Elasto-plastic model to determine the maximum force for shaft straightening process

Modelo elasto-plástico para determinar la fuerza máxima para un proceso de enderezamiento de un eje

J. C. Galvis¹, H. E. Maury², and R. J. Hernández³

ABSTRACT

This paper shows the development of a mathematical model based on elasto-plastic behavior of materials in order to determine the bending moment and the maximum straightening force required to repair a bent shaft. The importance of this work is because many machines use shafts to transmit power and movement, and these elements can suffer permanent deflections during operation cycles. An appropriate repair process can increase the life of these elements significantly. Knowing the mechanics for a straightening process is essential for design, processes and maintenance engineers in charge of systems that use these elements (shafts).

Keywords: Elasto-plastic model, straightening process, solid mechanics.

RESUMEN

Este artículo muestra el desarrollo de un modelo matemático basado en el comportamiento elasto-plástico del material con el objeto de determinar el momento de flexión y la fuerza máxima de enderezamiento requerida para reparar un eje doblado. La importancia de este trabajo radica en que muchas máquinas usan ejes para transmitir potencia y movimiento, y estos elementos pueden sufrir deflexiones permanentes durante sus ciclos de operación, de tal modo que un proceso adecuado de reparación puede incrementar la vida de estos elementos significativamente. Saber la mecánica del proceso de enderezamiento es esencial para los ingenieros de diseño, manufactura y mantenimiento a cargo de sistemas que usan este tipo de elementos (ejes).

Palabras clave: Modelo elasto-plástico, proceso de enderezamiento, mecánica de sólidos.

Introduction

The most used mechanical element to transmit power and movement is a shaft. They are used in all kind of industrial systems. During operation they can suffer deflections. In order to increase their life cycle, shaft straightening reparations are needed. The main parameter for straightening machines is the maximum force that can be applied in the process. It is important to know the mechanics of the straightening process; some studies used experimental data (Schleinzer, et al., 2001; Srimani, et al., 2005) and numerical models (Jing, et al., 2013; Wu, et al., 2000). Some authors have studied the process and how to control the stages based on material response (Li, et al., 1999; Seung-Cheol, et al., 2002). An important phenomenon is the residual stress (Schleinzer, et al., 2001) and the springback of the material. In many cases they studied the effect on test bars and rails. The idea with this work is to develop a model based on the elasto-plastic behavior of the material and the geometric characteristics of the bent shaft. This will be an important tool for design straightening machines, and also to improve reparations processes.

Proposed model

In order to develop a mathematical model of a bent shaft deformation based on geometry and mechanical behavior of materials, some assumptions have to be made. The final goal is to compute the force necessary to straighten a bent shaft.

First, the bending moment required to straighten the shaft will be computed with Equation (1) (Ugural, and Kenster, 1995).

\[ M_f = \int_{y_2}^{y_b} \sigma(y) b(y) \, dy \]  

Where, \( y \) is the height of the cross section, \( b \) is the width, \( d \) is the diameter and \( \sigma \) is the normal stress. Figure 1

---

¹ Mechanical Engineer. Mathematician. Universidad del Norte, Colombia. Graduate Student (M.Sc. in Mechanical Engineering). Universidad del Norte, Colombia. Affiliation: Universidad del Norte, Colombia. E-mail: cgalvis@uninorte.edu.co

² Mechanical Engineer. Universidad del Norte, Colombia. Ph.D. in Mechanical Engineering. Universidad Politécnica de Cataluña, Spain. Affiliation: Professor, Universidad del Norte, Colombia. E-mail: bmaury@uninorte.edu.co

³ Mechanical Engineer. Universidad del Norte, Colombia. M.Sc. in Mechanical Engineering, Universidad de los Andes, Colombia. Ph.D. in Mechanical Engineering University of Texas at Austin, USA. Affiliation: Universidad Autónoma del Caribe, Colombia. E-mail: roque.hernandez@uac.edu.co
illustrates the normal stress distribution in the cross section of the shaft.

\[ b(y) = 2\sqrt{(d/2)^2 - y^2} \quad (2) \]

The normal stress, as a function of height, is a piecewise continuous and bounded function. The fact that it is a continuous piecewise function is due to the transition from elastic flow, determined by Hooke’s law, to plastic flow which can be approximated by the Ludwik equation (Lubliner, 2008); and it is bounded by 0 (no stress applied) and sut (ultimate strength). Then, the normal stress as a function of height is given by:

\[
\begin{align*}
\sigma(y) &= \begin{cases} 
\frac{Ey}{R}, & 0 \leq y \leq y_f \\
\sigma_{ys} + K \left( \frac{y}{R} - \frac{\sigma_{ys}}{E} \right)^n, & y_f < y \leq d/2
\end{cases} \quad (3)
\end{align*}
\]

Where \( E \) is the Young modulus of the material, \( R \) is the average radius of curvature of the shaft in the cross-section analyzed (Where \( R \neq \infty \) due to shaft deflection), \( \sigma_{ys} \) is the yield strength of the material, \( K \) is the strain hardening coefficient of the material, \( n \) is the strain hardening exponent of the material and \( y_f \) is the height which divides the elastic and plastic region in the cross section.

On the other hand, once the shaft is deformed plastically, the stress-strain curve changes (Strain hardening - Cold work) and the new function \( \sigma'(y) \) has to be computed. The material tries to recover its original shape (This phenomenon is called “springback”). This recovery is approximately linear with a slope equal to the Young modulus of the material. Also the material suffers a strain hardening due to plastic deformation and the new yield stress \( \sigma'_{ys} \) is equal to the strain that caused the deformation observed in the component. Figure 2 shows this process.

\[
\begin{align*}
\sigma'(y) &= \begin{cases} 
\frac{Ey}{R'}, & 0 \leq y \leq \varepsilon R' \\
\sigma_{ys} + K \left( \frac{y}{R(y)} - \frac{\sigma_{ys}}{E} \right)^n, & \varepsilon R' < y \leq d/2
\end{cases} \quad (4)
\end{align*}
\]

Where \( R' \) is the radius of curvature after deformation with stress level \( \sigma'_{ys} \) and the load still applied, \( \varepsilon \) is the strain corresponding to \( \sigma'_{ys} \) and \( R(y) \) is the radius of curvature of the shaft at height \( y \). This latter term is also dependent on the variation of the stress \( \sigma' \) but this is a function of \( y \), then \( R \) ends a function of \( y \).

Now, the values of \( R' \) and \( \varepsilon \) have to be computed. The strain \( \varepsilon \) is determined using equation (5).

\[
E(\varepsilon - \varepsilon') = \sigma_{ys} + K \left( \varepsilon - \frac{\sigma_{ys}}{E} \right)^n \quad (5)
\]

Where \( \varepsilon' \) is the strain corresponding to the springback, and in this case it is given by:

\[
\varepsilon' = \frac{d/2}{R} \quad (6)
\]

Now \( R' \) can be calculated using Equation (7).

\[
\sigma'_{ys} = \sigma_{ys} + K \left( \frac{d/2}{R'} - \frac{\sigma_{ys}}{E} \right)^n \quad (7)
\]
The stress corresponding to \( R(y) = \infty \) is denoted by \( \sigma_1 \). It is important to take into account the material springback again, so an extra stress (overbending) has to be added, this extra stress is denoted by \( \sigma_2 \). Figure 3 illustrates both stresses.

\[
\frac{M_f}{4} = \frac{E}{R^2} \int_0^{\infty} \left( y^2 \sqrt{(d/2)^2 - y^2} \right) dy
\]

\[
+ \int_{R^2}^{\infty} y \sqrt{(d/2)^2 - y^2} \left( \sigma_2 + \sigma_{ys} + K \left( \frac{y}{R} - \frac{\sigma_{ys}}{E} \right)^n \right) dy
\]

For the design of straightening machines is imperative to know the maximum force required to straighten a shaft. In order to compute the maximum force is necessary to define the maximum diameter \( d_{max} \), the maximum length of the shaft \( L_{max} \) and the maximum radius of curvature \( R_{max} \). The latter can be calculated as follows using Equation (12) and Equation (13):

\[
R_{max, \text{permissible}} = \frac{d/2}{\epsilon_{\sigma_{ut}}'}
\]

\[
\epsilon_{\sigma_{ut}}' = \frac{\sigma_{ys}}{E} + \frac{\sigma_{ut} - \sigma_{ys}}{E} y^n + \frac{\sigma_{ut}}{E}
\]

Finally, the maximum force required for a straightening process (this is the force the machine has to applied to the shaft) is given by:

\[
F_{\text{max,required}} = L_{max} \frac{M_{\text{max,required}}}{d_{max}}
\]

### Results and discussion

To validate the model a case was selected. The material is the AISI 4340 steel. The stress-strain curve for this material is presented in the Atlas of Stress-Strain Curves (Atlas of Stress-Strain Curves, 2002). To determine the strain hardening coefficient \( K \) and the strain hardening exponent \( n \) for this steel a linear regression was used:

\[
\sigma = K \epsilon^n \leftrightarrow \ln \sigma = \ln K + n \ln \epsilon = a_0 + a_1 \ln \epsilon
\]

Where, \( a_0 \) and \( a_1 \) are the unknown coefficients for the linear regression. Table 1 shows the data used in the regression.

<table>
<thead>
<tr>
<th>( \epsilon )</th>
<th>( \sigma ) (Mpa)</th>
<th>( \ln \epsilon )</th>
<th>( \ln \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1166,666667</td>
<td>-4.6051709</td>
<td>7.06190596</td>
</tr>
<tr>
<td>0.02</td>
<td>1216,666667</td>
<td>-3.91202310</td>
<td>7.10387016</td>
</tr>
<tr>
<td>0.03</td>
<td>1236,68889</td>
<td>-3.5065579</td>
<td>7.12015455</td>
</tr>
<tr>
<td>0.04</td>
<td>1261,11111</td>
<td>-3.21887582</td>
<td>7.13974844</td>
</tr>
<tr>
<td>0.05</td>
<td>1283,33333</td>
<td>-2.99573227</td>
<td>7.15721614</td>
</tr>
<tr>
<td>0.06</td>
<td>1290,33333</td>
<td>-2.81341072</td>
<td>7.16265586</td>
</tr>
</tbody>
</table>

Source: Atlas of Stress-Strain Curves, 2002

The results for \( K \) and \( n \) are 1515.55 MPa and 0.0567871, respectively. Table 2 resumes all data used.
Finally, the maximum force required for this case is 11767 kN (1200 Ton). This force is in the range of power of the machines designed for this size (1000-3000 Ton).

Conclusions

A mathematical model was developed for straightening process of a bent shaft based on material behavior and geometric characteristics. The model uses the elasto-plastic mechanics of deformation. The model was validated with AISI 4340 steel which is a material highly used in machine elements. The maximum force calculated for this case is in the range of the applications for the geometric and material characteristics. This model is an important tool for design new straightening machines, and also to develop better reparation and maintenance techniques and processes. Future work should focus in more experimental data in order to adjust the model.

References


Table 2. Data to test the model

<table>
<thead>
<tr>
<th>d_max (mm)</th>
<th>l_max (m)</th>
<th>E (GPa)</th>
<th>σ_ut (MPa)</th>
<th>σ_ys (MPa)</th>
<th>K (MPa)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>3</td>
<td>205</td>
<td>1300</td>
<td>1124</td>
<td>1515</td>
<td>0,057</td>
</tr>
</tbody>
</table>

Source: Authors