Research Article/Civil and Sanitary Engineering

# Simulation Model for Indirect Tensile Test of Asphalt Mixtures via Discrete Elements

# Modelo de simulación de ensayos de tracción indirecta en mezclas asfálticas mediante elementos discretos

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#### **ABSTRACT**

This paper describes the planning, development, programming, and validation of a computational model based on the discrete element method, which was implemented to simulate, considering specific mechanical parameters in two dimensions, the indirect tension on cylindrical specimens of asphalt mixtures. The proposed software was developed in the Visual Basic.Net programming language, aiming to generate source code and an execution environment that is user-friendly and easy to understand while allowing for improvements or adaptations. The indirect tensile strength test of the analyzed asphalt mixtures was approximated based on compression forces and diametral deformation relationships, according to the regulations of the Colombian National Road Institute. As a complement to this project, the computational model was validated, comparing its simulated results against experimental data on manufactured asphalt materials typically used for the road infrastructure of Colombia's south-west. The simulated values fell within the order of magnitude and trend of the experimental results for the analyzed asphalt material, so it was concluded that the mathematical and computational model can reasonably replicate laboratory data.

Keywords: discrete element method, indirect tensile strength test, asphalt materials, Brazilian test, road infrastructure

#### **RESUMEN**

Este artículo describe la planificación, el desarrollo, la programación y la validación de un modelo computacional basado en el método de elementos discretos, el cual fue implementado para simular, considerando parámetros mecánicos específicos en dos dimensiones, la tensión indirecta en especímenes cilíndricos de mezclas asfálticas. El software propuesto fue desarrollado en el lenguaje de programación Visual Basic.Net, con el propósito de generar un código fuente y un entorno de ejecución que fueran fáciles de usar y comprender, al tiempo que permitieran mejoras o adaptaciones. La resistencia a la tracción indirecta de las mezclas asfálticas analizadas se aproximó a partir de fuerzas de compresión y relaciones de deformación diametral, de acuerdo con la normativa del Instituto Nacional de Vías de Colombia. Como complemento de este proyecto, se validó el modelo computacional mediante la comparación de sus resultados simulados con datos experimentales de materiales asfálticos fabricados y típicamente empleados en la infraestructura vial del suroccidente colombiano. Los valores simulados se ubicaron dentro del mismo orden de magnitud y la misma tendencia de los resultados experimentales para el material asfáltico analizado, por lo que se concluyó que el modelo matemático y computacional puede replicar razonablemente los datos de laboratorio.

Palabras clave: método de elementos discretos, ensayo de tracción indirecta, materiales asfálticos, ensayo brasileño, infraestructura vial

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#### Introduction

Extreme climate demands, increased traffic volume and vehicle load, and the need for environmental mitigation in road construction [1] are new technical challenges for engineers regarding road infrastructure projects [2]. Addressing these challenges is important when considering the relationship between road infrastructure communities' socioeconomic development.

Globally, asphalt pavements represent a large percentage of the transport infrastructure built in the world [3]. The traditional analysis and characterization of asphalt mixtures involves long tests, high experimental costs and specialized equipment [4], which may imply some limitations when developing road infrastructure projects. An alternative to this is the development of numerical simulations.

A bibliographic exploration regarding numerical asphalt mixtures simulation showed an important increase in research based on the discrete element method (DEM) over the last 20 years. This is briefly summarized below.

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In 2006, [5] used the DEM to simulate the behavior of a highly idealized bituminous mixture subjected to uniaxial and triaxial compressive creep tests in the PFC3D commercial software. In 2009, [6] studied the behavior of layered rock samples subjected to the Brazilian test using a combination of the finite element method (FEM) and the DEM. In 2011, [7] used the DEM to simulate crack propagation during the direct tensile test in asphalt mixtures. This approach integrated the cohesive zone model and the PFC2D v. 3.1 software. In 2014, [8] employed a micromechanics-based three-dimensional DEM to capture the time-dependent behavior that is usually studied during dynamic shear rheometer analysis. In 2016, [9] sought to obtain reflection fatigue laws in MDC-2type asphalt mixtures by running DEM simulations in the PFC2D. In 2020, [10] used a two-dimensional viscoelastic and heterogeneous DEM to investigate the formation of block cracking in asphalt pavements subjected to thermal loading. In 2021, [11] presented a discrete element model for simulating the compaction process of asphalt mixtures using a gyratory compactor with complex constitutive laws. Finally, in 2025, [12] studied the dynamic response of the interaction between a vehicle and a deteriorated pavement, using the DEM as well as the PFC software.

The above-presented studies aimed to discontinuously simulate (by means of DEM approximations) the mechanical behavior of diverse materials like asphalt mixtures, hydraulic concrete, and rock, in addition to different physical phenomena studied in the field of road and geotechnical engineering.

In more recent studies, [13] used micromechanical modeling via the 3D DEM method to study bituminous materials by representing mineral aggregates with realistic particle shapes. [14] evaluated the fatigue behavior of asphalt mixture cores in DEM modeling using tomography images to build a mesoscopic model in the PFC software. [15] investigated the uniaxial compression and indirect tensile test of asphalt mixtures based on laboratory tests and discrete element simulation in order to study the effect of size on the strength of asphalt mixtures from a microscopic perspective. Finally, [16] conducted a review of the idealized model generation methods, image models, and user-defined models currently used to generate DEM models of asphalt mixtures, together with their constituent contact parameters and main applications.

Moreover, [17] extended the incremental generalized Kelvin contact (GK) model to enable its application in the fracture analysis of asphalt mixtures using the VirtualPM3DLab software. The authors proposed two damage models: brittle and bilinear softening. They found that the proposed calibrated damage models could successfully reproduce the time-dependent behavior, maximum stress, and crack path observed in experiments. Moreover, the proposed GK-bilinear model was the best fit for the known behavior of the analyzed asphalt and mastic mixture under stress.

[18] proposed a cohesive viscoelastic-elastoplastic damage model using the DEM to capture the behavior of asphalt concretes under different loading rates and durations. Validations were performed in the laboratory, including relaxation, creep, and bending tests, demonstrating that the proposed approach overcomes the limitations of the DEM under various loading conditions.

In general, recent studies have shown that the DEM is a thorough and efficient representation of the mechanical behavior and failure processes of real asphalt mixtures. Additionally, the computer simulations carried out in the referenced studies employed commercial software with closed-source code, as is the case with [19], [20], [21], [22], who used PFC3D. In addition, [23] used PFC2D, and [24] employed EDEM v2020.1.

This research aimed to implement a computational model based on the numerical DEM to simulate the indirect tensile strength of dry samples of a two-dimensional (2D) asphalt mixture, with the purpose of preliminarily predicting and evaluating the performance and behavior of such materials under certain service conditions. The analytical simulation conditions of the proposed computational model are consistent with experimental requirements in Colombia, as stipulated in the relevant regulations, i.e., INV E786-13 [25]. For the numerical simulation and the tensile evaluation, the asphaltic material of the cylinders was separated into its main volumetric components. A specific mechanical and rheological behavior was considered regarding its elements and the asphaltic mastic (fine aggregate, filler mineral, and asphaltic cement). The same principles were applied to the numerical representation of the plate meant for transmitting the load to the laboratory sample. The mineral aggregate of the asphalt mixture and the transmission loads were mechanically modeled in an elastic, non-linear manner, while the asphaltic mastic was modeled with a typical viscoelastic behavior.

Likewise, the proposed approach considered a crack model of the asphaltic material, which allowed observing the internal fracture and the appearance and evolution of fissures on the numerical samples, caused by the separation between the different constituent materials bonded on the bituminous material.

The validation of the computer model presented herein employed actual experimental data from indirect tensile test applied to various types of local asphalt mixtures. This was done to evaluate the efficiency of the applied model in providing exact and satisfactory results.

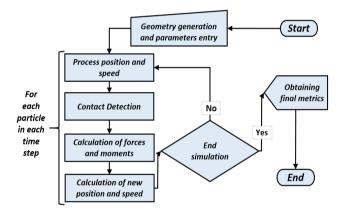
# Methodology

#### Discrete element model

In 1971, Peter A. Cundall developed the distinct element method to simulate mechanical particle system behavior in the context of rock mechanics and masonry wall fracture. Later, in 1979, as an initial approach [26], the method was applied to the modeling of solid and granular bodies through discrete particle assembly in the field of geomechanics. Today, and with the evolution of computers, the method has been improved, acquiring new perspectives that enable the study of a broader range of phenomena and environmental and engineering problems.

In recent years, the DEM has been widely used in road engineering, given its applicability in solving problems of non-continuity and large deformations in road materials, as suggested in [27].

The DEM represents the mechanical behavior of a system made from a group of particles, typically dispersed at random, where each particle moves independently while interacting with others based on proximity. The DEM algorithm is based on three basic processes: i) detection of connected particles, ii) strength and momentum assessment, iii) equations to calculate the speed and location of each particle. Fig. 1 shows the typical flow of the DEM.



**Figure 1.** General flowchart for DEM analysis **Source:** Authors

### Movement equations

The transitional and rotational movement in the simulated DEM is given by the resulting strength and the moment acting upon each particle. This is described by the Newton-Euler equations for solid body dynamics. In 2D models, each particle has three independent degrees of freedom: two translations and one rotation. Thus, any discrete element *i* is simulated according to Newton's laws.

The forces involved are either tension or compression, and those acting on each of the discrete particles within the simulated environment are a result of the interactions with other particles or nearby elements and the conditions governing the model. Therefore, the vectorial form of any discrete element *i* is as follows:

$$F_{i} = \sum_{c=1}^{Nc} F_{i}^{c} + F_{i}^{ext} + F_{i}^{damp}$$
 (1)

$$T_{i} = \sum_{c=1}^{Nc} \left( r_{i}^{c} * F_{i}^{c} + q_{i}^{c} \right) + T_{i}^{ext} + T_{i}^{damp}$$
 (2)

where  $F_i^{\it ext}$  and  $T_i^{\it ext}$  are forces and moments associated with the external load;  $F_i^{\it c}$  denotes the contact strength of nearby interacting elements and other obstacles;  $F_i^{\it damp}$  and  $T_i^{\it damp}$  are the resulting force and momentum of the shockabsorbing system;  $r_i^{\it c}$  is the connecting vector towards the center of the particles of the *i*-th element in contact c;  $N_c$  is the number of particles in contact; and  $q_i^{\it c}$  is the torque from the non-related torsion with the tangential strength (Fig. 2).

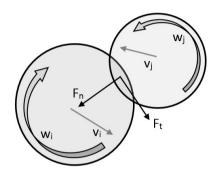
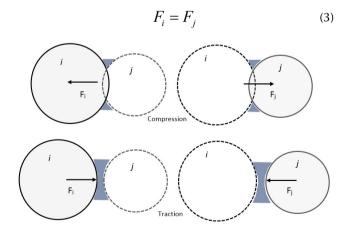


Figure 2. Resulting force breakdown and contact forces in normal and tangential components

Source: Authors

Once discrete element contact is detected, an algorithm calculates the associated forces at the contact point. The interactions between two solids (Fig. 3) for the particles i and j can be represented by the contact forces  $F_i$  and  $F_j$ . According to Newton's third law, this satisfies Eq. (3).



**Figure 3.** Contact between discrete elements **Source:** Authors

#### Integration of the movement equations

For simulation methods based on discontinuity (e.g., DEM), explicit integration is recommended [5], [28], given its low computational cost, ease of logic, and optimal precision.

This implies second-order differential equations that can be numerically included in a time-stepping manner via velocity Verlet integration, *i.e.*, the algorithm selected for our computer model.

At each time step, the state of the particles is updated through Newton's second law, using the aforementioned integration method. With this information, the position and speed conditions of each particle are updated in terms of its current position and velocity, mass, and external forces, regardless of other particles or elements.

The Verlet algorithm's scheme for calculating the integration of time according to the transitional movement of a particle *i* at the *t*-th, is expressed below.

$$v_i \left( t + \frac{1}{2} \Delta t \right) = v(t) + \frac{1}{2} a(t) * \Delta t \tag{4}$$

$$x_{i}(t + \Delta t) = x(t) + v\left(t + \frac{1}{2}\Delta t\right) * \Delta t$$
 (5)

$$a_i \left( t + \Delta t \right) = \frac{F_i}{m_i} \tag{6}$$

$$v_i\left(t+\Delta t\right) = v\left(t+\frac{1}{2}\Delta t\right) + \frac{1}{2}a\left(t+\frac{1}{2}\Delta t\right) *\Delta t \quad (7)$$

where  $X_i$ ,  $V_i$ , and  $A_i$  correspond to the position, speed, and transitional acceleration of the particle under analysis.

Likewise, the scheme for time integration regarding the rotating movement of particle i at the t-th time step is as follows:

$$w_i \left( t + \frac{1}{2} \Delta t \right) = w(t) + \frac{1}{2} \dot{w}(t)^* \Delta t \tag{8}$$

$$\theta_i (t + \Delta t) = \theta(t) + w \left(t + \frac{1}{2} \Delta t\right) * \Delta t$$
 (9)

$$\dot{W}_i \left( t + \Delta t \right) = \frac{T_i}{I_i} \tag{10}$$

$$w_i\left(t + \Delta t\right) = w\left(t + \frac{1}{2}\Delta t\right) + \frac{1}{2}\dot{w}\left(t + \frac{1}{2}\Delta t\right) * \Delta t \quad (11)$$

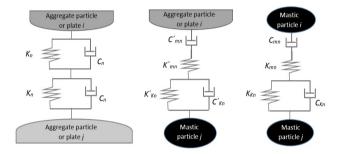
where  $\theta_i$ ,  $w_i$ , and  $\dot{w}_i$  correspond to the orientation angle, angular speed, and angular acceleration of the studied particle's movement.

Mechanical models for the normal and tangential contact of asphalt mixture components

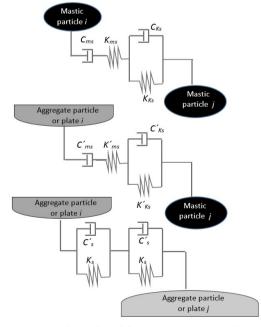
The compression contact between particles, as well as that in the normal and transversal directions, depends on the overlapping particles' typology and is governed by the assumed foundational fracturing of the asphalt mixture. Therefore, if contact compression is generated between two particles of the mineral compound (e.g., gravel or thick sand representing the incongruous phase of the asphalt mixture) or elements representing transmission load plates, both the tangential and normal contact can be considered to be elastic and nonlinear, with damping for energy dissipation. They are mechanically modeled by means of a Maxwell-type element re-fed into a parallel spring and piston [28].

However, if the compression contact between two particles involves discrete elements from the asphaltic mastic (regarded as the continuous phase of the asphalt mixture), the normal and tangential compression contact is considered to be in combined viscoelastic form, thus representing a combined model (*i.e.*, the Burgers model), as suggested in studies [29], [30], and [31].

Figs. 4 and 5 show the meso-mechanic form considered for the possible configurations of normal compression contact between the diversity of particle types into which the asphalt mixture was divided for analysis.



**Figure 4.** Meso-mechanical model representing normal compression contact between the representative particles of the asphalt mixture **Source:** Authors



**Figure 5.** Meso-mechanical model representing tangential compression contact between the representative particles of the asphalt mixture **Source:** Author

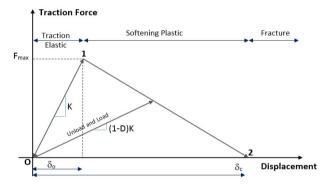
## Constitutive model for simulating material fractures

In this study, the constituent materials considered for modeling the discrete elements of the asphalt mixture and the load plate were as follows:

- Aggregate particles
- Asphaltic mastic particles (fine aggregate, filler, and asphaltic cement), as suggested in [32]
- Steel particles representing the load transmission instrument

Consequently, the physical phenomena associated with the fracture of materials—where inter-particle contacts exhibit a mechanical bonding capable of transmitting tensile forces, as addressed in this study—and the interactions between aggregate particles and the asphalt mastic, or between the mastic and the aggregates, are represented using a bilinear cohesive fracture model [33].

The tensile deformation response pattern considered has favorable characteristics from a logical and computational standpoint, as it models the beginning and spread of the adherent material in a gradual and incremental manner while evaluating the adherence force between the agglutinated particles as they separate from each other. The respective coherence force is defined by a relation between the traction force and the corresponding displacement. Fig. 6 shows the tensile displacement curve for damage modeling.



**Figure 6.** Tensile displacement for the bilinear cohesive fracture model **Source:** Adapted from [34]

This model considers a degrading tightness parameter  $\,D\,$  that varies from 0 to 10 and is correlated with the contact tension state:

$$D = \frac{\delta_c * (\delta_{max} - \delta_o)}{\delta_{max} * (\delta_c - \delta_o)}$$
 (12)

If the adherent particles reach critical separation, adhesion is lost, generating discontinuities within the material and, therefore, fractures.

The contact forces between particles are determined based on the following group of equations:

When  $F \leq F_{max}$ :

$$F^n = K^n * \delta^n \tag{13}$$

$$F^t = K^t * \delta^t \tag{14}$$

When  $F > F_{max}$ :

$$F^{n} = (1 - D) * K^{n} * \delta^{n}$$

$$\tag{15}$$

$$F^{t} = (1 - D) * K^{t} * \delta^{t}$$

$$\tag{16}$$

where:

- K<sup>n</sup> and K<sup>t</sup> are the normal and transversal hardness of the contact tension
- $\delta^n$  and  $\delta'$  denote the elastic displacement in the normal and tangential directions, respectively
- ullet  $\delta_o$  is the maximum displacement, linked to the maximum contact resistance
- $\delta_{max}$  represents the maximum displacement sustained at contact during the analyzed period
- $\delta_c$  means the accumulated elastic displacement associated with full separation or critical displacement
- $F_{max}$  denotes the tensile resistance and failure of contact adhesion between particles

## Proposed computational model

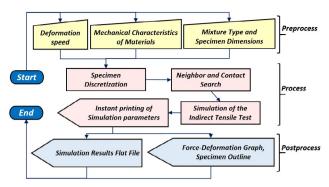
The logic used to elaborate the DEM computational model was written in the Visual Basic.Net programming language.

All statements associated with the variables, functions, procedures, arrays, and data structure, as well as the corresponding interface, were intended for computational simplicity, ease of processing, visual aesthetics, and usability. An additional goal was to facilitate the understanding and use of the source code by anyone with an average knowledge of algorithms and programming.

Thereupon, the proposed algorithm for simulating the indirect traction of asphalt mixtures comprises three main stages:

- Preprocessing: entry of the mechanical parameters for different materials, the geometric conditions of the model sample, and the testing speed, given as the sample loading rate.
- Processing: discretization of the asphaltic material's virtual specimen, the actual simulation, and the graphic representation and the real-time visualization of the simulation parameters and results.
- Post-processing: sharing the final numerical outputs and graphs for the results yielded by the computational representation.

Fig. 7 presents the general flowchart of the elaborated code, which reflects the conceptual model discussed in this work.



**Figure 7.** General flowchart of the computational model for simulated indirect tension testing

Source: Authors

The reasoning behind the computational modeling of the geometric conditions of the simulated asphaltic material involves using the algorithm to generate a subdivision with a finite number of discrete elements, in order to elaborate a numerical prototype that ensures unified mass magnitude and porosity conditions similar to those considered for asphalt mixtures in laboratory settings.

Considering that the geotechnical materials typically used in road infrastructure exhibit a random particle distribution, the discretization process begins by tracing each discrete element according to the geometric dominance. These elements are linked to a particular type of material while following a random numerical process that considers the granulometric distribution associated with the specific type of asphalt mixture in the data entered by the user.

Fig. 8 shows the discretization of a dense hot mixture sample 101.6 mm in diameter, using 4180 discrete elements with a particle diameter of 1.50 mm.

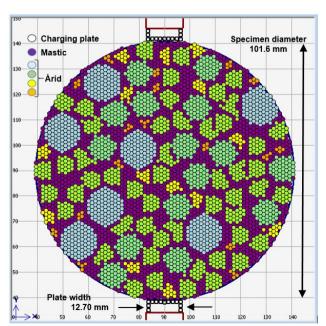
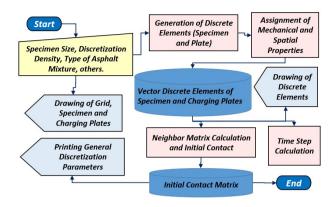


Figure 8. Discrete case of a dense asphalt mixture with a diameter of 4 in

Source: Authors

Fig. 9 shows the flowchart the discretization algorithm.



**Figure 9.** Flowchart of the algorithm for discretizing the geometric dominance of asphaltic material sample

Source: Authors

Fig. 10 details each step in the execution of the proposed computational model. The flowchart of the algorithm deals with the indirect tensile simulation of a given asphaltic sample. This process implies the continuous and systemic execution of the following logical sub-procedure:

- Application of a vertical load of diametral compression according to the discrete elements' coordinates at the top and bottom charge plates.
- Tracing and update of the overlaps between the sample's discrete elements and the load plates, in order to obtain the force and moments of each particle.
- Specific data update of the kinematic and geometric data for each element (acceleration, speed, positioning), and update of the graphical interface based on spatial information.
- Calculation and storage of the applied tensile and compression forces and the vertical and diametric deformations of the material sample.
- Identification of the generated fracture or breakage conditions of between adhering particles.
- Tracing the resulting sample parameters regarding material deformation and volumetric and gravimetric conditions
- Generation of a matrix for the other control parameters aimed at visualization and storage—or of an output matrix for such purpose.

The developed iterative process stops automatically when 10% of the maximum strength of the specimen is reached, keyword query inside the simulation, widely identified by the application source code during the respective analysis period. However, the user can end the simulation at any given time but must save all resulting information beforehand.

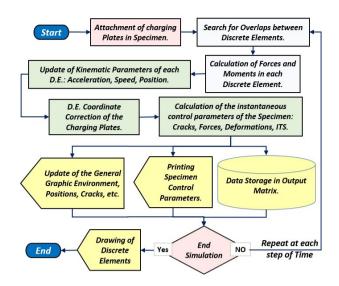


Figure 10. Flowchart for the algorithm used to simulate indirect transition

Source: Authors

## Conditions for validating the computational model

With the purpose of evaluating the numerical model, we compared the simulation results against those of experiments conducted in the laboratory for different types of local asphalt mixtures. To broaden the model verification spectrum, the analyzed mixtures exhibited different mechanical conditions, *i.e.*, a high module MAM-25 mixture (Colombian nomenclature), a modified semi-dense mixture modified with the MSC-19 polymer, and a MDC-19 heated dense mixture with conventional 60/70 asphalt. These mixtures are commonly used in Colombian road infrastructure.

The results regarding indirect tensile strength are presented in Table I. These were obtained while considering dry average conditions. The corresponding variation coefficient is also included.

**Table 1.** Experimental results regarding indirect tensile strength for the local asphalt mixtures analyzed

Asphaltic material	Experimental		
	Average ITS (kPa)	Variation Coefficient (%)	
MAM-25	1351.67	0.64	
MSC-19 modified	744.0	7.94	
MDC-19 conventional	554.57	5.58	

Table II presents the basic mechanical parameters included as entry data for each of the asphaltic samples used for validating the computational model.

Table II. Mechanical properties of the asphalt mixtures analyzed

Asphalt mixture MAM-25		MSC-19	MDC-19	Units	
Viscoelastic Parameters (Burgers)					
E <sub>1</sub>	600 000	350 000	300 000	kPa	

n <sub>1</sub>	1800	900	800	kPa-s
$E_{\!\scriptscriptstyle 2}$	80 000	45 000	40 000	kPa
$n_2$	35	28	25	kPa-s
Asphaltic binder				
Density	1020	1018	1029	kg/m3
Penetration	20.0	55.3	65.0	mm/10
Asphaltic content (weight)	4.6	4.7	5.2	%
Fracture paramete	rs (mastic/ma	stic and masi	tic/aggregate	interphase)
Snmax	5000	2250	1600	kPa
Kno	10 000	5000	4000	kPa/mm
Gn	1875	675	440	kPa-mm
Ssmax	2500	1125	800	kPa
Kso	5000	2500	2000	KPa/mm
Gs	937.5	337.5	220	kPa-mm

With the purpose of obtaining meaningful results, a total of three simulations were executed for each type of mixture. The aim was to represent the median conditions stipulated by standardized Colombian regulations (INV E786-13), albeit with reasonable simulation times and efficient computational use. As such, we employed a 275 mm/min (4.58 mm/s) loading rate and an acceleration factor of 5X, equal to the normal speed stated by INVIAS. Therefore, the simulations were planned to last two or three days, instead of 10 to 15, and were conducted while following operational norms on a standard desktop computer.

The specimen geometry and asphalt mixture characteristics used in the simulations were defined for the smallest nominal diameter allowed for standardized testing, *i.e.*, 101.6 mm. This involved a minimal number of discrete elements, allowing for efficient modeling conditions, short execution times, and low memory storage requirements. The specimen thickness defined for the calculations was 1.0 mm.

The element size established for sample discretization was a particle diameter of 1.50 mm. With both references (sample and particle size), the maximum number of discrete elements was 4180.

#### Validation results

The indirect tensile strength results obtained from simulating the different asphaltic samples under the above-presented conditions are shown in Table III. They are consistent with similar works such as [20], [35], and [36].

**Table III.** Indirect tensile strength results for the local asphalt mixtures analyzed

	Type of			
mixture Simulation		MSC-19mod	MDC-19	
1		1272.91	737.01	504.65
2		1287.51	720.65	570.00

3	1505.28	865.40	660.47
Average	1355.23	774.35	578.37
Standard deviation (kPa)	130.15	79.27	78.25
Variance coefficient (%)	9.60	10.24	13.53

Source: Authors

The average indirect tensile strength of the asphaltic materials considered in our model's validation are presented in Table IV.

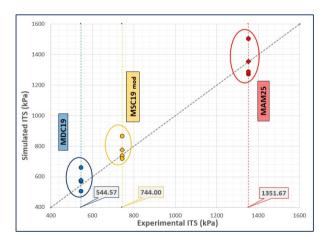
**Table IV.** Simulation and experimental results regarding the indirect tensile strength of the analyzed samples

Type of asphaltic material	Simulation		Experimental	
	Average ITS (kPa)	C.V. (%)	Average ITS (kPa)	C.V. (%)
MAM-25	1355.23	9.60	1351.67	0.64
MSC-19 modified	774.35	10.24	744.0	7.94
MDC-19 Conventional	578.37	13.53	554.57	5.58
Coefficient of determination $(R^2) = 0.943$				

Source: Authors

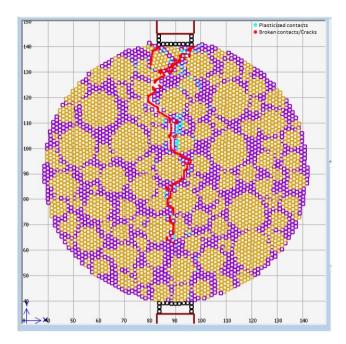
#### Discussion

Fig. 11 shows the scatter of the values obtained from the indirect tensile strength simulations vs. the experimental results. Similar values were obtained in [20], [35], and [36].



**Figure 11.** Scatterplot of simulated *vs.* experimental results for the asphaltic materials used for validating our computational model **Source:** Authors

As an example, Fig. 12 presents the instantaneous state of a numerical specimen after a simulated indirect tensile test. The elasticity of some particle contacts, the development of fissures, and the trajectory of fractures on the asphaltic material agree with the results of [36].



**Figure 12.** Typical specimen after the simulations **Source:** Authors

Our mathematical and computational model reasonably replicates the laboratory data. This is in line with that reported by [13] and [15], who validated the reliability of the DEM via laboratory tests involving uniaxial compression and indirect tensile strength. This work distinguishes itself from previous research because of the software used, which was developed using the Visual Basic.Net programming language, allowing for improvements or adaptations, unlike other works such as [14] and [21], which employed commercial software.

#### **Conclusions**

The results obtained from simulating the indirect tensile strength of the analyzed asphaltic materials are consistent with our expectations regarding the experimental results. It can be inferred that our mathematical and computational model comes reasonably close to laboratory results, which makes it a reliable tool, as evidenced by its R<sup>2</sup> of 0.943.

For the three simulated asphaltic materials, low variation coefficients were observed (less than 13.5%). For the same input data, the model exhibited a low scatter of the results, indicating that the logical algorithm is optimally precise when compared to the laboratory results.

Under the conditions presented in this work, the model's numerical and computational accuracy was evaluated with regard to the input data for each asphalt mixture.

Below are some of the technical and methodological limitations of this study.

- The discretization density and size of the analyzed elements was constrained with regard to the model's logical levels, given its high computational cost in terms of execution time and storage requirements.
- In terms of execution times and computational efficiency, the numerical model was considered within the framework of an exclusively bi-dimensional approach.
- With the purpose of delimiting the scope of this study, the logical model was developed to exclusively include asphaltic specimens under dry conditions.
- Given the characteristics of the local mixtures analyzed, it was not possible to find references in the literature regarding their viscoelasticity and fracture models. The special conditions of such tests for research are not specified in the national regulations for the elaboration and control of asphalt mixtures.

The benefits and limitations of the proposed solution are limited to the physics of the numerical model. As future work, diverse samples should be tested using the software in order to validate and improve the model for practical applications.

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#### **Author contributions**

Author 1: conceptualization, data curation, formal analysis, investigation, software, validation, writing (original draft, review, and editing).

Author 2: conceptualization, formal analysis, funding acquisition, investigation, methodology, project administration, resources, software, supervision, visualization, writing (original draft, review, and editing).

#### Conflicts of interest

The authors declare no conflict of interest.

## **Data availability**

The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable re-quest.

## **Use of artificial intelligence (AI)**

No AI was used during the research.

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