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Assessment of groundwater level variations using multivariate statistical methods

Evaluación de cambios en el nivel freático mediante métodos estadísticos multivariados

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ABSTRACT

Fluctuation of groundwater level induces changes in pore-water pressure of soil. However, this variation is not considered for underground constructions. This article explores the application of a statistical method to evaluate the groundwater level variation in geotechnical designs. The methodology included: (i) data collection, (ii) statistic formulation, and (iii) statistic data analysis. We collected information from the technical studies of the project "Metro de Bogotá", and selected four boreholes spanning 160 m, approximately, where the 1° de Mayo metro station will be built, in the south of the city. We used groundwater level readings reported by different piezometers for 30 days and data variance was assessed using a multivariate statistical method: analysis of repeated measures profiles. Results present a procedure to estimate the groundwater level fluctuation during a short monitoring period. We concluded that the analysis of repeated measures profiles allows estimating the groundwater level variation under a significance level 1-α.

Keywords: Analysis of repeated measures profiles, boreholes, infrastructure projects, Metro of Bogotá city.

RESUMEN

La fluctuación del nivel freático induce cambios en la presión de poros del suelo. Sin embargo, esta variación no se contempla en construcciones subterráneas. Este documento explora la aplicación de un método estadístico para evaluar la variación del nivel freático en diseños geotécnicos. La metodología incluyó: (i) recolección de datos, (ii) formulación estadística y (iii) análisis estadístico de datos. Se recopiló información de los estudios técnicos del proyecto "Metro de Bogotá". Se seleccionaron cuatro sondeos que abarcan 160 m, aproximadamente, donde se construirá la estación 1° de Mayo, al sur de la ciudad. Se utilizaron lecturas de nivel freático reportadas por varios piezómetros, durante 30 días y la variabilidad de los datos se evaluó utilizando el método estadístico multivariado: análisis de perfiles de medidas repetidas. Los resultados presentan un procedimiento para estimar la fluctuación del nivel freático durante un período corto de monitoreo. Se concluyó que el análisis de perfiles de medidas repetidas permite estimar la variación del nivel freático bajo un nivel de significancia 1-α.

Palabras clave: Análisis de perfiles de medidas repetidas, Metro de la ciudad de Bogotá, proyectos de infraestructura, sondeos.

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Introduction

Geotechnical explorations often require to register the position of the water along the soil profile. This registration is a parameter known as groundwater level. The identification of groundwater level allows calculating the effective stress. Likewise, such reference point is used to establish the possible drainage conditions of the soil in the structure design. Moreover, groundwater level depends on the hydrogeological conditions of the ground (Gonzalez de Vallejo and Ferrer, 2011) and its variation is conditioned by the weather and the hydraulic properties of soil in the hydraulic parameters of the soil layers (Ruge, Da Cunha, Colmenares, and Mendoza, 2017).

However, in many cases, the variability of this state is not contemplated as part of the design process. Osterberg (2004) states that better exploration and sample practices are necessary to improve the building quality. Hence, in order to avoid extra laboratory tests, the groundwater

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level variation must be included in the designs of underground constructions (Tristá, Sotolongo, Cristía, and Fernández, 2016).

The variation of water content affects the stiffness of the material and increases its strains during reload stages (Molina-Gómez, Camacho-Tauta, and Reyes-Ortiz, 2016). In addition, fluctuations in the groundwater level induce variations in the lateral pressure, and those changes can be considered in the design of retaining wall construction (Ruge, 2014). Ausilio and Conte (2005) affirm that the groundwater level position is an issue when computing the bearing capacity of shallow foundations. Therefore, the groundwater level may affect the stability of geotechnical structures, such as tunnels.

Nevertheless, the study of the effect of the groundwater level variation has acquired relevance, especially, in the slope stability analysis. Reddi and Wu (1991) and Cascini, Calvello, and Grimaldi (2010) proposed a model to derive the time-dependent shear strength along the main slip surfaces. Conte and Troncone (2011) developed a method, based on a simple sliding-block model, to estimate the probability of failure in slopes, induced by the increment of pore water pressure during the rising of groundwater level. During seismic events, water under the surface controls the saturation degree, which can affect the soil strength due to the liquefaction phenomena (Soares and Viana da Fonseca, 2016).

The fluctuation of the groundwater level can be assessed, mainly, by statistical methods. Zhao, Li, Zhang, and Wang (2016) used a regression model to calculate the position of the water table. They validated the results using field measurements and found that the equation can predict the water level variations, with good precision. Han et al. (2016) implemented a groundwater level modelling framework through the coupling of two spatial and temporal clustering techniques. In addition, their procedure used self-organizing map technique to identify spatially homogeneous clusters of groundwater level piezometers. Yoon et al. (2016) predicted the long-term groundwater level fluctuation using a time series model and artificial neural network to evaluate the effect of rainfall on the soil.

In this study, we applied the analysis of repeated measures profiles to the estimation of the groundwater fluctuation. The remainder of this paper has four sections. The first section corresponds to the compilation of theoretical background and the description of the statistical method. The second section shows the data source and describes the soil composition of the study case. The third section refers to the validation of the null hypothesis of the technique. Finally, the fourth section presents the analysis and conclusions of this research.

Repeated measures profiles

Multivariate analysis allows to solve problems based on analytical criteria, which include all the variables involved (Gatingon, 2013). Hence, those methods can help to interpret better any type of information (Johnson and Wichern, 2007). Statistics methods do not only analyze numerical information, but they can also interpret graphical data with a quantitative approach. One of those methods is the repeated measures profiles, which analyzes graphical results.

Through a repeated measures profiles design, it is possible to estimate the variation of a response variable subjected to different treatments. As stated by Tabachnick and Fidell (2013), the data set may come from a dependent variable measured several times under the same pattern, i.e. the same independent variable is considered. This technique is an application of multivariate analysis of variance (MANOVA), where all samples (n) are measured within a fixed or constant variable. It focuses mainly on the comparison of variances, hence, the mean vectors of an specific treatment measured at the same level are compared (Friendly, 2010).

We evaluate three null hypotheses, according to Davis (2002). Those hypotheses are parallelism, flatness and coincidence. Harrar and Kong (2016) affirm that the technique seeks to respond the following questions: (*i*) whether there is interaction effect between-subjects and within subject factors, (*ii*) whether there is a between-subject factor effect, and (*iii*) whether there is a within-subject factor effect. Figure 1 presents a graphical representation of the null hypothesis.

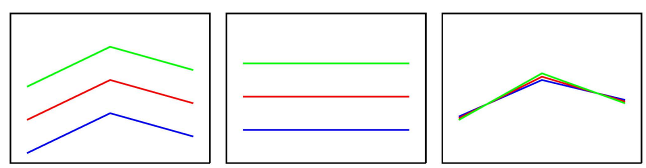

Figure 1. Null hypotheses: (a) parallelism; (b) flatness; (c) coincidence.
Source: Authors

Mathematically, the profiles of repeated measures analyze the variance or covariance of the data (Johnson and Wichern, 2007). Therefore, the method compares the matrix of the slope parameters. Timm (2004) suggested Equations (1-3) to describe the null hypotheses (H_0).

$$H_{0_1}: \begin{bmatrix} \mu_{1,1}-\mu_{2,1} \\ \mu_{2,1}-\mu_{3,1} \\ \vdots \\ \mu_{p-1,1}-\mu_{p,1} \end{bmatrix} = \begin{bmatrix} \mu_{1,2}-\mu_{2,2} \\ \mu_{2,2}-\mu_{3,2} \\ \vdots \\ \mu_{p-1,2}-\mu_{p,2} \end{bmatrix} = \begin{bmatrix} \mu_{1,3}-\mu_{2,3} \\ \mu_{2,3}-\mu_{3,3} \\ \vdots \\ \mu_{p-1,3}-\mu_{p,3} \end{bmatrix} \quad (1)$$

$$H_{0_2} : \mu_{slope} = \begin{bmatrix} \mu_1 - \mu_2 \\ \mu_2 - \mu_3 \\ \vdots \\ \mu_{p-1} - \mu_p \end{bmatrix} \quad (2)$$

$$H_{0_3} : \mu_{1,1} + \mu_{2,1} + \cdots + \mu_{p,1} \neq \mu_{1,2} + \mu_{2,2} + \cdots + \mu_{p,2} = 0 \quad (3)$$

where μ represents the media of the measurement in each repetition; p the number of the measurement and i the number of the profile.

Moreover, all the hypotheses validation has been programmed in several statistical software, due to the amount of information collected. Through those routines, it is possible to calculate the variance and significance level between data. Those criteria can be estimated via p-value, which is a parameter used to reject or not reject the null hypothesis in any statistical model (Habiger, 2015). In addition, the p-value provides results with a confidence level $1 - \alpha$. Therefore, if the p-value is lower than α, the null hypothesis is rejected (Wackerly, Mendenhall, and Scheaffer, 2008). However, it is important to establish previously if the hypothesis must be rejected or not. Bulut and Desjardins (2017) stated that the repeated measures profiles are parallel, flatness and coincident when the p-value $< \alpha$.

Data collecting

Information comes from the geotechnical exploration of the site designated as 1° de Mayo metro station. Such station is part of the most important infrastructure project in the city: the metro of Bogotá. For this project, an exhaustive ground identification, which included 15 months of geological-geotechnical exploration was performed. The Institute for Urban Development of the city affirms that they hired the drilling of 563 boreholes in the 27 km of the metro line (IDU, 2015a). Each perforation had 50 m depth and were approximately 100 meters from each other. Likewise, the studies identified the physical, mechanical and dynamical properties of the subsoil, through more than 2000 laboratory tests. The data used in this article comes from the records reported in the geotechnical study for the metro of Bogotá (IDU, 2015b) for the boreholes SE1-25, SE1-26, SE1-27 and SE1-28. Figure 2 presents the localization of the boreholes.

According to the information provided in the Decree 523 of 2010, "Microzonificación Sísmica de Bogotá", the soil of the area in Figure 2 corresponds to an alluvial material (Secretaría General de la Alcaldía Mayor de Bogotá D. C., 2010). Molina-Gómez, Moreno-Anselmi, and Arévalo-Daza (2016) explain that this type of soil has medium to high load-carrying capacity, low compressibility, medium liquefaction susceptibility and could be unstable in open excavations. In addition, Caicedo, Mendoza, López, and Lizcano, (2018) indicate that these deposits are located in plain areas composed of loose to compacted clayey sands. Table 1 shows the soil lithology of the zone based on the classification proposed by the Unified System of Soils Classification (USCS).

Figure 2. Location of the boreholes of the 1° de Mayo Station.
Source: Authors adapted from Google Earth®

Table 1. Soil classification and its physical properties

Depth (m)	SE1-25	SE1-26	SE1-27	SE1-27
2,4-3,0	ML	SM	SC	SM
6,0-6,6	SM	CL	SC	SM
9,0-9,6	CL	CL	CL	CL
11,4-12,0	MH	CL	CL	CL
20,0-20,6	SM-SC	SC	SM	CL-ML
24,0-24,6	CL	SM	CL	SC
29,4-30,0	SM	CL	CL	CL

Source: Authors adapted from IDU (2015b)

The zone for the 1° de Mayo metro station is located in the area of Tunjuelo River, which is a basin in the locality of Kennedy. The underground water flow of such river moves in southwest to northeast direction along 73 km distance. In addition, the slope of this aquifer changes 15 to 3, with an average slope of 5 at the south of Bogotá. According to geological-geotechnical reports for the first line of metro (Oteo-Mazo, 2015), the piezometric level of the aquifer is between 2560 and 2540 masl. Figure 3 presents the hydrogeological profile of the 1° de Mayo Station.

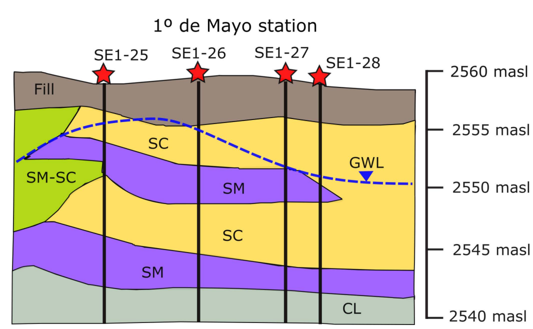

Figure 3. Hydrogeological profile of the 1° de Mayo Station.
Source: Authors

In the area for the 1° de Mayo Station construction, piezometers were positioned after drilling at the four research points for the future metro station (Figure 4). Piezometric readings were registered daily. For this research, readings at the same specific time were selected, in order to ensure repeated measures in the entire exploration site.

Figure 4. Piezometric space for groundwater level reading into borehole SE1-26.
Source: IDU (2015b)

Results and analysis

We used ***RStudio*** to process the data, which is a free statistical software based on an object-oriented algorithm. This software allows data plotting and information modelling by several techniques. In addition, ***RStudio*** has different packages to estimate the data variance using different multivariate techniques.

In this research, we used *profile*R package, which was proposed by Bulut, Davison, and Rodriguez (2017). Computation procedure covers an experiment design, including calculation of variance through sums squares matrix and vector products of the Equations (1-3). This tool provides a set of multivariate methods and data visualization options to implement profile analysis and cross-validation techniques described by Bulut (2013) and Davison and Davenport (2002). Likewise, it includes routines to perform criterion-related profile analysis, profile analysis via multidimensional scaling, moderated profile analysis, profile analysis by group, and a within-person factor model to derive score profiles. In addition, it allows to compare simultaneously the effect of treatments by univariate techniques as the Hotelling's T^2 test.

Furthermore, we used four different techniques to assess the parallelism of groundwater level along the boreholes. Those statistics were Pillai trace, Wilks' Lambda, Hotelling-Lawley trace and Roy's largest root. The aforementioned statistical models were described by Molina-Gómez et al., (2016). We validated the null hypothesis using the application of the procedure proposed by Bulut and Desjardins (2017), which evaluates the profiles of repeated measures with the *profile*R package.

Based on the exploration records, we selected thirty different measures of groundwater level during thirty different days. The measures started on March, 2014 and finished on April, 2014. During the monitoring period, there are no values of groundwater level at the surface. Topographic records indicate that all the research points and piezometers are at the elevation position, 2558 masl. Figure 5 shows the descriptive exploration of the groundwater level readings in the research points.

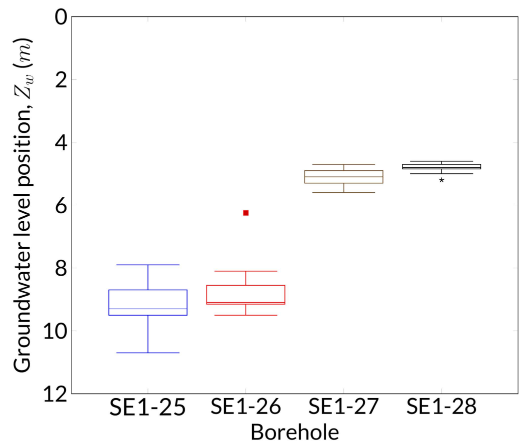

Figure 5. Box plot of the measures.
Source: Authors

Results revealed an interaction of information between the boreholes SE1-25 with SE1-26 and SE1-27 with SE1-28. We found similar inter-quartile distribution in all measures. However, we identified an outlier in SE1-26 and SE1-28, which is the first measure in SE1-26, and a change during monitoring in SE1-28.

In order to satisfy the homogeneity of variances assumption, using the median, we performed the Levene's test before executing the analysis of repeated measures profiles. The null hypothesis of this test assesses if all the populations' variances are equal and the alternative hypothesis considers that at least two of them differ. Results showed that the groundwater variances of the profiles are homogenous, under a confidence level of 99 (α = 0,01). If data satisfy the homogeneity condition, it is valid to apply the analysis of repeated measures profiles technique. Table 2 presents the Levene's model outcomes.

Table 2. Results of the homogeneity of variance evaluation

Source	DF	*F*-test	p-value
Boreholes	3	3,6153	$1{,}53 \times 10^{-2}$
Residuals	116		

Source: Authors

Figure 6 presents the repeated measures profiles of the groundwater measurements. From a qualitative viewpoint, we observed that all the profiles remain constant along the readings. Nevertheless, we confirmed the outlier of Figure 6 and a decreasing groundwater level during day

23, as well as a lower fluctuation in SE1-28. Possible reasons for the variation are the soil permeability and the material heterogeneity. According to the soil classification presented in Table 1 and the contrast with the typical values of permeability coefficient presented in the literature by Warrick (2003), it is possible to affirm that the soil has intermediate hydraulic conductivity and could experience fluctuations during short periods of time.

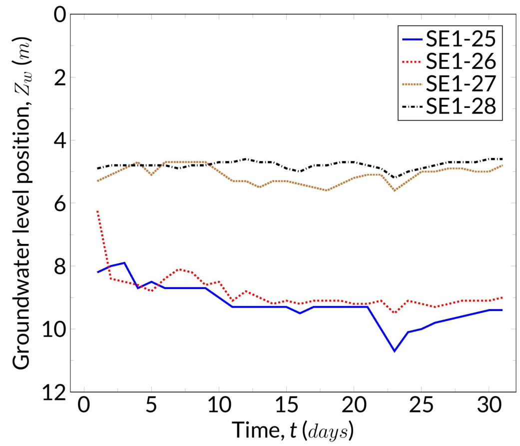

Figure 6. Groundwater level profiles.
Source: Authors

Table 3 presents the analysis of the repeated measures results. We assessed the three null hypotheses of the method, under a confidence level of 99 (α = 0,01). The value of α selected is based on the experiments design in the literature by Kuehl (2000) and Ramachandran and Tsokos (2009). Since the p-value obtained is less than α, there is statistical evidence to reject all the hypotheses (Habiger, 2015; Wackerly et al., 2008) under a confidence level of 99. Therefore, results suggest that the profiles of groundwater level are flat (horizontal), but not parallel and neither coincident.

Statistical results showed that the groundwater level of the aquifer remains constant during the period of study. In addition, under the area for the station construction, the water flows through stratified zones where the piezometric readings change from a research point to the other due to the effect of the equivalent hydraulic conductivity of the layered soil (Dulcey-Leal, Molina-Gómez, and Bulla-Cruz, 2018). However, for underground constructions, like the metro, it is necessary to ensure a constant groundwater level using a set of monitoring/control procedures, in order to avoid additional pore-water pressures that can increase soil lateral stresses. In this context, flatness hypothesis (horizontality) is the most important null hypothesis, H_{02}, since it establishes the uniformity of groundwater level during the monitoring period.

Conclusions

This article addressed a statistical assessment of groundwater variations for underground constructions. Results present a quantitative procedure to estimate the uniformity

Table 3. Validation results of null hypotheses and variance

Hypothesis Tested:	Multivariate Test	*F*-test	*p*-value
H_{01}: Profiles are parallel	Wilks' Lambda	54 767,19	$1,22 \times 10^{-51}$
	Pillai trace	230,55	$3,96 \times 10^{-28}$
	Hotelling-Lawley trace	819 581	$3,14 \times 10^{-55}$
	Roy's largest root	313 737	$4,76 \times 10^{-31}$
H_{02}: Profiles are flat (horizontal)	–	56,44	$2,03 \times 10^{-2}$
H_{03}: Profiles are coincidental	–	1 449 329	$2,17 \times 10^{-16}$

Source: Authors

of the groundwater level position. We analyzed four different research points using data collected for the design of the infrastructure project "Metro de Bogotá". The points are continuous and cover a distance of 160 m. From results, the following conclusions were drawn:

1. We obtained a profile of repeated measures. We analyzed data that came from the geotechnical studies of the "Metro de Bogotá" project, including thirty different measures of groundwater level position. By visual inspection, not all the boreholes presented variations in the groundwater level position, except in day 23. However, we identified that the plot does not indicate the degree or quantitative value of the variation of such level.

2. We implemented a multivariate graphical statistical method, the analysis of repeated measures profiles, and assessed the variation of groundwater level for underground constructions. We evaluated the null hypotheses of the repeated measures analysis. Outcomes showed that the profiles are flat (horizontal), but are not parallel and not coincident under a confidence level of 99 (α = 0,01). Thus, we found no variations of groundwater level in a period of 30 days, which indicates no probable pore-water pressure build-up and no increments in the lateral stress of the soil during the station construction.

3. Analysis of repeated measures profiles showed that, when assessing groundwater variations, the most important null hypothesis is H_{02}, which determines the horizontality of the profiles. Hence, the measurements of the four boreholes have a between-subject factor effect. In this way, we found that there are no variations of the groundwater level. Therefore, we can affirm that the groundwater level is statistically uniform, even if we observed some variations graphically.

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