changes in induced draft fans, and the torsional vibration is in turn caused by frequency conversion driving technology due to dynamic electromechanical coupling. Several alternatives to eliminate the torsional vibration of the shaft were considered in (Qingjie *et al.*, 2020), finding that it is only possible to displace its resonance region and reduce its amplitude but not eliminate it.

M and K (2023) propose a MATLAB-based methodology to prevent machine downtime. This methodology consists of continuously monitoring the rotor-bearing system with regard to vibration responses obtained through data collection and the fast Fourier transform (FFT) analyzer. This work, however, only proposes the detection of the fault and makes some suggestions. In our work, conducting vibration monitoring in the rotor-bearing system was imperative to identifying the primary vibration modes and establishing their similarity to the operating frequency. This understanding was crucial to define an effective solution. To facilitate our vibration analysis, we utilized the MATLAB software and integrated the Fourier spectrum of the bearing's rotating behavior in both the horizontal and vertical directions.

Numerous research projects such as that of Manish *et al.* (2015) have employed the FEM for the modal analysis of rotating industrial equipment. Some works, beyond relying solely on computational approaches, also incorporate physical models to validate their findings, as demonstrated by Čorović and Miljavec (2020) and Noureddine and Noureddine (2022). In Manish *et al.* (2015), a comprehensive analysis of the vibration modes of a centrifugal fan was undertaken, focusing on the acquisition of the first ten vibration modes. This extensive examination aimed to validate that these modes did not closely align with the fan's operating speed or frequency, thus avoiding vibration issues. The authors of this study emphasized the crucial nature of such analyses for rotating machinery,

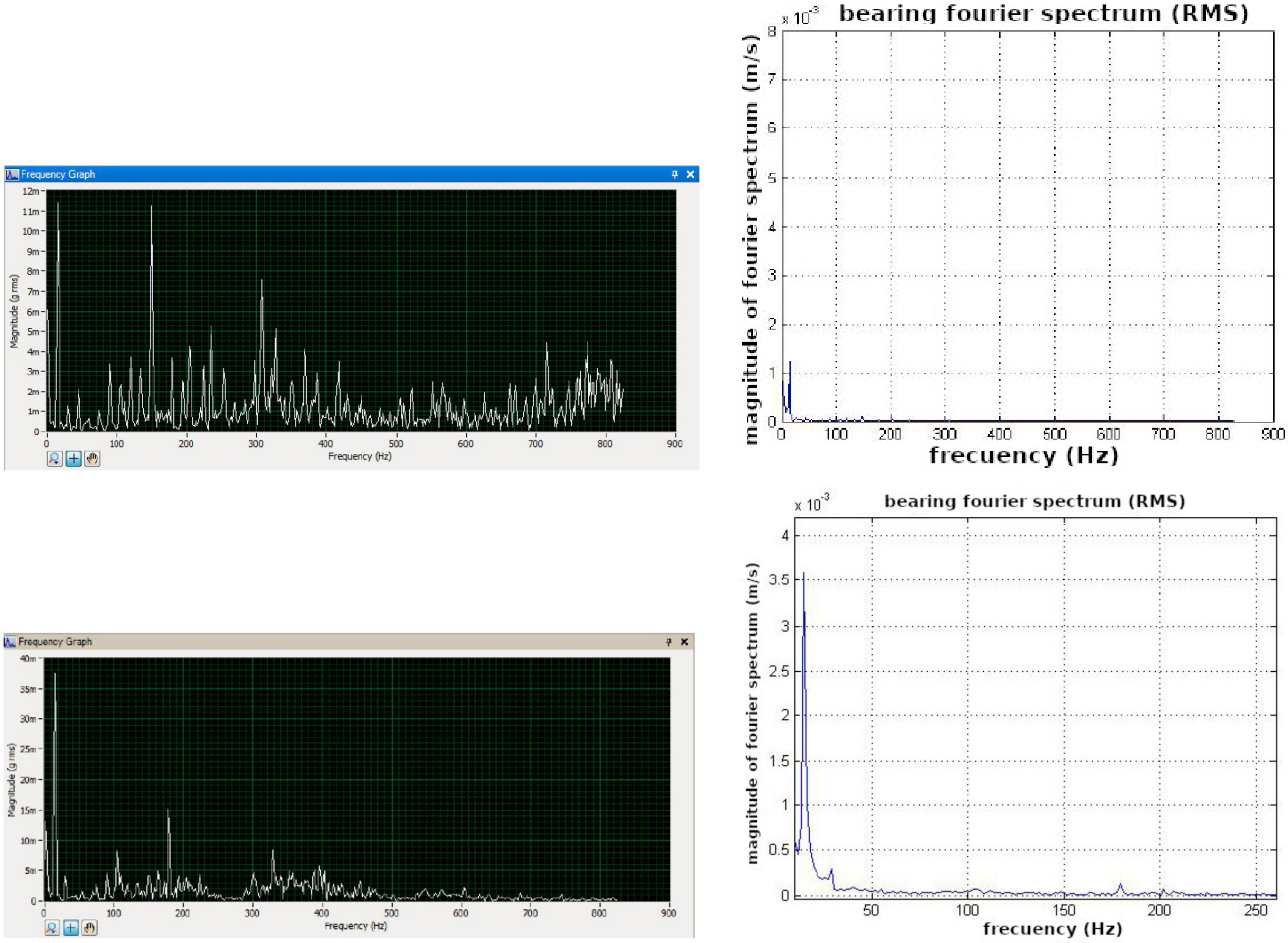


Figure 19. Fourier spectrum and integral of the Fourier effect in the vertical direction of the inner bearing, top; rear bearing, below. Induced-draft fan 1A.
Source: Authors

stressing its indispensability in predicting design outcomes and enhancing performance. In the context of our project, we conducted a similar analysis, acquiring data on the first six vibration modes. This document presents findings up to the fourth mode (Figure 22). Our study revealed a proximity between the first natural vibration frequency and the operating frequency of the induced-draft fan. This correlation provides enough evidence to attribute the excessive vibration issues to the alignment of the first natural vibration frequency with the fan's operating frequency. Based on our results, in the subsequent vibration modes, the values move away from their operating frequency, as in Manish *et al.* (2015), Čorović and Miljavec (2020), and Noureddine and Noureddine (2022). These modes increase their numerical value as shown in Figures 21 and 22.

In the study conducted by Trebuna *et al.* (2014), a methodology similar to ours was developed. Their approach involved finite element analysis, strategically placing accelerometers in both rotor bearings, and conducting static impact tests. These measures were implemented in order to scrutinize excessive vibration in two air extraction fans. As in our case, both fans were of the same model, resulting in the same behavior. One distinctive aspect of the work by Trebuna *et al.* (2014) was its focus on air extraction fans with mixed-flow characteristics, combining both flow directions (axial and radial). In this vein, three accelerometers were required for instrumentation: one of the accelerometers was placed in the axial direction of the rotor, while, for the horizontal and vertical directions, two accelerometers were used to assess the radial flow effects. Given that the air extraction fans operated within a speed range of 400-1 000 rpm, modal and experimental analyses were conducted at distinct frequency values in orded to capture the nuances of their behavior.

Similarly, Tarek *et al.* (2018) employed accelerometers positioned in all three directions. In an effort to rectify fan behavior issues, their maintenance department opted for a comprehensive replacement of the rotor, encompassing the shaft, fan, and roller bearings. In contrast, in our project, we placed the accelerometers in the two necessary directions: horizontal and vertical. This approach was prompted by the radial flow characteristics of the induced-draft fan under study. Through our experimental and modal analyses, we identified a distinctive pattern: the fan exhibited an operating frequency closely aligned with the first natural frequency of the system. Consequently, we recommend a design modification strategy centered around altering the rotor's diameter, including the replacement of the fan rotor itself.

Our methodology was shaped by a combination of factors, including an assessment of ineffective previous approaches, our team's extensive experience, and a thorough analysis

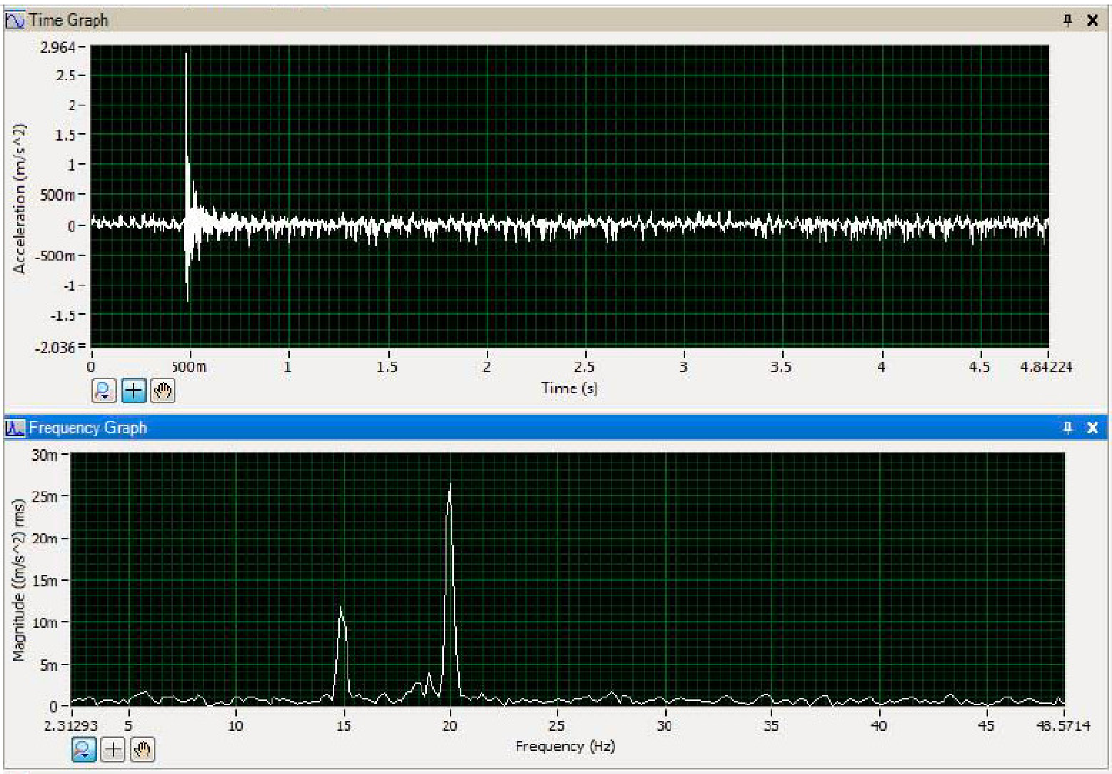
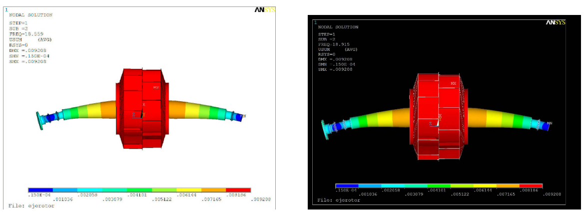


Figure 20. Results of the impact test. 1B induced-draft fan.
Source: Authors



(a) (b)

Figure 21. First mode of vibration of the original model without bearings: (a) natural frequency of 18.559 Hz, 1026 steel; (b) natural frequency of 18.915 Hz, 4340 steel
Source: Authors

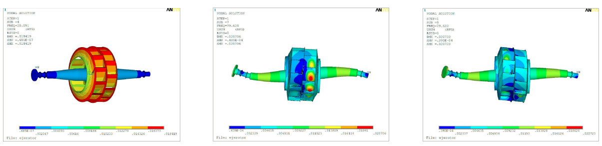


Figure 22. Main vibration modes of the fan rotor with a 1026 steel central shaft and original diameter dimensions. Left: second mode of vibration, with a frequency of 20.091 Hz. Center: third mode of vibration, with a frequency of 79.628 Hz. Right: fourth mode of vibration, with a frequency of 79.632 Hz.
Source: Authors

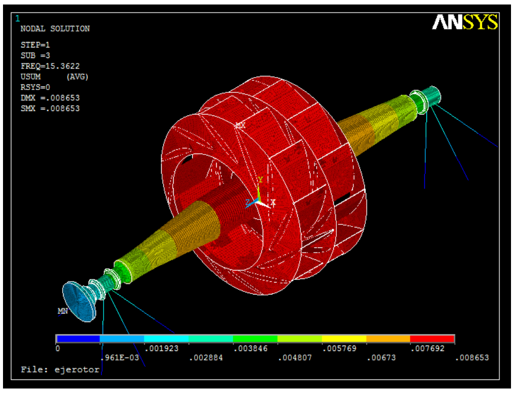


Figure 23. First mode of vibration of the original model with bearings, with a natural frequency obtained of 15.3622 Hz.
Source: Authors

of the induced-draft fan's performance. We also considered and explored alternative solutions to tackle the studied issue.

We proposed a solution based on instrumentation and computer simulation to address the excessive vibration issue, which had not been considered by companies in the public and private sectors and company maintenance personnel. The root cause of the problem was the need for more analysis and communication between the fan manufacturer and the company's engineering and maintenance area, as they were operating in areas close to the fan's resonance, *i.e.*, they failed to consider that the first natural frequency of the system would be close to its operating frequency in the plant.

To support our proposal, Table 4 should be considered, as it shows that an increase in the rotor's diameter raises the system's natural frequency, causing it to move away from the operating frequency or speed. This finding aligns with turbomachinery principles, as it is advisable for the natural frequencies to exceed the operating one.

Conclusions

In this work, mechanical vibratory analysis was conducted to address the issue of excessive vibrations in two induced-draft fans at a thermal power plant in Mexico. Excessive vibrations occurred suddenly and frequently, causing continuous shutdowns to balance the rotors, thus generating economic losses.

The research methodology comprised four key processes: a) analyzing previous solutions to the problem, b) conducting a literature review encompassing background and standards, c) performing experiments by instrumenting accelerometers at crucial points in the fans with the purpose of determining their natural frequencies (a crucial component for subsequent vibration analysis), and d) validating the natural frequencies through modeling and computer simulation. This validation allowed defining a solution to mitigate the problem. It is worth adding that the computer simulation was based on finite element analysis.

Based on the data obtained from both experimentation and simulation, the following was observed:

- The first natural frequency of the system is closely aligned with the operating frequency of the induced-draft fans. Consequently, the system is close to mechanical resonance, resulting in excessive vibration.
- After reviewing the available literature and considering the essential operation of the fans under analysis, implementing a variable frequency drive to adjust the operating speed was not considered a viable option.

In light of the results, it is imperative to modify the fans' geometry. This is necessary because the first natural frequency is close to the operating frequency or impeller speed. In this context, the most practical approach would involve modifying the shaft's cross-section to one with a larger diameter. Although the expenses associated with this solution may be substantial, it remains a necessary measure to rectify the initial design flaws and address the ongoing issues.

Future work could leverage real-time monitoring, computer modeling, and simulation tools, along with the integration of deep learning methodologies. These approaches are poised to offer crucial insights into the behavior of systems, aiding in identifying potential faults and developing corresponding solutions. In this case study, establishing a database lays a foundation for applying deep neural network learning in forthcoming projects. Another improvement would be enhancing the accuracy of sensitivity measurements, similarly to the methods outlined in this study, which could be further advanced through techniques such as speckle interferometry (Dhiya *et al.*, 2023; Jesús *et al.*, 2024).

Author contributions

Erick-Alejandro González-Barbosa conceived the idea and conducted preliminary research. José Juan Vázquez Martínez conducted further research. Fernando Jurado-Pérez oversaw the evaluation process and provided research supervision. Héctor Castro Mosqueda contributed to the methodology, software development, and original draft writing. Francisco Javier Rodríguez Ornelas conducted the literature review, formal analysis, and data collection processes. José-Joel González-Barbosa assisted with data collection and supervision, and he offered valuable feedback. All authors contributed to the manuscript and approved its final version for publication.

Conflicts of interest

The authors declare that they have no conflict of interest

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