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Inter-Laboratory Testing Program for the Physical Characterization of Guamo Sand

Programa inter-laboratorio de ensayos para la caracterización física de la arena del Guamo

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ABSTRACT

In soil testing, assessing physical properties is essential for accurately characterizing sands. However, testing results can vary depending on the experimental procedures used and their implementation. A round-robin exercise facilitates the simultaneous analysis of the reproducibility and replicability of the standard methods used to characterize the properties of a specific material. This paper presents the outcomes of the first inter-laboratory testing initiative (i.e., a round-robin exercise) aimed at assessing the results variability of the physical characterization of a sandy soil. Guamo sand, widely utilized in local research and engineering projects in Colombia, was the focus of this study. 11 national academic laboratories participated in the program, conducting seven replicates of grain size distribution, solids specific gravity, and maximum and minimum void ratio tests. The data provided by all participants were analyzed and interpreted using statistical techniques. The results revealed significant differences between the data collected for each physical property, which can be attributed to the intrinsic variability of this sand's natural origin and to the use of diverse testing procedures. These comparisons offer valuable practical insights into the discrepancies between the testing methodologies employed by the participants for soil characterization, and they constitute a comprehensive database for future research or practical applications.

Keywords: round-robin testing, laboratory tests, standards and codes of practice, statistical analysis, sands

RESUMEN

En los ensayos de suelos, la evaluación de las propiedades físicas es esencial para caracterizar arenas con precisión. Sin embargo, los resultados de los ensayos pueden variar según los procedimientos experimentales utilizados y su implementación. Un ejercicio tipo *round-robin* facilita el análisis simultáneo de la reproducibilidad y la replicabilidad de los procedimientos estándar utilizados para caracterizar las propiedades de un material específico. Este artículo presenta los resultados de la primera iniciativa de ensayos inter-laboratorios (i.e., un ejercicio *round-robin*) cuyo objetivo fue evaluar la variabilidad de los resultados en la caracterización física de un suelo arenoso. La arena de Guamo, ampliamente utilizada en proyectos de investigación e ingeniería en Colombia, fue el foco de este estudio. 11 laboratorios académicos nacionales participaron en el programa, realizando siete réplicas de ensayos de distribución de tamaño de grano, gravedad específica de sólidos y de relación de vacíos máxima y mínima. Los datos proporcionados por todos los participantes fueron analizados e interpretados utilizando técnicas estadísticas. Los resultados revelaron diferencias significativas entre los datos recopilados para cada parámetro físico, atribuibles a la variabilidad intrínseca del origen natural de esta arena y a la utilización de diversos procedimientos de ensayo. Estas comparaciones ofrecen valiosas perspectivas prácticas sobre las discrepancias entre las metodologías de prueba empleadas por los participantes para la caracterización del suelo y constituyen una base de datos integral para futuras investigaciones o aplicaciones prácticas.

Palabras clave: ensayos *round-robin*, ensayos de laboratorio, estándares y normativa de ensayos, análisis estadístico, arenas

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Introduction

Round-robin testing (RRT) programs involve collaborative testing conducted by multiple specialized laboratories. The objectives of RRT include assessing the reproducibility and replicability of a particular test or procedure, reviewing new experimental processes, and validating standards for certification or updating purposes in engineering practices. In addition, RRT programs allow assessing the variability of a testing method and improving the characterization of materials and products. Typically, these programs involve various entities, notably universities and research institutes that are specialized or recognized in specific disciplines. RRT execution is the primary strategy to evaluate and identify differences between procedures and results. In addition, the advantage of RRT lies in conducting measurements under the same experimental conditions to determine the properties controlling the behavior of the same type of material [1]. Thus, many collaborations focus on specific tests or parameters – physical, chemical, or mechanical – as shown by [2]. Furthermore, cooperative research and comparison may lead to developing or modifying classical experimental methods or proposing new test procedures [3].

RRTs are organized by one of the participating laboratories or groups responsible for coordinating activities and compiling results. Slight variations in any empirical processes often result in poor reproducibility estimates of collaborative studies [4]. Hence, if critical parameters are identified during RRT, pilot tests should be conducted with specialized setups within the same laboratory. These preliminary tests enable the examination of methods' susceptibility to minor changes in system conditions before formal collaboration [5]. Consequently, collaborative programs between research centers aimed at identifying the accuracy and precision levels of soil analytical methods are becoming increasingly common [6].

In some cases, a set of randomly selected samples is analyzed using an unsystematic sampling method to obtain statistically homogeneous data during RRT development. However, this technique is only applied to data meeting the assumption of variance homogeneity [7]. For soils, which are heterogeneous materials due to their geological formation process, efforts should be made to establish uniform test protocols to ensure consistent characterization procedures. Representative data are employed, and statistical methods are used to determine significant differences between the test procedures or protocols evaluated in the RRT.

Accurate characterization of soil properties is crucial for both practical engineering applications and academic research. However, the inherent variability of natural soils, coupled with differences in experimental procedures and their implementation, often leads to discrepancies in test results. These inconsistencies can undermine the reliability of soil characterizations, particularly when the results are used as benchmarks for construction projects or research studies. In geotechnical engineering, several RRTs have evaluated the behavior of different soils using various procedures. These RRTs include assessing the liquefaction resistance of Japanese sands through cyclic triaxial tests [8]; the stress-strain behavior of Toyoura sand, Fujinomori remolded clay, and soft sedimentary rock [9]; the shear wave velocity of Toyoura sand, measured while employing bender elements [10]; the measurement and suction control of sand, kaolin, and bentonite mixtures [11]; the critical state line of Coimbra sand [12]; and the critical state line of a gold mine tailings by triaxial compression testing [13]. These RRTs have allowed drawing conclusions regarding the differences between the procedures used by participating laboratories. Additionally, they have contributed to identifying factors inducing bias in the tests and increasing variability between the measured properties in the studied soils.

Guamo sand is a reference sand in Colombia, locally akin to Toyoura and Ottawa sands, that is frequently used to calibrate constitutive models for geotechnical research [14], [15], [16]. This sand is sourced from alluvial deposits in the department of Tolima. Given its alluvial origin, the geotechnical properties of this sand are expected to be dependent on sampling location. However, the inherent variability and statistical metrics of Guamo sand's geotechnical properties have not yet been extensively studied. The objective of this paper is to quantify statistical measures related to the inherent variability of the geotechnical properties of Guamo sand.

As previously noted, there has been a notable absence of RRT programs specifically focused on assessing the physical properties of sands. Addressing this gap, the Geotechnical Research Group of Universidad Militar Nueva Granada (Colombia) initiated the first inter-laboratory testing program to evaluate the reproducibility and accuracy of measurements of key physical properties of a representative national sandy soil, *i.e.*, Guamo sand. This material, extensively used in Colombia for research and practical applications, was selected due to its importance. The physical properties of Guamo sand examined in this study include grain size distribution, specific gravity of solids, and soil packing, as indicated by the maximum and minimum void ratios.

This article presents the outcomes of this pioneering inter-laboratory testing program, conducted as a round-robin exercise, with the following objectives: (i) to outline the organization and execution of the testing program for evaluating the reproducibility and accuracy of measurements of physical properties of sands; (ii) to compile and present the experimental results from the 11 participating laboratories; and (iii) to quantitatively analyze the differences between the datasets using statistical methods. The findings herein offer valuable insights into the discrepancies between different testing procedures. In addition, the comprehensive database established through this collaborative testing program can serve as a useful resource for validating the physical properties of Guamo sand in future research or practical applications. This initiative underscores the importance of collaborative efforts in advancing the understanding of soil behavior and refining testing methodologies to accurately characterize soil properties.

Materials and methods

Description of Guamo sand

Guamo sand originates in the Luisa River in the department of Tolima, Colombia, and it is characterized as a sandy soil of alluvial origin. This particular sand is extensively used in Colombia for practical and research applications due to its widespread commercial availability and consistent gradation [17], [18]. The practical applications of Guamo sand include its use in assessing soil density and unit weight through the sand-cone method, as outlined in ASTM D1556M-15e1 [19]. Moreover, researchers have used this soil to investigate mechanisms related to the instability of granular soils, as evidenced by [20].

Mineralogically, Guamo sand consists predominantly of quartz, accounting for approximately 99% of its composition [21], [22], with its particles' morphology predominantly characterized as sub-angular to sub-rounded [15], [23], [24]. The friction angle at the critical state (φ'_{cs}) of this soil has been reported by [16] and [25] to range between 31 and 34º. Table I summarizes the physical parameters documented in previous research studies on Guamo sand. These index properties include the coefficient of curvature (Cc), the coefficient of uniformity (Cu), the average particle size (d_{50}), the effective size (d_{10}), the specific gravity of solids (Gs), the maximum void ratio (e_{max}), and the minimum void ratio (e_{min}). The values presented in this table indicate variability among the reported physical parameters, emphasizing the importance of accurate characterization or establishing a reliable database that contains the properties of this soil.

Organization and methodology of the inter-laboratory testing program

The Geotechnical Research Group of Universidad Militar Nueva Granada (UMNG) launched a collaborative testing program that brought together specialized geotechnical characterization laboratories to estimate and compare the physical parameters of Guamo sand as reported by various institutions. This effort marks the first instance of this work by geotechnical specialists in Colombia. Invitations

Table I. Physical parameters of Guamo sand as reported in the literature

Reference	Cc	Cu	d_{50} (mm)	d_{10} (mm)	Gs	e_{max}	e_{min}
Arias (2006) [26]	-	2.04	0.51	0.11	-	1.00	0.52
Patiño (2006) [27]	-	0.65	0.56	0.16	2.63	0.83	0.50
Gómez (2010) [28]	-	-	0.43	0.07	2.66	1.00	0.52
Jiménez (2011) [25]	1.21	2.73	0.54	0.14	2.62	0.92	0.55
Camacho-Tauta *et al.* (2014) [29]	1.35	1.67	0.55	0.75	2.70	0.95	0.77
Bermúdez and Ruiz (2015) [23]	1.05	2.44	0.55	0.15	2.70	1.00	0.59
Tique (2016) [15]	0.63	3.05	0.54	0.24	2.64	0.97	0.60
Dulcey-Leal *et al.* (2018) [17]	0.69	1.76	0.49	0.19	2.70	0.90	0.63
Molina-Gómez *et al.* (2019) [21]	1.29	1.23	0.50	0.20	2.70	0.93	0.57
Ruge *et al.* (2019) [22]	0.93	1.34	0.43	0.23	2.70	0.90	0.58
Ramos-Cañon *et al.* (2022) [14]	2.44	1.05	0.53	0.21	2.71	1.02	0.59

Source: Authors

were extended to academic and commercial laboratories renowned for their expertise in soil mechanics and geotechnical engineering. While 11 academic laboratories from Colombian universities were accepted based on established criteria, unfortunately, no commercial laboratory responded positively to the invitation. All the data were compiled and statistically assessed by the RRT coordinators.

The coordinating group requested reporting seven replicates for each estimated parameter to assess the repeatability and variability of each test and reduce the probability of type I and II errors associated with random sampling of the replicates analyzed [4], [7]. However, no specific test method was set; on the contrary, it was indicated that each participant was free to choose the test method (standardized or non-standardized) that they deemed relevant to measure each of the aforementioned physical parameters. In addition, it was indicated that each laboratory was responsible for acquiring the material for the collaborative testing program. All laboratories, except Uniandes, UCC, and Unal-B (Table II), used material from the same supplier. For confidentiality and commercial reasons, the names of the material suppliers are not disclosed in this document and are represented by the numbers 1 and 2. Fig. 1 summarizes the methodology of the inter-laboratory testing program.

Table II presents the laboratories that participated in the Guamo sand characterization RRT program and the methods implemented by them to measure physical parameters. The physical characterization of Guamo sand involved an experimental program focused on measuring grain size distribution (GSD) curves (Fig. 1), Gs, e_{max}, and e_{min}. These parameters were selected because they influence the macro-mechanical behavior of sandy soils, as observed by [30], [31], [32], [33], [34], [35], [36], and [37].

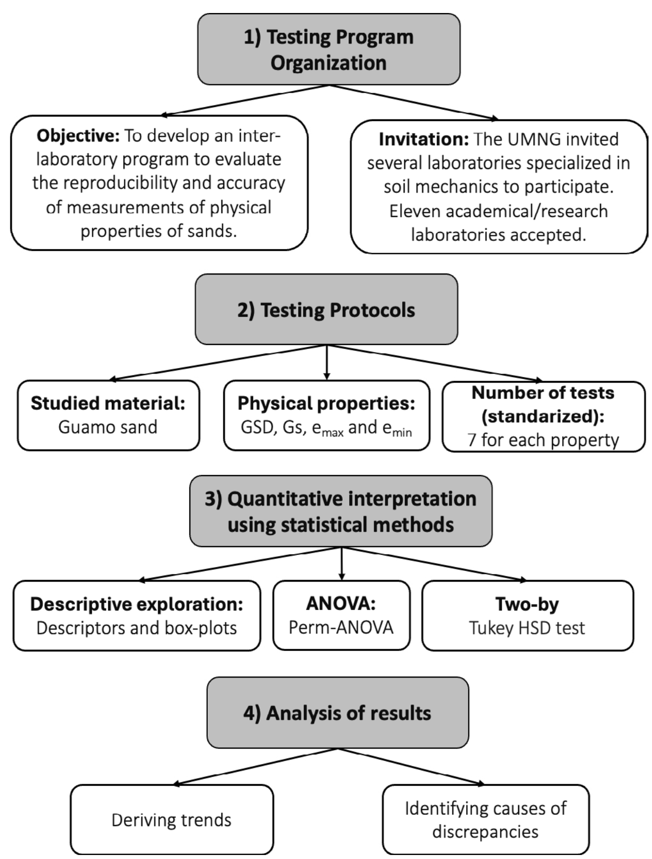


Figure 1. *Methodology of the interlaboratory testing program.*

Source: *Authors*

Table II. Data on participating laboratories and summary of test methods

Laboratory/University (Acronym)	Location	Material supplier	GSD	Gs	e_{max} / e_{min}
Universidad Militar Nueva Granada (UMNG)	Bogotá DC	1	123-13[a]	128-13[e]	136-13[h]
Pontificia Universidad Javeriana (PUJ-B)	Bogotá DC	1	123-13[a]	128-13[e]	136-13[h]
Universidad de los Andes (Uniandes)	Bogotá DC	2	78-95[b]	128-13[e]	92-95[i]
Pontificia Universidad Javeriana (PUJ-C)	Cali	1	D6913[c]	D854-14[f]	136-13[h]
Universidad Distrital Francisco José de Caldas (UDFJC)	Bogotá DC	1	123-07[d]	128-07[g]	136-07[j]
Universidad Industrial de Santander and Universidad del Magdalena	Bucaramanga/Santa Marta	1	123-13[a]	128-13[e]	136-13[h]
Universidad Nacional de Colombia (Unal-B)	Bogotá DC	2	123-13[a]	128-13[e]	136-13[h]
Universidad Católica de Colombia (UCC)	Bogotá DC	2	123-13[a]	128-13[e]	136-13[h]
Universidad Piloto de Colombia (UPC)	Bogotá DC	1	123-13[a]	128-13[e]	136-13[h]
Universidad Nacional de Colombia (Unal-Mz)	Manizales	1	123-13[a]	128-13[e]	136-13[h]
Universidad Francisco de Paula Santander (UFPS)	Cúcuta	1	123-13[a]	128-13[e]	136-13[h]

[a] INVIAS (2013). Determinación de los tamaños de las partículas de los suelos INV E–123–13.
[b] ICONTEC (1995). Método para determinar por lavado el material que pasa el tamiz 0.075 mm en agregados minerales NTC 78.
[c] ASTM International (2021). Standard test methods for soil particle-size distribution (gradation) using sieve analysis ASTM D 6913.
[d] INVIAS (2007). Determinación de los tamaños de las partículas de los suelos INV E–123–07.
[e] INVIAS (2013). Determinación de la gravedad específica de las partículas sólidas de los suelos y del llenante mineral, empleando un picnómetro con agua INV E-128-13.
[f] ASTM International (2014). Standard test methods for specific gravity of soil solids by water pycnometer ASTM D854.
[g] INVIAS (2007). Determinación de la gravedad específica de las partículas sólidas de los suelos y del llenante mineral, empleando un picnómetro con agua INV E-123-07.
[h] INVIAS (2013). Determinación de las masas unitarias máxima y mínima para el cálculo de la densidad relativa INV E-136-13.
[i] ICONTEC (1995). Determinación de la masa unitaria y los vacíos entre partículas de agregados NTC 92.
[h] INVIAS (2007). Determinación de las masas unitarias máxima y mínima para el cálculo de la densidad relativa INV E-136-07.
Source: Authors

Results and discussion

Grain size distribution

Fig. 2 presents the GSD curves obtained during the collaborative testing program. A visual inspection of the figure indicates no significant differences between GSD curves, suggesting that the material supplier does not influence these outcomes. However, a visual analysis shows that the results provided by Unal-Mz and UIS-UM have the highest dispersion among all the datasets (Figs. 2j and 2f, respectively). In contrast, the GSD curves from Uniandes, Unal-B, and UMNG show the slightest variations, reflecting higher consistency in their testing procedures (Figs. 2c, 2g, and 2a). Further analysis of the differences in GSD regarding the coefficients of curvature (Cc) and uniformity (Cu) is presented below. Cc and Cu were calculated using Eqs. (1) and (2), respectively:

$$C_u = \frac{d_{60}}{d_{10}} \quad (1)$$

$$C_c = \frac{d_{30}^2}{d_{60}\ d_{10}} \quad (2)$$

where d_{10}, $d_{30,}$ and d_{60} correspond to the particle sizes associated with 10, 30, and 60% of soil sample passing, respectively.

The results for Cc and Cu indicate that all Guamo sand samples can be classified as poor-graded sand (SP) according to the Unified Soil Classification System. This classification is based on the quantitative criterion that Cu should be less than 6, and Cc should not fall within the range of 1 to 3 [38].

Descriptive exploration

The statistical analysis of the RRT results included a descriptive exploration phase for each physical parameter, which was estimated using several observations (n = 77). The descriptors used were the mean value ($\overline{X}$), the standard deviation (s), the coefficient of variation (COV), the median ($\hat{X}$), the maximum value ($X_{(1)}$), the minimum value (X_n), the range of values ($X_n - X_{(1)}$), symmetry ($\hat{k}_1$), kurtosis ($\hat{k}_2$), and the standard error (ϵ). Table III presents the descriptive exploration results for Cc, Cu, Gs, e_{max}, and e_{min}, as reported in the collaborative testing program.

The COV was calculated separately to measure the dispersion variation of each property. The COV values in Table III could be utilized to prioritize the testing of each property and help to meticulously select geotechnical investigation scopes, allowing to perform further tests [39], [40]. These values reveal that the e_{min} and Gs results exhibit the highest and lowest variability, respectively. The 22% variability in the e_{min} results stems from the differences between the methods and procedures chosen for characterizing this physical parameter. The data highlight the need for improvements to testing procedures to assess e_{min}, where variability is more pronounced. In contrast, the low variability observed in Gs can be attributed to the stable mineralogical composition of Guamo sand, predominantly comprising silica minerals. This minimal variation suggests that, despite the diverse testing environments and techniques used, the inherent material properties of the sand remain consistently measurable, reinforcing the robustness of this parameter for geotechnical evaluations.

Furthermore, the low variation in the Gs results is due to the fact that 90% of the laboratories used the same standard procedure. The coefficients $\hat{k}_1$ and $\hat{k}_2$ indicate that the groups of the evaluated physical parameters follow asymmetric trends. For example, Cc, Cu, and Gs are rightward asymmetric due to positive $\hat{k}_1$ results, while e_{max} and e_{min} are leftward asymmetric due to negative $\hat{k}_1$ results. In addition, the results for $\hat{k}_2$ showed that the distributions of Cc, e_{max}, and e_{min} are leptokurtic, and that the distributions of Cu and Gs are platykurtic. The values of $\hat{k}_1$ and $\hat{k}_2$ suggest that, while some parameters exhibit deviations from normality, others indicate a broader spread in the data, which could impact the overall interpretation of the results. These tendencies provide preliminary insights into the normality of the data (in general, normally distributed data are symmetric), which will be further discussed below.

The descriptive exploration of the results also involved a graphical comparison using boxplots. Fig. 3 presents the boxplots of the physical Guamo sand parameters assessed. In the data presented therein, significant differences between the results reported by the participants are observed. Moreover, the boxplots identify outlier differences (or *outliers*) in all datasets. The outliers vary between participants and intra-parameters, which precludes confirmation of whether the test procedures were correctly applied in the physical characterization of the Guamo sand, as observed in the statistical analysis of granular materials by [39]. These outliers may indicate potential inconsistencies in the experimental procedures – or variations in standard implementation. Hence, more appropriate statistical methods will be implemented in the following sections, in order to quantify the differences between the results and subsequently group them. This approach will help to ensure that the physical properties of Guamo sand are characterized more accurately and consistently.

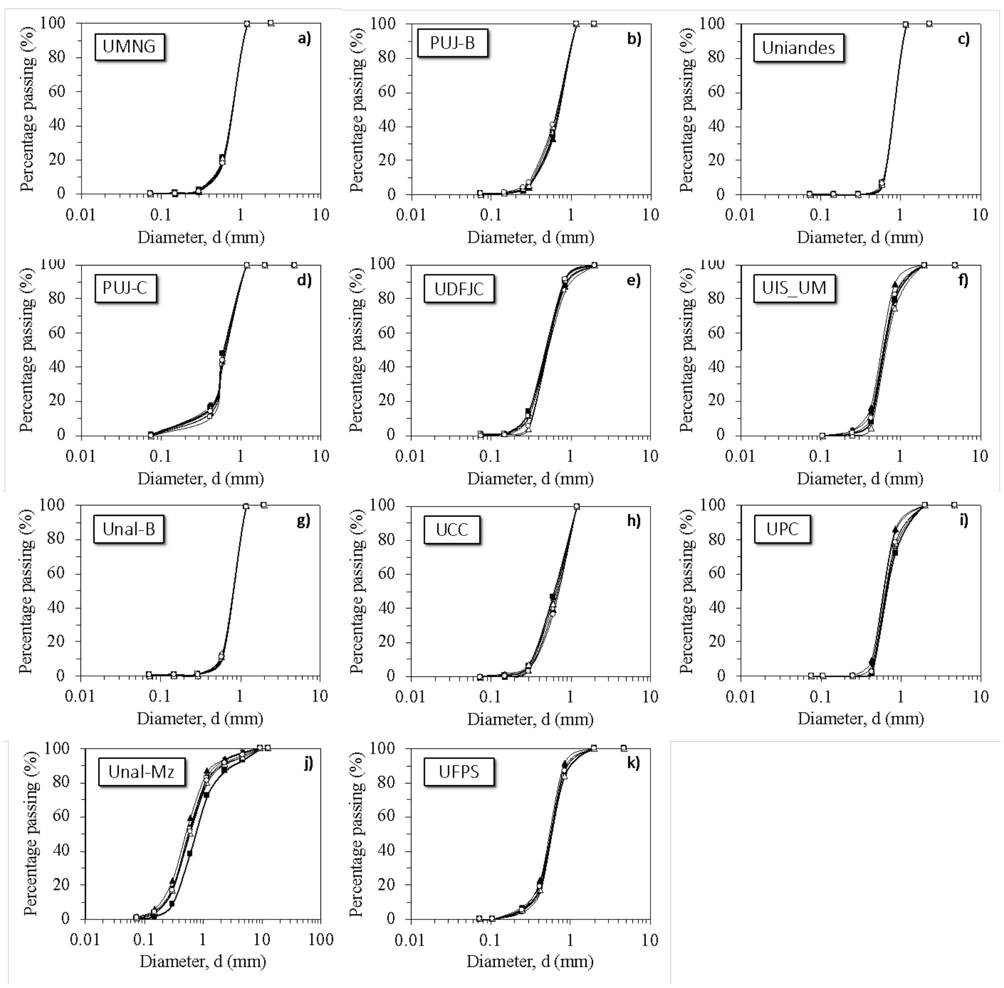
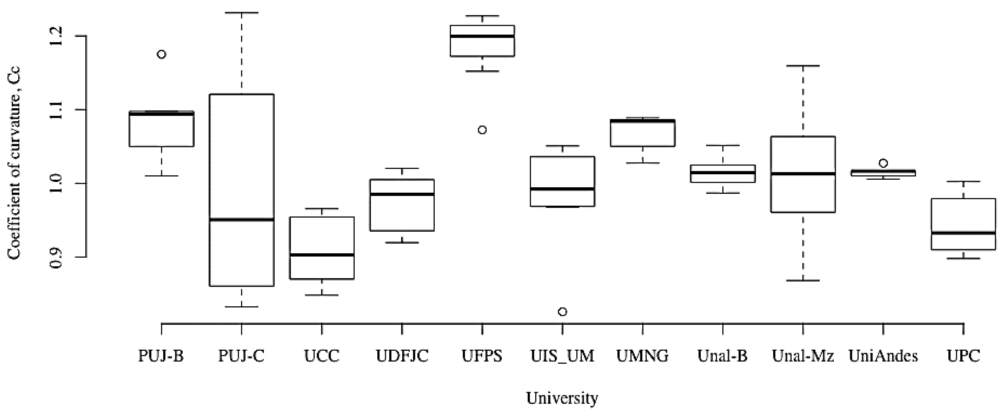


Figure 2. GSDs of Guamo sand as reported by the participants: a) UMNG, b) PUJ-B, c) Uniandes, d) PUJ-C, e) UDFJC, f) UIS_UM, g) Unal-B, h) UCC, i) UPC, j) Unal-Mz, and k) UFPS
Source: Authors

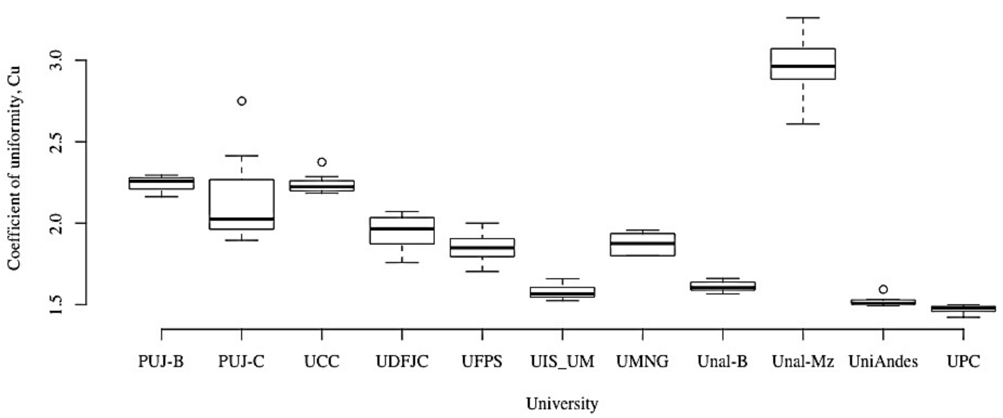
Table III. Descriptors of the physical parameters of Guamo sand

Variable	n	$\overline{x}$	s	COV	$\hat{x}$	$x_{(1)}$	x_n	$x_n - x_{(1)}$	$\hat{k}_1$	$\hat{k}_2$	ϵ
Cc	77	1.02	0.10	0.10	1.01	0.83	1.23	0.41	0.26	-0.26	0.01
Cu	77	1.95	0.34	0.17	1.88	1.42	3.26	1.84	1.09	0.67	0.05
Gs	77	2.69	0.07	0.03	2.68	2.58	2.88	0.3	1.12	0.89	0.01
e_{max}	77	0.91	0.06	0.07	0.90	0.77	1.04	0.27	-0.28	-0.01	0.01
e_{min}	77	0.63	0.14	0.22	0.66	0.35	0.81	0.46	-0.51	-1.09	0.02

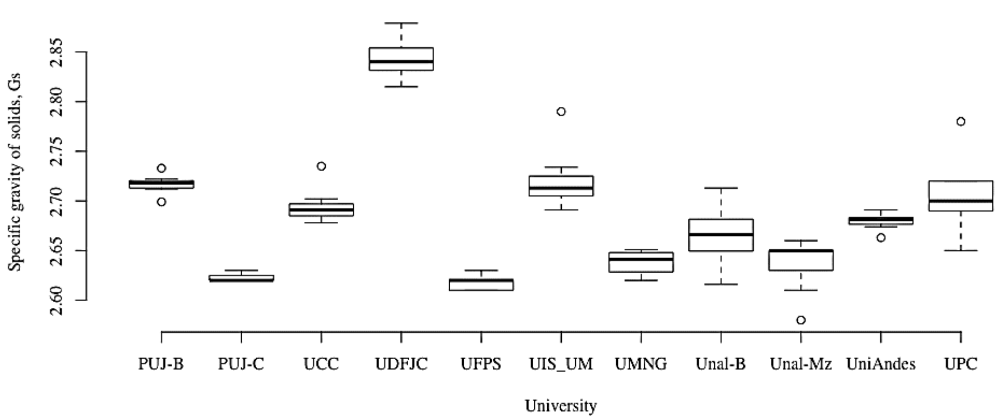
Source: Authors



(a)



(b)



(c)

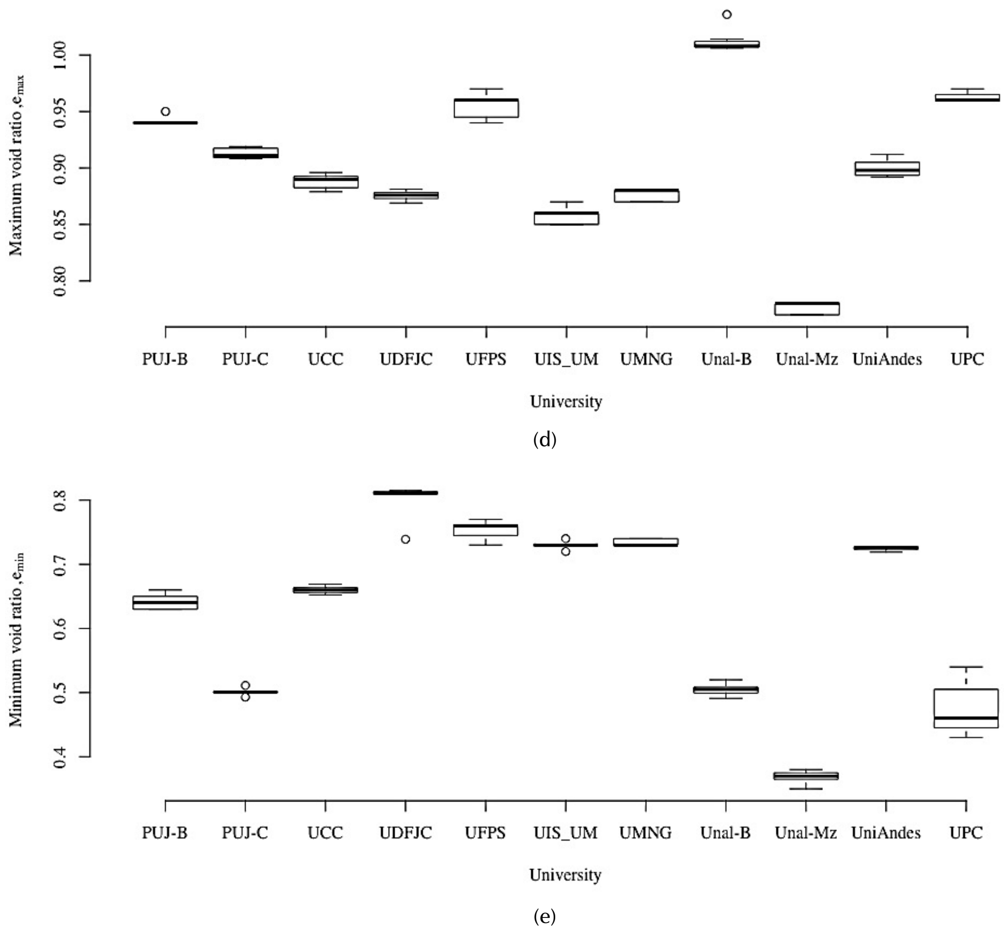


Figure 3. Boxplots for the values of a) the coefficient of curvature, b) the coefficient of uniformity, c) the solids specific gravity, d) the maximum void ratio, and e) the minimum void ratio.
Source: Authors

ANOVA analysis

All participants interpreted their data using RStudio [41], a freely available integrated tool for statistical analysis. A univariate analysis of variance (ANOVA) was employed to compare the results reported by the participants, aiming to assess the statistical equality between the population means of each set of variables (μ) while utilizing the hypothesis (H_0) described in Eq. (3).

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_n \tag{3}$$

The alternative hypothesis (H_a) in Eq. (4) allowed determining whether there were different means in at least one pair of datasets (or results provided by the participants):

$$H_a : \mu_1 \neq \mu_2 \neq \mu_3 \neq \ldots \neq \mu_n \tag{4}$$

In the collaborative testing program, a g = 11 group of data was established, with the same number of replicates or observations (n = 7), resulting in a total of N = 77 observations for each physical parameter. Based on this configuration, it was assumed that the data for each physical parameter of Guamo sand could be statistically analyzed under the criterion of a balanced experiment, as established by Woodward's design experiments [42]. Eq. (5) describes the response of the ANOVA model when comparing the observed values ($i = 1, 2, 3, \ldots, n$) for the datasets reported by each participant ($j = 1, 2, 3, \ldots, g$) with regard to each physical parameter studied.

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \tag{5}$$

where μ is the mean of all random observations, τ_i represents the group effect, and ϵ_{ij} corresponds to the model error or residuals. The validation of H_0 involved calculating the mean of each set of values reported by each participant ($\overline{y}_{i.}$) as well as the mean of the total of all values ($\overline{y}_{..}$). This was done by using Eqs. (6) and (7) [43]:

$$\overline{y}_{i.} = \frac{1}{n}\sum_{j=1}^{n}(y_{i.}) \tag{6}$$

$$\bar{y}_{..} = \frac{1}{N}\sum_{i=1}^{g}\sum_{j=1}^{n}(y_{ij}) \quad (7)$$

where $y_{i.}$ represents the sum of the data for each dataset, and y_{ij} is the value of each individual observation. In addition, using Eqs. (8) and (9), the ANOVA procedure performed herein considered the degrees of freedom for the results reported by the participants (df_p) and those of the model residuals (df_e).

$$df_p = (g-1) = (11-1) \quad (8)$$

$$df_e = g(n-1) = 11(7-1) \quad (9)$$

The sums of squares and mean squares of each dataset (SS_p, and MS_p) were obtained using Eqs. (10) and (11), while those of the residuals (SS_e and MS_e) were calculated via Eqs. (12) and (13).

$$SS_p = n\sum_{i=1}^{g}(\bar{y}_{i.} - \bar{y}_{..})^2 \quad (10)$$

$$MS_p = \frac{SS_p}{N-1} \quad (11)$$

$$SS_e = \sum_{i=1}^{g}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2 - n\sum_{i=1}^{g}(\bar{y}_{i.} - \bar{y}_{..})^2 \quad (12)$$

$$MS_e = \frac{SS_e}{n-N} \quad (13)$$

Then, F_0 (representing the F-value of the F-distribution) was calculated to evaluate H_0 through Eq. (14).

$$F_0 = \frac{MS_p}{MS_e} \quad (14)$$

The last step of the ANOVA involved calculating the upper percentile of the distribution (F_{df_p, df_e}), which is equivalent to F_α. In this study, a significance level of 95% was assumed (*i.e.*, $1-\alpha=0.95$) for all statistical analyses. Therefore, if $F_0 > F_\alpha$ or *p*-value < 0.05, H_0 was rejected, as reported by Hassan *et al.* [44]. Table IV presents the ANOVA results for the physical parameters of Guamo sand.

Table IV. ANOVA results

Data group	Degrees of freedom	Sum of squares	Mean square	F value	*p*-value
Cc	10	0.38	0.36	7.65	5.6×10^{-8}
Residuals of Cc	66	0.33	0.0049	–	–
Cu	10	13.46	171.53	80.61	2.0×10^{-16}
Residuals of Cu	66	1,.1	0.0002	–	–
Gs	10	0.29	0.03	55.72	2.0×10^{-16}
Residuals of Gs	66	0.03	0.0002	–	–
e_{max}	10	0.28	0.03	555	2.0×10^{-16}
Residuals of e_{max}	66	0.01	0.0001	–	–
e_{min}	10	1.40	0.14	474	2.0×10^{-16}
Residuals of e_{min}	66	0.02	0.0002	–	–

Source: Authors

The findings in Table IV suggest notable differences between all data groups in relation to the physical parameters of Guamo sand, However, assessing and validating the assumptions of normality and variance homogeneity of the ANOVA residuals before affirming this conclusion is essential. This study evaluated said assumptions through the Shapiro-Wilk and Levene tests. The results are presented in Tables V and VI.

Table V. Normality test results

Data group	Shapiro-Wilk	p-value
Cc	0.94	0.0014
Cu	0.87	8.46×10^{-7}
Gs	0.92	0.0002
e_{max}	0.96	0.0111
e_{min}	0.68	4.62×10^{-7}

Source: Authors

Table VI. Results obtained in the variance homogeneity test

Data group	Degrees of freedom	F value	p-value
Cc	10	4.27	0.0001
Cu	10	2.65	0.0088
Gs	10	1.48	0.1674
e_{max}	10	0.76	0.6654
e_{min}	10	3.05	0.0031

Source: Authors

The Shapiro-Wilk and Levene tests revealed that not all datasets met the normality and variance homogeneity criteria. Therefore, in this study, a permutational analysis of variance (perm-ANOVA) was employed to validate the differences between the results obtained for the physical parameters. The perm-ANOVA is a non-parametric technique that does not require validating the normality and homogeneity of variance [45]. It addresses data randomization through permutation in order to rearrange the data Np times under the H_0 hypothesis of an ANOVA [46]. In other words, the perm-ANOVA allows calculating the number of times that $F_0 > F_\alpha$ by randomly rearranging the data Np times. This technique can be applied in experiments with relatively few observations [47]. [48] used a perm-ANOVA to compare parameters of gradation curves in granular materials, demonstrating its applicability to soils. In this study, the perm-ANOVA was applied with Np = 5000 – as recommended by [49] – using the *RVAideMemoire* package [50]. Table VII presents the results.

Table VII. Perm-ANOVA results

Data group	Degrees of freedom	Sum of squares	Mean square	F value	*p*-value
Cc	10	0.38	0.36	7.65	0.0019
Residuals of Cc	66	0.33	0.0001	–	–
Cu	10	13.46	171.53	80.61	0.0005
Residuals of Cu	66	1.11	0.0002	–	–
Gs	10	0.29	0.03	55.72	0.0009
Residuals of Gs	66	0.03	0.0002	–	–
e_{max}	10	0.28	0.03	555	0.0042
Residuals of e_{max}	66	0.01	0.0001	–	–
e_{min}	10	1.40	0.14	474	0.0031
Residuals of e_{min}	66	0.02	0.0002	–	–

Source: Authors

The results in Table VII confirm the significant differences between the results obtained by the participants for the physical properties of Guamo sand (*p*-value<0.05). This provides clear statistical evidence that the mean values of the physical parameters differ significantly between participants. Such discrepancies highlight the need for improved standardization and uniformity in testing methodologies, in order to ensure more consistent and comparable results in future studies.

One limitation of ANOVA methods is their inability to pinpoint which treatments have distinct means (in this context, the treatments correspond to the participants) [43]. Thereupon, a multiple comparisons analysis was conducted to determine which means deviated from the others concerning each physical parameter. This analysis employed the HSD (honestly significant difference) test introduced by [51]. This test computes the overall mean of all the data and juxtaposes the confidence intervals of the data reported in pairs. Essentially, it compares the mean for each pair of participants. These intervals follow a skewed range distribution (q), as estimated through Eq. (15).

$$q = \frac{\overline{y}_{max} - \overline{y}_{min}}{S\sqrt{2/n}} \tag{15}$$

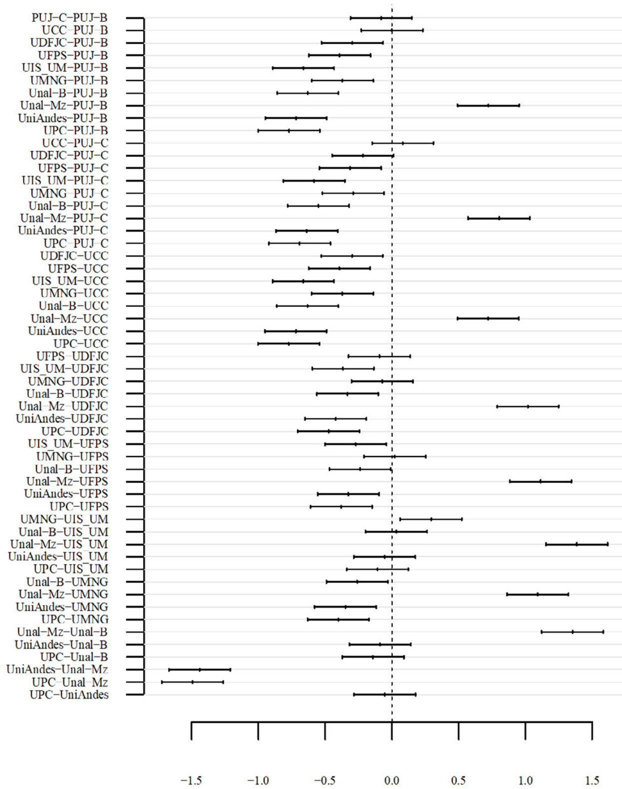
In this equation, $\overline{y}_{max} - \overline{y}_{min}$ is the difference between means, and S corresponds to the variance. Eq. (16) outlines the calculation of the confidence limits used in the Tukey HSD test for a significance level of 1– α (currently 95%).

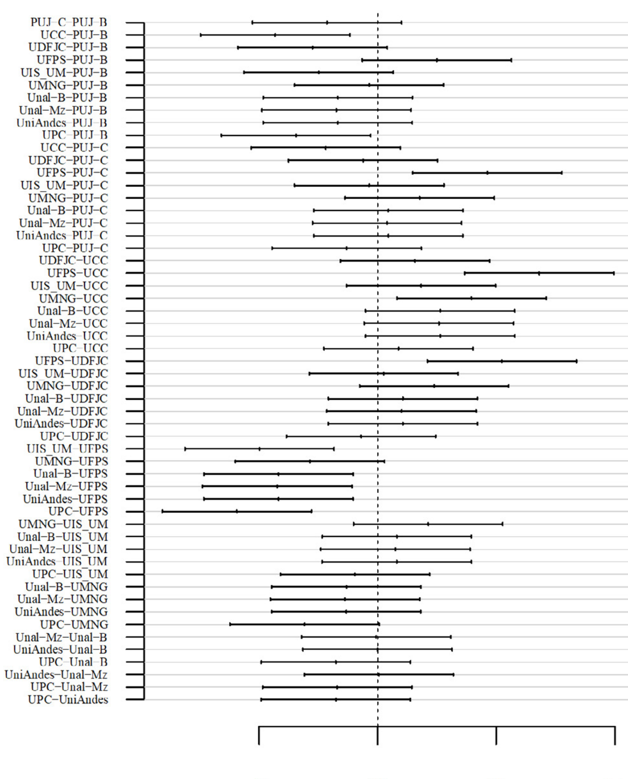
$$\overline{y}_i - \overline{y}_j \pm \frac{q_{\alpha;k;N-k}}{\sqrt{2}} S\sqrt{\frac{2}{n}} \quad for \quad i, j = 1,\ldots,k \quad i \neq j. \tag{16}$$

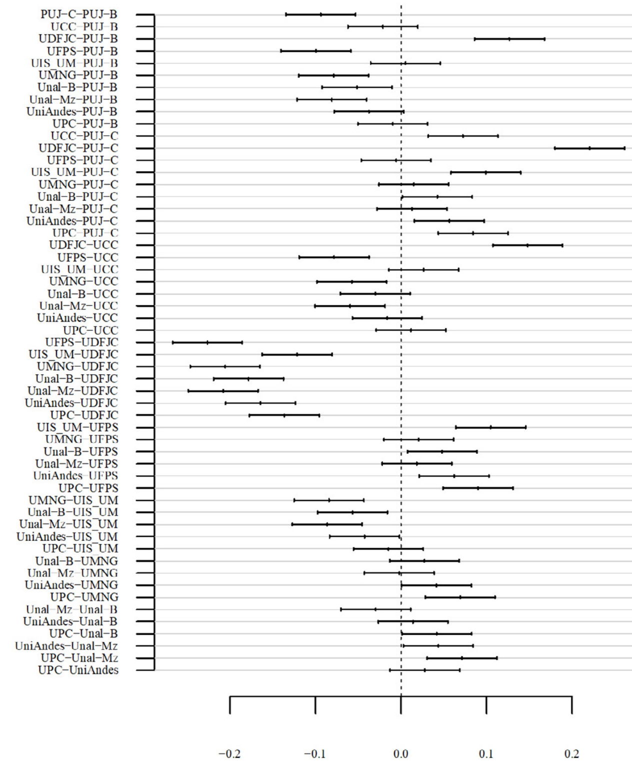
Thus, comparing the confidence intervals allows identifying the participants whose reported values exhibit statistically significant differences. Fig. 4 presents the confidence intervals obtained from the Tukey HSD test for all physical parameters of Guamo sand. Interpreting these results involves approximating the confidence intervals to a common point (zero), making it easier to visually assess which participants' results deviate significantly from the others. When a confidence interval does not overlap with these zero points, it indicates a statistically significant difference from the mean values of the other participants. This visual approach is crucial for highlighting variations in testing methods or material handling across laboratories and underscores the need for standardization to minimize discrepancies, as detailed below.

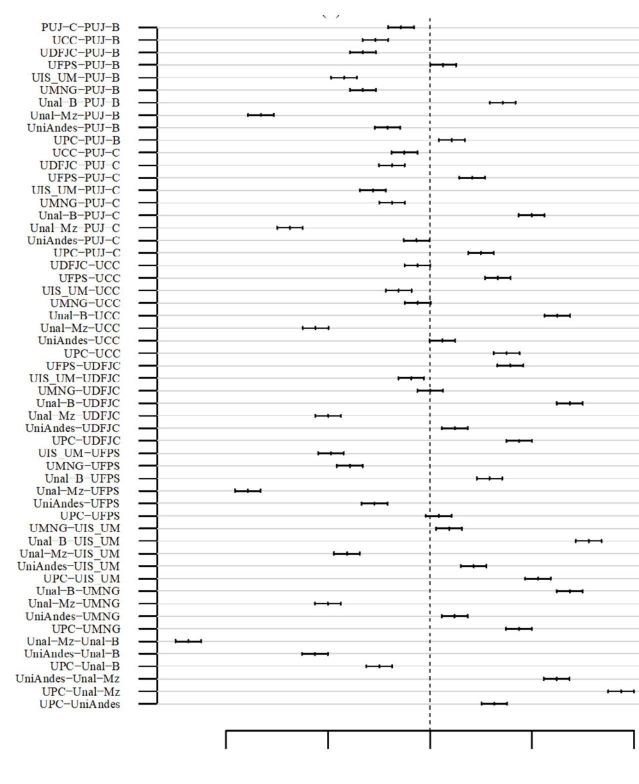
The multiple comparisons performed using the confidence intervals from the Tukey HDS test indicate that the Cu and Gs datasets show the lowest differences among the RRT participants. It was observed that, for Cu, the results of UFPS differed significantly from those of the other participants. In contrast, for Gs, differences were noted in the results of Unal-Mz and UDFJC when compared to the other datasets. The low discrepancies between Cc and Cu are attributed to the intrinsic variability of the Guamo sand samples tested. The differences in the Gs results are acceptable and fall into the typical ranges of other literature-reported siliceous sands of alluvial origin, which generally vary from 2.6 to 2.7 [31], [33], [34]. However, the results reported for UDFJC fall outside said ranges, revealing that this participant may have applied the wrong testing procedure. This confirms that the Gs results reported by the other RRT participants match the typical values of different sands, indicating consistent characterization.

On the other hand, the e_{max} and e_{min} datasets reported the most significant divergence, particularly for e_{min}. This effect is primarily attributed to the correct application of test procedures and the differences between the standardized methods for estimating these physical parameters. The likely sources of errors include variations in potential energy (controlled by the height of grain fall) and particle crushing when assessing e_{max} and e_{min}, respectively. Similar results were reported by [52] when comparing e_{max} and e_{min} values for different sands obtained through various test methods. These authors also highlighted the effect of loose sand compaction on e_{max} results, as well as that of particle crushing on e_{min} values. The high variability of the datasets may be due to the above-mentioned factors. However, the intrinsic variability of Guamo sand can be identified in the e_{max} and e_{min} results [30], [31], [37], [53], so the high variability can be attributed to a combination of procedural differences and the inherent characteristics of the sand, emphasizing the need for rigorous testing protocols and an understanding of sandy soils' natural variability.









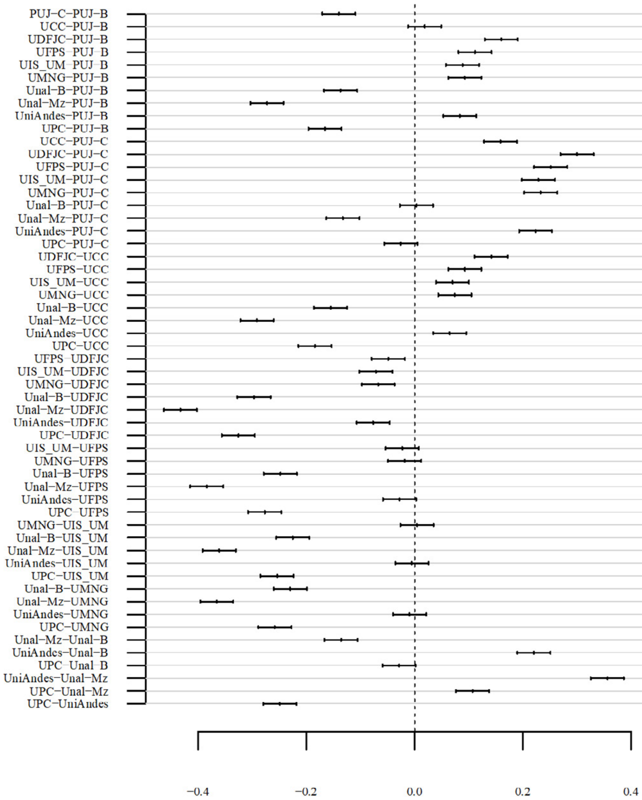


Figure 4. Confidence intervals from the Tukey HDS test for a) the coefficients of curvature, b) the coefficients of uniformity, c) the solids specific gravity, d) the maximum void ratio, and e) the minimum void ratio
Source: Authors

The results indicate that, when the same standardized procedure is consistently applied across different laboratories, the mean values of the measurements tend to align closely, as shown in Fig. 4d. This can also be demonstrated by the Gs assessment and results, which exhibited the lowest variability. This consistency suggests that the discrepancies observed in other parameters, such as e_{max} and e_{min}, are primarily due to variations in the testing methods rather than the intrinsic properties of the material. In light of this, the authors strongly recommend adopting a unified testing procedure across all laboratories when conducting physical characterizations of materials like Guamo sand. The likelihood of significant discrepancies can be minimized by standardizing methods, particularly in relation to parameters with higher sensitivity to procedural differences, such as e_{max} and e_{min}, leading to more reliable and comparable results across different studies and applications.

The discussion of the results is anchored in the limited references that are most relevant to this study. Although these references do not pertain directly to a joint round-robin exercise, they offer valuable analytical tools because the material under study remains the same. According to a simple descriptive analysis, all the research on Guamo sand shows an acceptable Cc range. Only one outlier is revealed in the study by [14], with a value of 2.44. As for Cu, all the values obtained are similar, around a median of 1.76. In this study, Cu reported a value of 1.95 on average.

Gs is a very convenient parameter because it has a restricted range of values, *i.e.*, generally between 2.55 and 2.85 in soils. Therefore, the possibility of finding outliers in the exploration is much higher. Almost all previous studies recorded a value of 2.70. In this RRT program, a value of 2.70 was reported. This parameter is the most reliable in performing a complete analysis of our results. According to the literature, the maximum and minimum void ratios also show reasonable and physically acceptable values. Only one of the values for e_{min} (0.77) reported by [29] revealed itself as an outlier, as the median took a value of 0.59. In our RRT, the estimated value was 0.63.

It is important to note that this study quantifies the uncertainty in the properties of Guamo sand without separating its inherent and epistemic components. Epistemic uncertainty is implicitly represented in sampling frequency, while inherent uncertainty is related to the alluvial deposit's spatial variability [54]. The contribution of each component to the properties' overall uncertainty is outside the scope of this paper.

Regular inter-laboratory comparisons should be implemented as part of ongoing quality controls. These comparisons would enable the early detection of deviations from standardized procedures and provide opportunities for corrective actions before significant discrepancies occur. Moreover, laboratories should maintain detailed records of the procedures followed, including any deviations or adjustments made during testing. A centralized review process can then be implemented to analyze these records, ensuring that any variations are understood and addressed in subsequent modifications of standard testing procedures for the physical characterization of sands.

Summary and conclusions

The collaborative testing program organized by the Geotechnical Research Group of Universidad Militar Nueva Granada was considered successful. The participants were able to estimate the values of the variables involved using the abovementioned procedures, analyzing them through different statistical techniques.

From the experimental data obtained in this collaborative testing program, the following conclusions were drawn:

- The analysis of the physical properties of Guamo sand provided significant insights into its variability and characterization among the participants. The examination of COV values exhibited notable differences, with e_{min} displaying the highest variability due to methodological disparities. Meanwhile, Gs exhibited minimal variability owing to a stable mineralogical composition, which was dominated by silica minerals.

- The perm-ANOVA results showed significant differences in the physical properties reported by the participants, prompting a multiple comparisons analysis through the Tukey HSD test. The study revealed that the Cu and Gs datasets exhibited the lowest participant discrepancies.

- As highlighted by previous studies, the e_{max} and e_{min} datasets exhibited the most significant divergence, which can be attributed to methodological differences and potential errors in test procedures.

Considering that reliability-based geotechnical analysis and design necessitate knowledge of the uncertainty regarding soil properties, the results presented in this paper provide important statistical information that will enable the use of uncertainty propagation methods during the constitutive modeling of Guamo sand.

The findings and conclusions of this collaborative testing program offer valuable insights for future research using Guamo sand and underscore the importance of standardized testing procedures and methodological consistency for geotechnical characterizations.

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Author contributions

Authors 1, 2, and 3: writing (original draft, review, editing). *Authors 1 and 2:* conceptualization, data curation, methodology, supervision, visualization. *Authors 4, 6, 9,10, and 20:* supervision, resources, investigation, writing (review and editing). *The rest of the authors:* resources, investigation.

Conflicts of interest

The authors declare no conflict of interest related to this research.

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