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Comparative Study of Theoretical and Real Deflection of Simple and Reinforced Concrete Joists

Estudio comparativo de la deflexión teórica y real de viguetas de concreto simple y reforzado

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ABSTRACT

The objective of this research is to determine the real deflection of a concrete joist and correlate the result with theoretical deflection, which is based on a stress vs. deformation model which was proposed by Mander *et al.* (1988) for monotonic loads of reinforced and non-reinforced concrete. The construction of a concrete joist does not result in a 100% homogenous, isotropic, and linearly elastic element, since its production depends on many conditions, such as aggregate selection, water, cement manufacturing, tests performed for mixture design, the operator in charge of the mixture, and the construction of the joist. Therefore, research was carried out on the variation of real reflection with respect to theoretical calculations. To this effect, 30 simple-concrete and 30 reinforced-concrete joists were elaborated. They were tested by measuring their maximum deflection and comparing it to its theoretical counterpart. To calculate the theoretical deflection, a curvature moment diagram was elaborated with the Rect_Mom software by Restrepo and Rodríguez (2012), which uses the model by Mander *et al.* (1988). Experimental results showed a greater deflection than the one reported by theoretical calculations.

Keywords: joist deflection, inelastic behavior, curvature moment

RESUMEN

Esta investigación tiene como objetivo determinar la deflexión real de una vigueta de concreto y correlacionar el resultado con la deflexión teórica, que está basada en el modelo esfuerzo vs. deformación propuesto por Mander *et al.* (1988) para cargas monótonas de concreto reforzado y no reforzado. La construcción de una vigueta con concreto no conforma un elemento 100 % homogéneo, isótropo y linealmente elástico, ya que su fabricación depende de muchas condiciones como la elección de los agregados, el agua, la fabricación del cemento, los ensayos realizados para el diseño de mezclas, el operario a cargo de la mezcla y la construcción de la vigueta. Por ello se investigó la variación de la deflexión real con respecto a los cálculos teóricos, para lo cual se fabricaron 30 viguetas de concreto simple y 30 viguetas de concreto armado, que se ensayaron midiendo la deflexión máxima y se comparándola con la teórica. Para el cálculo de la deflexión teórica se elaboró el diagrama de momentos de curvatura con el programa Rect_Mom de Restrepo y Rodríguez (2012), que utiliza el modelo de Mander *et al.* (1988). Los resultados experimentales mostraron una deflexión mayor que los cálculos teóricos.

Palabras clave: deflexión en viguetas, comportamiento inelástico, momento de curvatura

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Introduction

The calculation of deformations or deflections in reinforced concrete elements subjected to bending is important because these elements must have an adequate rigidity to eliminate any deformation along a structure, which constitutes a risk for its resistance or operation under service conditions (Carrillo and Silva-Páramo, 2016; Carrillo, Cárdenas Pulido, and Aperador, 2017).

Likewise, reinforced concrete elements are used when there is a deficit in any of the properties of a structure due to a new state of charge during its useful life. These increased loads generally result from the state of service for which these deflections must be controlled (Falope, Lanzoni, and Tarantino, 2019)

It is important to know the responses and resistant mechanisms present in a reinforced concrete element when subjected to different kinds of stress. These factors can be measured through experimental tests, which allows verifying the theories formulated by standards or studying other theories (Chiorean and Buru, 2017).

Carrying out this type of study is important because it will help us have a clearer perspective of real deflection (depending on compressive strength, applied load, reinforcement and geometry), which will allow adjusting the calculations. It is important to verify deformations in structural elements with little inertia to ensure the operation of the structure as a whole and not endanger human life (Hemn Qader, Dishad Kakasor, and Abdulkhaleq, 2020).

Regarding deflections and cracking, adequate service behavior can be achieved in beams with less than the maximum allowable by standard E 060 (L 300). For 200 x 300 mm beams with 290 cm of free length between supports and 30% of redistribution in negative steel, deformations of 7,90 mm of deflection and 0,35 mm of cracking were obtained (Ministerio de Vivienda, Construcción y Saneamiento, 2009).

For the comparison of both general models and simplified formulae, experimental data are required which adequately represent the magnitudes of the most significant variables of the structural elements with deformation problems, as well as sufficient complementary data to allow the theoretical analysis of the problem (Purushothama Raj and Ramasamy, 2012).

The only way to rationalize force and displacement factors is by quantifying the relationships of resistance and structural ductility through analytical studies and experimental tests, determining design forces and displacements in a more rational way (related theories) and contemporary trends in building code (Carrillo, Blandón Valencia, and Rubiano, 2013; Ismail *et al.*, 2018).

Materials and methods

This research was experimental. Several mixtures were designed, whose compressive strength varied from $f'c$ = 280 kg/cm^2 up to 400 kg/cm^2. The aggregates were from the quarries of Tres Tomas, Pátapo, and Batangrande, which are located in the Lambayeque region, as well as from Talambo, near Chepén, which belongs to the region of La Libertad. Using type I cement, the physical properties of these mixtures were studied for the purpose of preparing concrete mixtures. Simple and reinforced concrete joists were manufactured with dimensions of 15 cm x 15 cm x 53,5 cm, which were flexurally tested at 7, 14, and 28 days. At the same time, specimens were produced to obtain compressive strength at the same ages as the joist. A mix was designed for each quarry, and 02 joists were tested for each break, with 10 joists at 7, 14, and 28 days for both simple and reinforced concrete. The flexion of a total of 60 joists was therefore tested (Alhajri, Tahir, Azimi, Mirza, and Ragaee, 2016).

Figure 2 shows the molds used to manufacture the joists, Figure 3 shows the specimens of the tested joists, Figure 4 shows the bending test of the joist, and Figure 5 shows the break of the joist after being tested.

Figure 1. Joist formwork.
Source: Authors

Figure 2. Joist reinforcement.
Source: Authors

Figure 3. Simple and reinforced joists.
Source: Authors

Figure 4. Beam flexure test.
Source: Authors

Figure 5. Joist fissure.
Source: Authors

The resistance design method, together with the use of higher resistance concrete and steels, has allowed the use of relatively slim elements. Consequently, deflections and deflection cracking have become more severe problems than they were a few decades ago (Hemn Qader, Dishad Kakasor, and Abdulkhaleq, 2020; Luo, *et al.*, 2019).

One of the best ways to reduce deflections is by increasing the cant of the members, but the designers are always under pressure to keep the members with the cant as low as possible. Another solution is improving the quality of the material's resistance to deformation, in other words, increasing the elasticity modulus of the material. For this reason, it is necessary to have an adequate calculation of deflections, so as not to affect the resistance or functionality of the analyzed structure. If the designer decides not to use the minimum thicknesses given in Table 1, then he or she will be forced to determine the actual deflections, which must not exceed the values in Table 2.

Table 1. Cant or minimum thickness of non-prestressed beams or reinforced slabs in one direction unless deflections are calculated

	Minimum thickness or cant h			
Elements	**Simply supported**	**With a continuous end**	**Both ends continuous**	**Cantilever**
	Elements that do not support or are linked to divisions or other types of non-structural elements susceptible to damage due to large deflections			
Solid tiles in one direction	$\frac{\ell}{20}$	$\frac{\ell}{24}$	$\frac{\ell}{28}$	$\frac{\ell}{10}$
Beams or slabs ribbed in one direction	$\frac{\ell}{16}$	$\frac{\ell}{18.5}$	$\frac{\ell}{21}$	$\frac{\ell}{8}$

Source: Table 9.1 of Peruvian Building Standard E 060 (Ministerio de Vivienda, Construcción y Saneamiento, 2009).

Elastic methods are used to obtain and determine the equations to define the slope and elastic curve of a beam (Hamrat *et al.*, 2020).

The lateral part of the surface of a deformed beam is called the elastic, deformed, or elastic curve of the beam. It is the curve that forms the longitudinal axis, which at the beginning was straight. As shown in Figure 6, it is in this section that we can deduce the elastic curve, which also allows us to determine the deflection of any point based on its length or X coordinate.

The left end is the origin of the x axis, directed according to the initial direction of the beam without deforming, and the positive y axis upward. The deformations are so small that it is not possible to distinguish between the initial length and the projection of its already deformed length. Therefore, the elastic curve is very flat and its slope at any point is also very small.

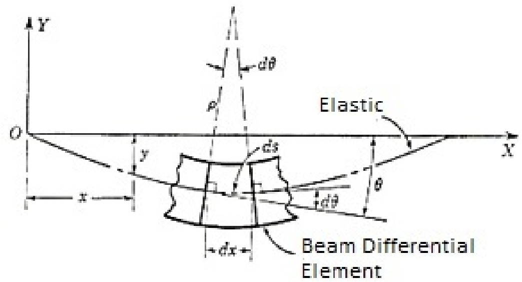

Figure 6. Elastic curve of a beam.
Source: Singer and Pytel (1994)

Table 2. Maximum allowable deflections

Item type	**Considered deflection**	**Deflection limit**
Flat roofs that do not support or are linked to non-structural elements susceptible to damage due to large deflections	Immediate deflection due to live load	$\ell/180°$
Floors that do not support and are linked to non-structural elements susceptible to damage due to large deflections	Immediate deflection due to live load	$\ell/360°$
Floors or ceilings that support and are linked to non-structural elements susceptible to damage due to large deflections	The part of the total deflection that occurs after the union of the non-structural elements (the sum of the long-term deflection due to all permanent loads, and that of immediate deflection due to any additional live load)	$\ell/480$
Floors or ceilings that support or are linked to non-structural elements not susceptible to damage due to large deflections.		$\ell/240$

Source: Table 9.2 of Peruvian Building Standard E 060 (Ministerio de Vivienda, Construcción y Saneamiento, 2009).

Some structural analysis problems can be solved using linear analysis, but the geometric nonlinearity, the nonlinearity due to the behavior of the material, and the nonlinearity due to the boundary conditions change when posing and solving non-linear problems (Beléndez, Neipp, and Beléndez, 2002; Ismail *et al.*, 2018).

Theoretical deflection

To make a prediction of the experimental displacement, the deflection was calculated from the analytical calculations described below.

The joists were modeled using the SAP2000 software with frame elements that can be used to model beams, columns, braces, and trusses in planar and three-dimensional structures. Nonlinear material behavior is available through frame hinges and includes the effects of biaxial bending, torsion, axial deformation, and biaxial shear deformations.

A frame element is modeled as a straight line connecting two points, and each element has its own local coordinate system for defining section properties and loads, as well as for interpreting output, as shown in Figure 7.

Figure 7. Joist modeled with frame elements in the SAP2000 program.
Source: Authors

To consider inelastic behavior, a plastic hinge was located in the center of the span, which is the point where the greatest deformation occurs.

When nonlinear properties are present in the element, they only affect nonlinear analyses. Linear analyses starting from zero conditions (the unstressed state) behave as if the nonlinear properties were not present. Linear analyses using the stiffness from the end of a previous nonlinear analysis use the stiffness of the nonlinear property as it existed at the end of the nonlinear case (Alhajri *et al.*, 2016; Luo *et al.*, 2019)

Each hinge represents concentrated post-yield behavior in one or more degrees of freedom. Hinges only affect the behavior of the structure in nonlinear static and nonlinear time-history analyses.

Since the predominant behavior was bending, the M3 ball joint type was used, which was also in the program's library. Hinge properties can be computed automatically from the element material and section properties according to Federal Emergency Management Agency FEMA-356 or American Society of Civil Engineers ACSE 41-13 criteria (FEMA and ASCE, 2000). For our case, the properties were entered manually and obtained from the curvature moment diagram of the joist section.

To obtain the curvature moment diagram, the Rect_Mom software (Rodriguez and Restrepo, 2012) was used. This application uses the model proposed by Mander *et al.* (1988) shown in Figure 8 for concrete modelling. This behavior allows the effect of the interaction between concrete and reinforcement bars by introducing tension reinforcement into the softening side of the curve (Sinaei, Mohd Zamin, and Mahdi, 2011).

Likewise, for the reinforcing steel, the Mander model was used, which is shown in Figure 9. This model considers three zones: the elastic zone, the creep zone, and the strain hardening zone.

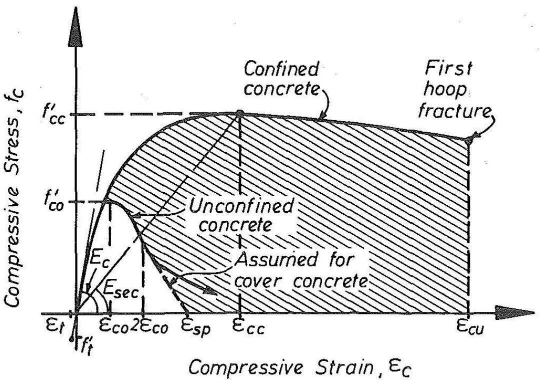

Figure 8. Stress-strain model proposed for monotonic loading of confined and uncofined.
Source: Mander *et al.* (1988)

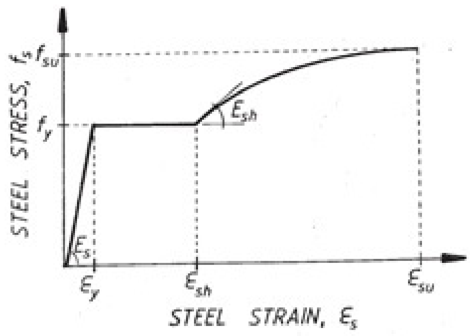

Figure 9. Monotonic stress-strain curve typical of a rebar.
Source: Mander *et al.* (1983)

Real Deflection

For the experimental deflection, the Peruvian Technical Standard NTP 339.079 testing method was applied to determine the flexural strength of concrete in simply supported beams with loads in the center of the span (Comisión de Normalización y Fiscalización de Barreras Comerciales no Arancelarias - INDECOPI, 2012).

The equipment to perform the test had to comply with the requirements of the sections based on the verifications, corrections, and the time interval between verifications. The mechanism by which loads are applied to the specimen employed one load application block and two specimen support blocks. The load was to be applied perpendicularly to the upper face of the beam, in such a way that eccentricities could be avoided (Comisión de Normalización y Fiscalización de Barreras Comerciales no Arancelarias - INDECOPI, 2012).

The specimens on which the tests were carried out had to be prepared according to the test method indicated above to meet the required compressive strength. The beam had a free span between supports approximately three times its height, with a tolerance of 2%. The lateral faces of the beam formed right angles with the upper and lower face. All surfaces were smooth and free of any porosity, according to Figure 10 (Comisión de Normalización y Fiscalización de Barreras Comerciales no Arancelarias - INDECOPI, 2012).

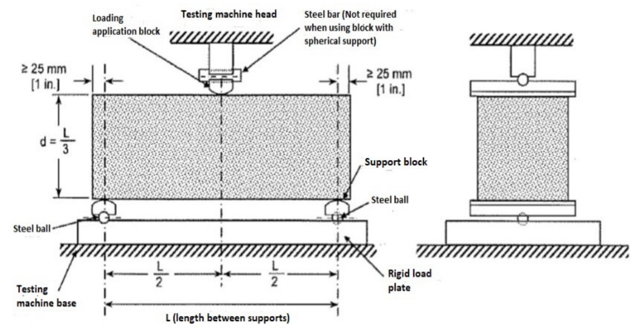

Figure 10. Diagram of a suitable device for concrete flexural testing by the mid-point load method and the application of the double integration method in a simply supported beam.
Source: Comisión de Normalización y Fiscalización de Barreras Comerciales no Arancelarias - INDECOPI, 2012.

During the test, the beam had to be loaded continuously and without impacts. The load must be applied at a constant speed until breakage is reached (Comisión de Normalización y Fiscalización de Barreras Comerciales no Arancelarias - INDECOPI, 2012).

Justification of the investigation

The calculation of the deflections of a beam is carried out according to the theory of elasticity, considering concrete as a linearly elastic material, even though the deflections of the beam are actually due to the nonlinearity of the material, which is the result of external factors such as its composition (cement, aggregates, water), the preparation of the concrete mixture, and the construction of the beam. To date, these conditions cannot be mathematically modeled and are part of the calculation of the theoretical deflection.

Results

This section presents the results of the analysis performed in the laboratory and the correlation between theoretical and real deflection, as shown in Tables 3, 4, and 5. The test was performed for 3 different cure times: 7, 14, and 28 days. The elasticity modulus was calculated, as indicated by the Peruvian building standard E 060 and the inertia of the cross section (Ministerio de Vivienda, Construcción y Saneamiento, 2009). Likewise, with the load resulting from the test, the theoretical deflection was calculated. After reviewing the results of the theoretical and actual deflection, we concluded that there is a very wide difference between the two values. We verify that the theoretical calculations do not reflect the actual deformation of the element, which is due to the nonlinearity of concrete.

The obtained moment curvature diagram shown in Figure 11 is simplified by using bilinear approximation in the SAP2000 program; the abscissa is multiplied by the plastic length to express it as a function of rotation and reduced only to the part plastic diagram.

Table 3. Results of compressive strength, elasticity modulus at 7, 14 and 28 days of the tested concrete and cross section of the joist (area = 176,7 cm^2, joist cross-section = 225 cm^2, and length between supports = 45 cm)

Quarries name	Time (days)	Applied load (kg)	Compressive strength f'_c (kg/cm2)	Elasticity Module $1\,500\,0\sqrt{f'_c}$ (kg/cm2)
Olmos	7	35 289	199,71	211 978,91
	14	42 309	239	231 894,37
	28	51 685	292	256 320,11
Talambo Chepen	7	43 466	246	235 265,81
	14	46 738	264	243 721,15
	28	50 899	288	254 558,44
Tres Tomas	7	32 924	181	201 804,36
	14	44 762	250	237 170,82
	28	49 585	281	251 445,82
Patapo	7	38 612	218	221 472,35
	14	46 122	261	242 332,42
	28	61 408	347	279 419,04
Batan Grande	7	40 290	228	226 495,03
	14	57 432	325	270 416,35
	28	71 216	403	301 122,90

Source: Authors

Table 4. Inertia, length between supports, applied point load, theoretical and real deflection of the simple concrete joist (inertia = 4 218,75 cm^4, length between supports = 45 cm)

Quarries name	Punctual face (kg)	Elasticity Module $5\,000\sqrt{f'_c}$ (kg/cm2)	Theoretical deflection (mm)	Real deflection (mm)
Olmos	2 200	211 978,91	1,8577	2,300
	2 940	231 894,37	4,5484	5,200
	3 450	256 320,11	6,2003	6,400
Talambo Chepén	3 375	235 265,81	1,9251	1,800
	3 883	243 721,15	2,5851	2,900
	4 180	254 558,44	4,4996	3,700
Tres Tomas	1 150	201 804,36	2,2711	1,900
	1 370	237 170,82	3,2399	3,800
	1 550	251 445,82	7,3146	6,400
Pátapo	3 220	221 472,35	2,8526	3,150
	3 850	242 332,42	5,3213	4,950
	4 550	279 419,04	5,1367	5,780
Batangrande	2 180	226 495,03	1,5802	1,300
	2 986	270 416,35	4,3729	3,600
	3 504	301 122,90	6,5544	5,500

Source: Authors

Once the plastic hinge was defined and assigned, a point load was applied which was gradually increased according to the laboratory test until the failure of the section was reached. The values obtained are shown in Tables 4 and 5.

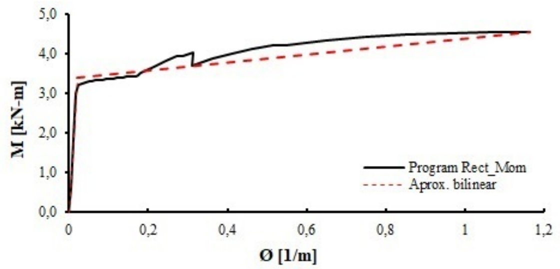

Figure 11. Moment-Curvature diagram for joist section.
Source: Authors

Table 5. Inertia, length between supports, applied point load, theoretical and real deflection of the reinforced concrete beam (inertia = 4 218,75 cm^4, length between supports = 45cm)

Quarries name	Punctual face (kg)	Elasticity Module $15\,000\sqrt{f_c'}$ (kg/cm2)	Theoretical deflection (mm)	Real deflection (mm)
Olmos	6 874	211 978,91	2,8900	2,500
	7 997	231 894,37	4,1607	4,300
	10 450	256 320,11	9,7037	8,400
Talambo Chepen	6829	235 265,81	4,2158	3,680
	8 354	243 721,15	11,3128	10,320
	9 359	254 558,44	13,2863	15,460
Tres Tomas	2980	201 804,36	5,7345	4,875
	3 780	237 170,82	5,1149	5,535
	4 150	251 445,82	7,4091	6,485
Patapo	7 650	221 472,35	7,2040	7,900
	8 985	242 332,42	10,6756	10,120
	10 658	279 419,04	11,4471	12,390
Batan Grande	3 150	226 495,03	7,8125	7,560
	3 390	270 416,35	5,7550	5,675
	3 431	301 122,8985	3,9517	4,875

Source: Authors

Figure 12 shows the diagram of the theoretical and real deformation of a simple concrete beam per quarry, where non-linear trends are observed between the theoretical and real deformation.

The correlation of the theoretical and real deflection of a simple concrete joist is shown in Figure 13, obtaining an Equation (1) of degree 4: $y = (-0{,}33764)\,x^4 + (0{,}01252)\,x^3 + (-0{,}71775)\,x^2 + (6{,}05057)\,x + 3{,}91200$, which was a good model of adjustment because it had an acceptable coefficient of determination ($R^2 = 0{,}9123$), because the 99% confidence interval included al the pairs of observed values, and because the result of the analysis of variance indicated that at least one coefficient of the polynomial model is significantly different from zero (p-value = $1{,}788e^{-07}$).

A very good positive correlation was found between the theoretical and actual deformation of a simple concrete beam (R = 0,9551).

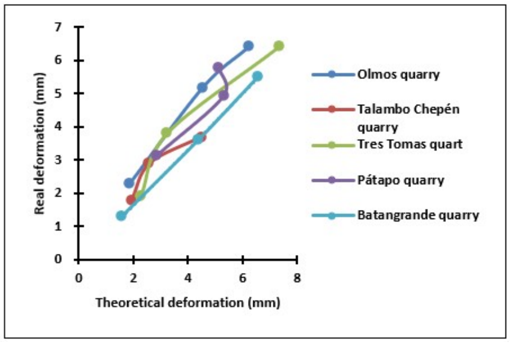

Figure 12. Diagram of the theoretical and real deformation of the simple concrete joist by quarry.
Source: Authors

According to the Shapiro Wilk test, residues originating from the order 4 polynomial model presented a normal distribution (p-value = 0,3857).

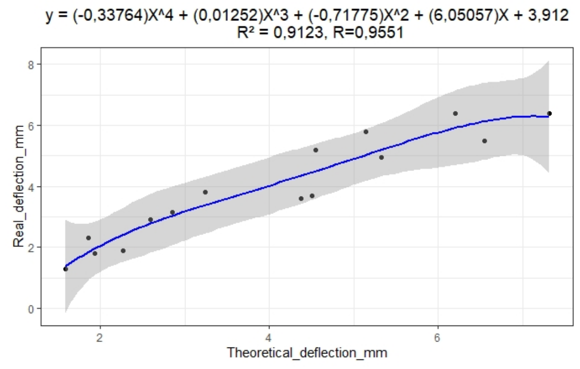

Figure 13. Correlation of theoretical and real deformation of a simple joist.
Source: Authors

Figure 14 shows the diagram of the theoretical and real deformation of a quarry-reinforced concrete beam, where non-linear trends are observed between the theoretical and real deformation, but these together will give rise to figure 15.

The correlation of the theoretical and real deflection of the reinforced concrete joist is shown in figure 15, obtaining an Equation (II) of degree 4: $y = (-0{,}0652)\,x^4 + (1{,}7485)\,x^3 + (1{,}7920)\,x^2 + (12{,}7575)\,x + 7{,}3383$, which was a good model of adjustment, because it had an acceptable coefficient of determination ($R^2 = 0{,}9686$), because the 99% confidence interval included al the pairs of observed values, and because the result of the analysis of variance indicates that at least one coefficient of the polynomial model is significantly different from zero (p-value = $1{,}788e^{-07}$).

A very good positive correlation was found between the theoretical and actual deformation of the reinforced concrete beam (R = 0,9842).

According to the Shapiro Wilk test, the residuals originating from the order 4 polynomial model were adjusted for the normal distribution (p-value = 0,2811).

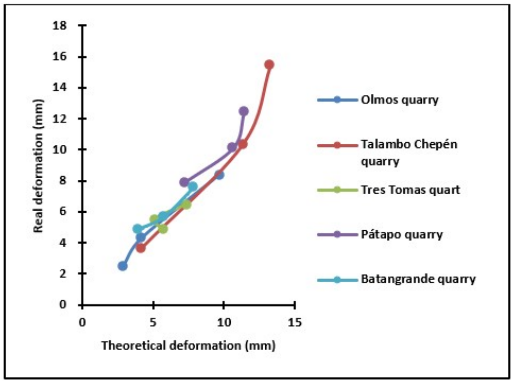

Figure 14. Diagram of the theoretical and actual deformation of a reinforced concrete joist by quarry.
Source: Authors

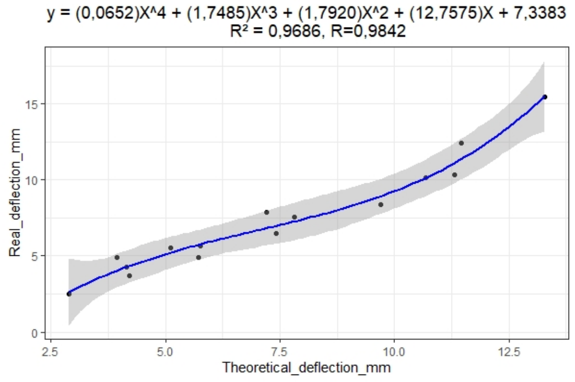

Figure 15. Correlation of theoretical and real deformation of a reinforced concrete joist.
Source: Authors

Conclusions

Based on the results obtained regarding the correlation of theoretical and practical deflection, we reached the following conclusions:

1. A very good positive non-linear correlation was found between the theoretical and actual deformation of both the simple and reinforced concrete beams.
2. The actual deflections for both the simple beam and the reinforced beam are greater than those calculated by the model proposed by Mander *et al.* (1988).
3. The evaluated analysis of the experimental results and the parameters calculated using the developed methodology is not within the expected ranges reported by the literature.
4. For subsequent work, it is recommended to make the stress-strain diagram of the joist to make a more detailed comparison of how it varies and where the divergence between the theoretical and actual deformation lies.
5. It was also concluded that the days of concrete curing influence the deflection of the joists, obtaining less resistance to deflection during the first weeks.

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