Optimal under voltage load shedding based on voltage stability index

Esquema óptimo de desastre de carga por baja tensión basado en índice de estabilidad de tensión

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ABSTRACT

This paper presents a methodology for under voltage load shedding using a metaheuristic optimization technique and a stability criterion. Two strategies are proposed to find the minimal size and location of load to shed for the recovery of normal operation conditions. The first one is based on a classical criterion for the under voltage load shedding, identifying the load to disconnect by considering bus voltage level; the second includes a simplified voltage stability index SVSI, which identifies critical buses in the system. The proposed methodology is implemented in an IEEE 14 bus test system, considering a heavy loading condition with and without contingency to validate its efficiency.

Keywords: Load shedding, voltage stability, particle swarm optimization.

Introduction

As power generation and load demand grow in an ever-increasing tendency, it has been widely reported that power systems are currently being run closer to the operation limits because of the lack of expansion of transmission networks as the loads grow, due mainly to environmental and economic constraints. This situation has conduced to more sensitive power systems, which are prone to voltage instabilities or collapses. This effect has been observed and reported in several power systems worldwide (IEA, 2005; Kundur, 1994). For those power systems, which reach the stability limits, strategies for guaranteeing the generation-demand balance are required to avoid collapses, minimizing non-supplied energy and optimizing energy efficiency.

Alternatives during an impending voltage collapse include Under Voltage Load Shedding (UVLS) schemes, which are applied when the operation of control and compensation devices, such as FACTS (Nguyen and Wagh, 2009; Greene, Dobson and Alvarado, 1997), turbine governors, automatic voltage regulators (Lerm and Silva, 2004), among others, are inefficient to reach a stable operating state after a disturbance or a contingency. UVLS is based on the possibility of disconnecting some loads (or percentages of load) after a severe disturbance, in order to relocate the operating point far from the critical voltage value (Kessel...
and Glavitsch, 1986) (Quoe et al., 1994). The immediate problem related to UVLS is the development of a strategy to define the amount of load to shed and its location, in order to save the system from a complete blackout.

According to the literature review, several schemes for the determination of load to shed by UVLS have been proposed, based on classical strategies that shed a constant percentage of load when the voltage is out of range. These can be inadequate for a particular or complex system (Laghari, Mokhlis, Bakar and Mohamad, 2013). These classical methods include homogeneous load shedding, centralized and decentralized load shedding (Pahwa, Scoglio, Das and Schulz, 2013; Mollah, Bahadornejad, Nair and Ancell, 2012; Niar et al., 1999; Klaric, Kuzle and Tomisa, 2005). Mathematical techniques such as linear programming (LP), nonlinear programming and the interior point method were for this purpose; however, these algorithms require approximations of the power system model to reduce the calculation time (Shen and Laughton, 1970). Metaheuristic computational techniques and their application in electric power systems are well known and have been used as an optimization tool in different applications, including fault recognition, oscillation control, planning, design and operation, among others. One of these techniques is Particle Swarm Optimization (PSO) (Zomaya and Oliveri, 2006; Kennedy and Eberhart, 1995), which were applied to improve load shedding automation. In Amraee, Ranjbar, Mozafari and Sadati (2007), an optimal load-shedding algorithm was developed for ULVS using two heuristic methods such as PSO and Genetic Algorithm. A new method based on sequential use of LP and PSO were used to minimize the load shedding in contingency conditions in Tarafdar and Galvanı (2011).

Methods for UVLS only consider bus voltage magnitude to define the location and amount of load to shed. However, even if voltages reach a safe operating value, it is well known that bus voltage magnitude is not an adequate indicator of the security of power system operating conditions, especially in modern power systems (Mozinia, 2007). Therefore, these UVLS strategies do not guarantee that the operating point after shedding is adequate in terms of voltage stability. A paper related with this is presented in Kanimozhi et al. (2014), where the minimization of the total load shed and the sum of a “New Voltage Stability Index (NVSI)” were considered as objectives to restore the power flow solvability. The same index was also integrated in a load shedding scheme (Sonar & Mehta, 2015), based on swarm intelligence-based optimization techniques.

In this paper, a methodology for UVLS is proposed, where UVLS is modeled as an optimization problem, solved using PSO. The objective function of the optimization problem includes a voltage stability criterion to guide the evolution process of the algorithm towards the determination of the amounts of load to shed and the improvement of the power system voltage stability. This finally leads to a reduction of the non-supplied energy and costs associated to load disconnection.

The structure of the paper is organized as follows. Section II presents the theoretical background related with voltage stability criteria and PSO algorithm. In Section III the methodology is exposed. Based on it, the simulation results are shown in Section IV, where the methodology is tested on an IEEE 14 bus system subject to several disturbances. Finally, in Section V, the main conclusions of our tests and future work are presented.

### Theoretical Background

**Voltage stability indices**

Voltage stability indices are mathematical tools to determine the proximity of a power system to an impending voltage collapse. These are usually formulated to indicate proximity to voltage collapse when its value is close to one, and secure operating points when its value is close to zero. In order to determine an indicator of risk of voltage instability on load buses, one index proposed in literature is Simplified Voltage Stability Index (SVSI) (Pérez, Rodriguez and Olivar, 2014). This index is based on the concept of relative electrical distance (RED), which is used to select the nearest generator to a specific load bus and also the association of electrical variables to improve its performance.

For a given system, the relation between the complex current ($I$) and voltage vectors ($V$) at the generator buses ($G$) and load buses ($L$) is represented by the admittance matrix, as is given in Equation (1):

$$
\begin{bmatrix}
IG \\
IL
\end{bmatrix} = \begin{bmatrix}
YGG & YGL \\
YLG & YLL
\end{bmatrix} \begin{bmatrix}
VG \\
VL
\end{bmatrix} = Y_{LL} \begin{bmatrix}
IG \\
IL
\end{bmatrix}
$$

(1)

Rearranging (1), (2) is obtained:

$$
\begin{bmatrix}
VL \\
IG
\end{bmatrix} = \begin{bmatrix}
Z_{LL} & FLG \\
KGL & Y_{GG}
\end{bmatrix} \begin{bmatrix}
IL \\
VG
\end{bmatrix}
$$

(2)

where $FLG = \frac{1}{Y_{LL}} Y_{LG}$ is a complex matrix that gives the relation between load and source bus voltages. The REDs (i.e. the relative locations of load buses with respect to the generator buses) are obtained from the FLG matrix and given in Equation (3) (Yesuratnam and Thukaram, 2007):

$$
RLG = \left[A \right] - \text{abs} \left[FLG\right] = \left[A \right] - \text{abs} \left[Y_{LL}^{-1} \left[Y_{LG}\right]\right]
$$

(3)

where $A$ is the matrix with size $(n-g) \times g$, $n$ is the total number of buses of the network, and $g$ is the number of generator buses. All of the elements of matrix $A$ are equal to unity. The information given by the matrix RLG can be used instead of path algorithms to obtain the electrical distances between load and generator buses (Pérez et al., 2014).
Once the nearest generator to a specified load bus is found with the RLG matrix, the voltage drop on the Thevenin impedance $\Delta V_i$ is estimated using Equation (4):

$$\Delta V_i = \sum_{b=1}^{n} |V_b - V_{b1}| \geq |V_i - V|$$

where $V_g$ and $V_i$ are the voltage phasors at the nearest generator and the analyzed load bus, respectively.

Due to simplifications in the development of this index, the inclusion of a correction factor is necessary to avoid loss of sensitivity to the critical point of the system. This factor, denoted as $\beta$, is calculated according to Equation (5):

$$\beta = 1 - \left(\max \left(\frac{V_i}{V_m}\right)\right)^2$$

The correction factor is associated with the highest differences of voltage magnitudes between two buses ($m$ and $l$), which can be obtained directly from PMU measurements in the analyzed power system under specific operating conditions. Considering the previous, SVSI is given in Equation (6):

$$SVSI_i = \frac{\Delta V_i}{\beta + V_i}$$

To consider a power system as voltage unstable, the proposed index SVSI must be close to unity (at the maximum loadability point) if and only if the voltage drop in the Thevenin impedance $\Delta V_i$ is equal to the voltage at the load bus, according to the formulation based on the maximum power transfer theory (Pérez et al., 2014).

**Particle Swarm Optimization**

Similar to other stochastic searching techniques, PSO is initialized by generating a population of random solutions, which is called a swarm. Each individual is referred as a particle and represents a candidate solution to the optimization problem. A particle in PSO, like any living object, has a memory that retains the best experience, which is gained during the exploration of the solution area. In this technique, a velocity vector is associated to each candidate solution of a $m$-th particle is saved as $pbest_i = (pbest_1, pbest_2... pbest_m)$ and the best previous experience of a group is defined as $gbestPSO$. The particle position and velocity are modified in each iteration through Equations (7-8):

$$v_{id}^{(t+1)} = \omega v_{id}^{(t)} + c_1 \text{rand}_1(0) \left( pbest_i - x_{id}^{(t)} \right) + c_2 \text{rand}_2(0) \left( gbest_{i} - x_{id}^{(t)} \right)$$

$$x_{id}^{(t+1)} = x_{id}^{(t)} + v_{id}^{(t+1)}$$

In an $n$-dimensional search space, the particle position and velocity can be represented as vectors $x_i = (x_1, x_2... x_n)$ and $v_i = (v_1, v_2... v_n)$ respectively. The best previous experience of a $i$-th particle is saved as $pbest_{PSO} = (pbest_1, pbest_2... pbest_m)$ and the best previous experience of a group is defined as $gbest_{PSO}$.

The particle position and velocity are modified in each iteration through Equations (7-8):

In Equations (7-8), $i=1,2,3,...,m$ is the particle index and $t$ is the iterations number, the constants $c_1$ and $c_2$ are weights that control cognitive and social components; $\omega$ is the inertia factor in each iteration, its value decreases according to Equation (9):

$$\omega = \omega_{max} - \frac{t}{T} \omega_{min}$$

The movement of a particle into a swarm according to the best experience of the group is illustrated in the Figure 1.
Formulation of load shedding optimization problem

The best load shedding location and the minimum load shedding amount during severe contingencies are solved in this paper as an optimization problem. The objective function contains a sensitivity factor to guide the optimization problem, and the problem is subject to constraints associated to power flow restrictions and element capabilities.

Objective functions

To obtain the load percentage corresponding to each busbar according with its voltage collapse sensitivity, two schemes for load shedding are developed. These schemes are a function of the load to be shed at each bus, denoted as $\Delta P_D$, and include a different sensitivity criterion for the objective function. The first one is related to the ULVS classical criterion based on the voltage level in the busbar, and the second one includes the simplified voltage stability index SVSI, which identifies critical buses in the system. The purpose is to determine the optimal quantities of active power to shed ($\Delta P_D$ are decision variables for this problem), according to the established sensitivity criterion. These schemes are explained below.

Scheme 1: Under voltage load shedding using voltage level in each bus

Bus voltage level has a straightforward relation to buses with considerable changes in its operational state after a disturbance. This is the main concept applied for classical load shedding schemes. The objective function in this case is defined by Equation (10):

$$\min \left\{ \sum_{i=1}^{NL} V_i \Delta P_D \right\}$$

where $\Delta P_D$ corresponds to load to be shed at bus $i$, $V_i$ is the voltage level in the bus $i$ and $NL$ is the total PQ busbar of the system. In this scheme, the lower voltage level of a busbar is the most susceptible of shedding.

Scheme 2. Under voltage load shedding using simplified voltage stability index (SVSI).

This stability index is included in the load shedding scheme in order to guide the algorithm in the load shedding distribution between the buses of the system, according to its contribution to voltage collapse. Therefore, the buses with higher value of SVSI will be better candidates for shedding.

The objective function to perform load shedding using SVSI is defined in Equation (11):

$$\min \left\{ \sum_{i=1}^{NL} \frac{\Delta P_D}{SVSI_i} \right\}$$

where $\Delta P_D$ corresponds to load shed, $SVSI_i$ to the voltage stability indicator of bus $i$, and $NL$ the total PQ busbar of the system.

Constraints of the problem

The load shedding algorithm is formulated in terms of both active and reactive power parameters ($P$ and $Q$, respectively). Therefore, it is necessary to consider power flow constraints Equations (12 -13).

$$P_G^i - P_D^0 + \Delta P_D = \sum_{j=1}^{N} \left| V_j \right| \left| V_i \right| \cos (\delta_j + \delta_i - \delta_i)$$

$$Q_G^i - Q_D^0 + \Delta Q_D = -\sum_{j=1}^{N} \left| V_j \right| \left| V_i \right| \sin (\delta_j + \delta_i - \delta_i)$$

Where the subscripts “$G$” and “$D$” are related to generation and consumption at bus $i$, respectively. Superscript “$0$” indicates initial state.

In order to ensure a sufficient distance to voltage collapse, a loading margin $\lambda_{min}$ is established according to Equations (14-15):

$$(1 + \lambda_{min}) (P_G^i - P_D^0 + \Delta P_D) = \sum_{j=1}^{N} \left| V_j \right| \left| V_i \right| \cos (\delta_j + \delta_i - \delta_i)$$

$$Q_G^i - (1 + \lambda_{min}) (Q_D^0 - \Delta Q_D) = -\sum_{j=1}^{N} \left| V_j \right| \left| V_i \right| \sin (\delta_j + \delta_i - \delta_i)$$

where the superscript “$c$” is related to the post-contingency state. The loading margin is explained in Figure 2. There, the power system behavior is shown before (blue line) and after a contingency (green line). When a contingency occurs (for instance, a line fault in the system), the point operation defined by point (1) moves to point (2), which is very close to the critical point (nose curve). To enhance power system security, load shedding must guarantee a loading margin $\lambda_{min}$, which is the distance between the new operational point after UVLS is applied (3) and the collapse point.

Other model constraints are associated to the voltage levels boundaries for both initial and stressed conditions, load shedding limits and fixed power factor, presented in Equations (16)-(19).

$$V_i^{min} \leq V_i \leq V_i^{max}, i \in N_L$$

$$\Delta P_D^{min} \leq \Delta P_D \leq \Delta P_D^{max}, i \in N_D$$

$$\Delta P_D^{min} \leq \Delta P_D \leq \Delta P_D^{max}, i \in N_D$$

$$\frac{\Delta P_D}{P_D^0} = \frac{\Delta Q_D}{Q_D^0}, \text{ fixed power factor}$$
Figure 2. Loading margin into the load shedding scheme.

Methodology

According to previous information, both load shedding schemes proposed are initialized if the voltage level at any bus of the system is under voltage threshold predefined by the user (in this paper it was established at 0.9 p.u). The ULVS scheme is carried out following the flow chart shown in Figure 3.

Considerations related to PSO algorithm

A vector of \( N \) components represents each particle for PSO algorithm, where \( N \) is the number of available loads to be shed: \( \{\Delta P_1, \Delta P_2, \ldots, \Delta P_i, \ldots, \Delta P_N\} \). The initial population is generated randomly, assigning a random number between zero and the 40\% of the total active power connected to the bus to each component of the particles.

If a particle reaches an infeasible solution, i.e., when one of the components is lower than zero or greater than the maximum active power shedding limit, a penalty factor is added to the objective function. Finally, as stopping criterion, each PSO execution is stopped when a fixed number of iterations is reached.

Results and discussion

Test description

The methodology previously presented is tested using the IEEE 14 bus test system (PSTCA, 2015), and PSAT as simulation tool. The test system is shown in Figure 4.

In order to test the effectiveness of the proposed methodology, this paper presents a study for the following operating conditions:

- Heavy loading at bus 14 without contingency
- Heavy loading with contingency (single line outage 3-2)

For each case, load shedding is triggered if voltage at any bus is lower than 0.9 p.u. The optimization problem sheds load until all buses have a voltage not lower than 0.95 p.u. The maximum amount of load to shed at each bus is 40\%
of the connected load at the bus at the moment when load shedding is triggered.

**PSO parameters**

PSO parameters for this application were obtained through exhaustive testing, choosing a set that led to the best results. These are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of particles</td>
<td>10</td>
</tr>
<tr>
<td>Maximum particle speed</td>
<td>0.002</td>
</tr>
<tr>
<td>Minimum particle speed</td>
<td>-0.002</td>
</tr>
<tr>
<td>Cognitive coefficient $c_1$</td>
<td>1.7</td>
</tr>
<tr>
<td>Social coefficient $c_2$</td>
<td>1.7</td>
</tr>
<tr>
<td>Maximum inertia $\omega_{\text{max}}$</td>
<td>0.9</td>
</tr>
<tr>
<td>Minimum inertia $\omega_{\text{min}}$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

For each loading factor, PSO algorithm was executed 20 times. Each execution is stopped after the algorithm has reached 100 iterations. A penalty factor $\alpha = 10^{10}$ is added to the objective function when an infeasible solution is reached. Each PSO execution is completed in two minutes.

**Heavy loading at bus 14 without contingency**

In this case, the loading at bus 14 is increased gradually until 4.3 times its nominal load. In this condition, the level voltage at that bus corresponds to 0.9 pu and, therefore, the load shedding schemes are executed. Figure 5 shows the minimum load shedding amount for each loading margin (from 0.01 to 0.1 p.u.).

![Figure 5](image1.png)

**Figure 5.** Load shed vs. loading margin for heavy loading without contingency.

According to Figure 5, if a higher loading margin is required, a greater load amount must be disconnected. In Figure 6, voltage magnitude at each bus considering a load margin of 0.1 is shown. Once load shedding procedures are applied, all voltage magnitudes are above the specified threshold of 0.95 p.u.

![Figure 6](image2.png)

**Figure 6.** Voltage level at each bus for $\lambda_{\min} = 0.1$.

In order to verify the power system stability after the execution of the proposed schemes on severe disturbances, a test was performed. The line 3-2 was disconnected and the power system loading is increased to 1.5 times its nominal value.

The minimum load shedding amount for each loading margin according to the proposed schemes is shown in Figure 8.

In this case, scheme 1 sheds less amount of load compared to scheme 2. However, both schemes follow the same tendency. Figure 9 shows the load shed corresponding to each bus for $\lambda_{\min} = 0.1$.

![Figure 7](image3.png)

**Figure 7.** Voltage stability index SVSI for each bus for $\lambda_{\min} = 0.1$.

**Heavy loading with contingency (single line outage 3-2)**

In order to show the effectiveness of the proposed schemes on severe disturbances, a test was performed. The line 3-2 was disconnected and the power system loading is increased to 1.5 times its nominal value.

The minimum load shedding amount for each loading margin according to the proposed schemes is shown in Figure 8.

In this case, scheme 1 sheds less amount of load compared to scheme 2. However, both schemes follow the same tendency. Figure 9 shows the load shed corresponding to each bus for $\lambda_{\min} = 0.1$.

According to Figure 9, either scheme assigns an amount of load for shedding to each busbar in the system considering the index used. The voltage levels before and after the
implementation of load shedding schemes are shown in Figure 10.

![Graph showing load shed vs. loading margin for heavy loading with contingency.](image)

**Figure 8.** Load shed vs. loading margin for heavy loading with contingency.

Figure 9 shows the load shedding amount for each bus for \( \lambda_{\text{min}} = 0.1 \).

![Bar chart showing load shedding amount for each bus for \( \lambda_{\text{min}} = 0.1 \).](image)

**Figure 9.** Load shedding amount for each bus for \( \lambda_{\text{min}} = 0.1 \).

Figure 10 depicts the voltage level for each bus for \( \lambda_{\text{min}} = 0.1 \).

![Bar chart showing voltage level for each bus for \( \lambda_{\text{min}} = 0.1 \).](image)

**Figure 10.** Voltage level for each bus for \( \lambda_{\text{min}} = 0.1 \).

The results show that voltage levels at all buses of the power system are improved after shedding, over the expected voltage level of 0.95 p.u. With the final purpose of analyzing the voltage stability before and after the implementation of load shedding schemes, a voltage stability study is performed using SVSI and the results are shown in Figure 11.

![Graph showing voltage stability index SVSI for each bus for \( \lambda_{\text{min}} = 0.1 \).](image)

**Figure 11.** Voltage stability index SVSI for each bus for \( \lambda_{\text{min}} = 0.1 \).

Conclusions

This paper approaches aspects related to load shedding in power systems, as an emergency strategy for avoiding voltage collapse. Due to the necessity to determine the minimal amount of load to shed, two methodologies for optimal under voltage load shedding considering particle swarm optimization are proposed; one of these including stability criteria in order to guide the optimization process to a solution where shed load is minimal and voltage stability is also improved.

According to the results obtained, both schemes of under voltage load shedding are efficient to increase the voltage level in every busbar of a system after its execution. However, after a voltage stability analysis of the results of each scheme, it is possible to conclude that load shedding considering the voltage level is no warranty of an adequate voltage stability condition. Then, the relevance of including scheme voltage stability index as SVSI into load shedding is demonstrated, as this consideration leads to a better solution in terms of voltage stability. As this methodology establishes a criterion for load shedding based on voltage stability indices, further work is focused on an online implementation, applying machine-learning techniques to reduce the computational effort associated to PSO calculations.

References


