A Multicriteria Approach to Integration and Evaluation of Investment Portfolios: The New York Stock Exchange

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Abstract: One of the problems investors often face is deciding on the stocks to include in an investment portfolio. Hence, this article seeks to select investment portfolios considering the 30 leading companies listed on the New York Stock Exchange (NYSE) of the Dow Jones Index. Portfolio selection in this index is carried out by generating a previous ranking of the shares with a novel approach that analyzes their performance using a multiple criteria hierarchical process (MCHP). The current research allows the evaluation of shares and optimizing a portfolio, while developing a procedure that supports a decision-making process for organizations or practitioners to invest in the stock market. The results of analyzing the NYSE generated a portfolio that can be used for interested investors. The main contribution of the application of MCHP and Markowitz model is related to the possibility of replicating the developed procedure to select portfolios in other stock markets.

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UN ENFOQUE MULTICRITERIO PARA LA INTEGRACIÓN Y EVALUACIÓN DE PORTAFOLIOS DE INVERSIÓN: EL CASO DE LA BOLSA DE VALORES DE NUEVA YORK

Resumen: uno de los problemas que a menudo enfrentan los inversores es la decisión sobre qué acciones incluir en un portafolio de inversión. Por ello, este artículo presenta una selección de portafolios de inversión con base en información de las 30 principales empresas listadas en la Bolsa de Valores de Nueva York (NYSE, en inglés) y el índice bursátil Dow Jones. La selección de portafolios en este índice se lleva a cabo mediante la conformación de un ranking de las acciones que es generado tras aplicar un enfoque novedoso que analiza su rendimiento empleando un proceso jerárquico multicriterio (MCHP, en inglés). Así, esta investigación permite evaluar las

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acciones y la optimización de un portafolio, al tiempo que desarrolla un procedimiento para la toma de decisiones por parte de organizaciones o profesionales interesados en invertir en el mercado de valores. Los resultados del análisis de la NYSE permitieron generar un portafolio que puede ser utilizado a manera de referencia por los inversionistas. El principal aporte de la aplicación del MCHP y el modelo de Markowitz es la posibilidad de replicar el procedimiento desarrollado para seleccionar portafolios en otros mercados de valores.

Palabras clave: ELECTRE-III, modelo de Markowitz, proceso jerárquico multicriterio, Bolsa de Nueva York, selección de cartera, bolsa de valores.

UMA ABORDAGEM MULTICRITÉRIO PARA INTEGRAR E AVALIAR PORTFÓLIOS DE INVESTIMENTO: O CASO DA BOLSA DE VALORES DE NOVA YORK

Resumo: um dos problemas que os investidores frequentemente enfrentam é decidir quais ações devem ser incluídas em uma carteira de investimentos. Portanto, este artigo apresenta uma seleção de carteiras de investimento com base nas informações das 30 principais empresas listadas na Bolsa de Valores de Nova York (NYSE, em inglês) e no índice de ações Dow Jones. A seleção de carteiras nesse índice é realizada por meio da formação de uma classificação de ações gerada após a aplicação de uma nova abordagem que analisa seu desempenho usando um processo de classificação multicritério (MCHP, em inglês). Assim, esta pesquisa permite a avaliação de ações e a otimização de um portfólio, ao mesmo tempo que desenvolve um procedimento de tomada de decisão para organizações ou profissionais interessados em investir no mercado de ações. Os resultados da análise da NYSE possibilitaram a geração de um portfólio que pode ser usado como referência pelos investidores. A principal contribuição da aplicação do MCHP e do modelo de Markowitz é a possibilidade de reproduzir o procedimento desenvolvido para selecionar porfólios em outros mercados acionários.

Palavras-chave: ELECTRE-III, modelo de Markowitz, processo hierárquico multicritério, Bolsa de Valores de Nova York, seleção de carteiras, mercado de ações.

INTRODUCTION

Investment decision-making corresponds to the process of analyzing alternatives for making investments. An investment strategy is considered the main element in the stock market study since stock market shares present a certain risk, where possible strategies to reduce uncertainty are hedging insurance or investment diversification (Buchner *et al*., 2017).

The investment portfolio allows obtaining different types of assets to achieve diversification. Markowitz's mean-variance model establishes an investment portfolio with various risk assets, reducing risk because of diversification without reducing its expected return (Markowitz, 1959). Recently, we can find some crucial criticisms towards Markowitz's model concerning the exclusion of investor preferences. The evidence focuses on portfolio optimization, where investors prefer portfolios behind the non-dominated frontier, although other portfolios dominate them to two objectives, such as expected return and risk (Ehrgott *et al*., 2004). It means that investors have specific preferences that are not reflected in non-dominated portfolios and, as a result, not all relevant information can be captured for an investment decision regarding return and risk. Nevertheless, it is possible to interpret this situation since the model considers only the aspects of the available information of the shares but does not consider some elements intrinsic to the investor's needs.

In a multicriteria analysis approach, it is essential to consider the decision-maker's preferences, such as portfolio selection, due the point of view of the investor's profile in the relative importance of criteria (Ehrgott *et al.*, 2004). A model that considers investor preferences, including other decision criteria besides profitability and risk, will show a portfolio closer to the investor's preferences. In this sense, analytical tools are required to meet the new demands in decision-making processes.

The portfolio selection for investment has been addressed as different problematic areas, such as sorting (Xidonas *et al.*, 2017), ranking (Basilio *et al.*, 2018), and portfolio problematics (Vetschera & de Almeida, 2012). With that in mind, this paper addresses the selection of shares as a two-stage approach (Mansour *et al.*, 2019) where, first, stocks are compared with a ranking method and, then, portfolios are optimized. The procedure is similar to that performed by Vetschera and de Almeida (2012).

This research is carried out through the adaptation of the hierarchical process of multiple criteria, similar to the process followed by Bernal *et al.* (2021), due to the natural hierarchy that the problem of selection of shares presents in its criteria, considering seven macrocriteria (groups of criteria): market, results operation, market value ratios, financial and economic profitability, liquidity, effectiveness, and dividends for the optimal evaluation of the different investment portfolios.

Consequently, this paper seeks to generate investment portfolios through the conjunction of two different methodologies for the selection of fair shares and the generation of portfolios aligned with investors' preferences. On the one hand, a multiple criteria decision-making methodology is applied to select shares based on investors' preferences and, on the other, the Markowitz model is implemented on the shares chosen to optimize the portfolio, thus seeking the highest return with the lowest risk.

The current work selects investment portfolios considering 47 financial indicators of 30 companies included in the Dow Jones index, as they are the most important in the New York Stock Exchange (NYSE). The current research could render practical implications for companies and investors when analyzing the stock market and its main contribution is related to the following elements:

- A procedure to support the decision-making process for investing in the stock market;
- The approach to generate portfolios using two different methodologies.

After this introduction, a literature review encompassing financial risk, investment portfolios, and multicriteria decision-making is presented. The methodology of the multicriteria hierarchical process and the hierarchical version of the ELECTRE III method are addressed in the section afterwards, to then delving into the analysis of the performance of shares implemented by companies using such method. The integration of the investment portfolio and the analysis of returns are presented next to open the space for the discussion of the ordering of stocks and portfolio selection. The last section presents the conclusions drawn for this study.

LITERATURE REVIEW

This section briefly reviews concepts required to understand the proposal for selecting shares of an investment portfolio concerning financial risk, portfolio theory, and the application of multicriteria models in decision-making.

Financial risk and investment portfolio management

As a world leader, the NYSE is where investors access capital and participate in global markets. NYSE is a unique model that minimizes execution risk and volatility in stock prices. At present, it remains the largest stock exchange in the world (the big board). A positive correlation between the Dow Jones index and the Chile stock exchange has a more significant correlation with income than company results (Chahuán-Jiménez, 2018). In recent years, the influence of experts and their preferences on investors' perception concerning risk and return has been more relevant. The latest findings consider the importance of behaviors in investment decisions (Škrinjarić & Slišković, 2020), as also addresses by other studies regarding the risk situation of the behavior of the decision-maker (Akbaş & Erbay Dalkiliç, 2021; Liesiö *et al.*, 2023; Mehrjerdi, 2022).

The importance of risk, analyzed by different authors, directly affects the company's financing decisions, given that the composition of the company's capital structure, the level of financial leverage, or the debt ratio directly affects the company's value. A well-known model in the financial field is the

Markowitz model, which contributes to the portfolio theory and proposed the concept of covariance and correlation (Markowitz, 1959).

The importance of decision-makers when defining an investment and setting up a portfolio allows maximizing profits and minimizing investors' risk. The main assumptions of the traditional methods for constructing portfolios point out the limitations for achieving each one of these objectives (Escobar, 2015), thus setting-up the scene for the contribution of financial institutions to carrying out more adequate advisory processes that respond more precisely to the needs of investor clients, with alternatives that adjust to their expectations and specific interests (Useche-Arévalo, 2015).

The importance of accounting information in financial statements allows identifying the necessary indicators for the construction of a business model in the integration of investment portfolios, for example, in the Egyptian stock market (Elselmy *et al.*, 2019), reflecting the importance of financial indicators of indebtedness, financial leverage, liquidity, growth, profit growth, bank interest rate, the market capitalization index, dividend payment, volatility, and the market beta coefficient. The theory of combined possibility and a model allows for considering trade-offs between investor preferences concerning several incommensurable objectives in an imprecise environment (Mansour *et al.*, 2019).

It is known that studying portfolio selection applied with multi-objective and multicriteria techniques in corporate social responsibility will support socially responsible investors in the search for a portfolio that can meet their expectations of maximizing returns and minimizing risk (Suárez *et al.*, 2018). On the other hand, it is also acknowledged that traditional tools do not consider that decision-makers face complex scenarios with a growing number of factors characterized by uncertainty (not only financial risk), the influence of different economic factors, social and environmental issues, and the existence of an increasing number of conflicting criteria that needs to be considered (Guerrero-Baena *et al.*, 2014).

Today, as expressed by Lin *et al.* (2022), a series of intelligent systems techniques are proposed as a solution to the portfolio selection problem: neural networks (Chaweewanchon & Chaysiri, 2022; Kovalnogov *et al.*, 2022), genetic algorithm (de Melo *et al.*, 2022; Jalota *et al.*, 2023), and decision trees (Ghahtarani *et al.*, 2019; Zhao *et al.*, 2022). However, these studies present techniques that require a certain degree of expertise, thus establishing barriers for their extended application in real-life situations.

Multicriteria models in decision making

This study presents a multi-objective approach that involves fuzzy parameters, where fuzzy numbers give the distributions of possibilities, and the investor's preferences are explicitly incorporated through the concept of satisfaction functions. The selection of an investment portfolio differently from the Markowitz model uses the MCDA method to evaluate portfolios for a multi-objective optimization problem (Greco *et al.*, 2013). In the work by Ehrgott *et al.* (2004), these authors propose a portfolio optimization model that applies the Markowitz mean-variance to the Standard and Poor's database.

On another note, a financial performance evaluation model is presented in the study by Aldalou and Perçin (2018), who combined the fuzzy AHP technique and the fuzzy TOPSIS to rank airlines from the Istanbul Stock Exchange. According to Escobar (2015), the AHP was applied with real data on the prices of the Colombian stock market's high and medium trading shares from 2007 to 2010. Moreover, a process with the DEA, AHP and TOPSIS techniques was carried out to analyze future stocks from the United States of America (Pätäri *et al.*, 2018).

The Turkish stock index was study applying the AHP-PROMETHEE and ranking companies based in their performance (Altınırmak *et al.*, 2016). On the other hand, the Tehran Stock Exchange was analyzed with PROMETHEE based on surveys, financial reports, and expert opinions (Albadvi *et al.*, 2007).

Principal components analysis and the subsequent PROMETHEE II method were applied to compare assets in terms of their performance in the financial indicators on the set of shares traded on the São Paulo stock exchange (Basilio *et al.*, 2018), in a study that generated the ordering of the shares, evaluated criteria on a single level, and presented the best shares to be deployed.

In relation with the outranking approach, the ELECTRE III is a well-known method that has been applied, for example, in the Portuguese Market Index to evaluate assets and, consequently, support the selection of a portfolio (Lima & Soares, 2013). In the same line, the study by Shabani Vezmelai *et al.* (2015) selected and classified 20 companies listed on the Tehran Stock Exchange (TSE) with the ELECTRE III method, comparing to the ranking offered by the TSE (Shabani Vezmelai *et al.*, 2015). For its part, the study by Boonjing and Boongasame (2017) proposed a combined portfolio selection using the same method to support small investors in their decision, generating a ranking that evaluates shares at a single level. The main difference between the current proposal and the studies mentioned above is that our work directly analyzes the interaction of the criteria for each category.

In other studies, the ELECTRE III was applied to sort a Pareto investment portfolio into eight different classes, each corresponding to a sector or industrial activity (Panagiotis *et al.*, 2009). In addition, Cambrainha and Fontana (2018) proposes a model to aid a group of decision-makers in establishing a portfolio of feasible shares that could balance water supply-demand strategies. Moreover, Fontana and Morais (2013) implemented a model to rehabilitate the greatest number of leakage points in a water network, applying Promethee V for the selection of a set of feasible alternatives, which were tested in a simulated network.

An additional study applying a hierarchical ELECTRE III method evaluated the Mexican stock market, including book values and market values decision criteria (Bernal *et al.*, 2021). The 120 companies represented as shares in the market were ranked in various criteria subgroups to explain their performance and investment potential. Here, 22 decision criteria (elementary criteria) were grouped into six subgroups.

The same hierarchical method was applied by Alvarez *et al.* (2020, 2022). The first work evaluated the competitive level of Mexican regions based on their performance on 10 primary factors from 100 indicators, while the second assessed the innovation capacity of 32 regions in Mexico under 52 decision decision-criteria. The region with the best performance and those with the highest and lowest competitive level were identified and described.

METHODOLOGY FOR PORTFOLIO SELECTION

The current section describes the methodological approach and its corresponding stages deployed to address investment portfolio selection. Figure 1 presents an overview of the two-stage approach.

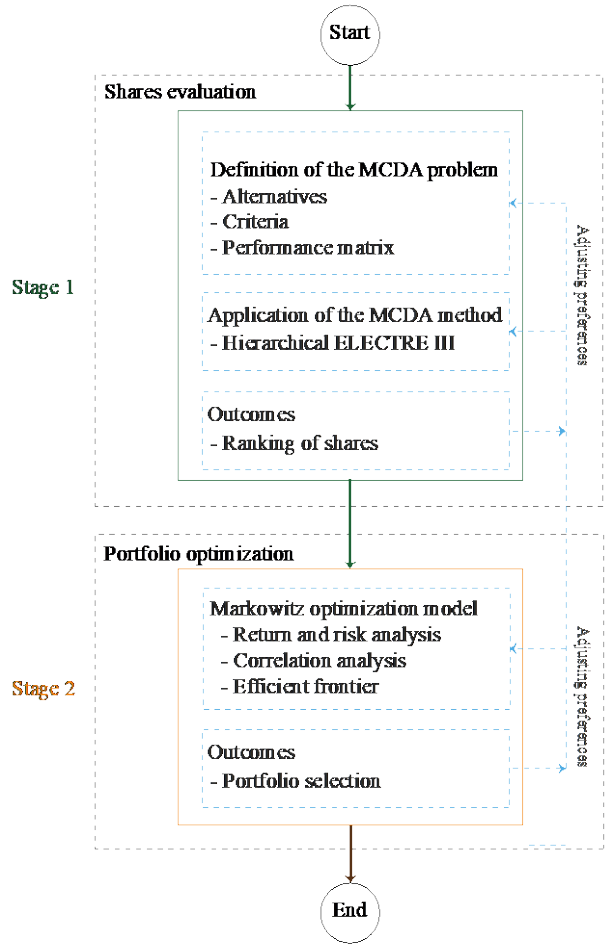


Figure 1. Methodological approach for investment portfolio selection. **Source:** authors.

In stage 1 the shares were evaluated using a MCDA methodology. It is defined as the multicriteria problem regarding problem definition, alternatives, and criteria. The hierarchical version of the ELECTRE III was applied, and the ranking of alternatives then becomes the output of this stage. The description of the MCDA method used in this stage is explained in the following section,

Stage 2 is meant for the construction of the portfolio and is based in the selection of a sample of shares from the ranking of alternatives from stage 1. The selected alternatives are the input of the Markowitz optimization model, where the return and risk objectives are assessed, and the efficient frontier identified. Finally, the optimal portfolio is shown to the decision-maker. The description of the optimization model is explained in the section titled “Capital asset pricing model.”

A multicriteria hierarchy process for share evaluation

The MCDA process develops the definition of a set of alternatives and a coherent family of criteria. Any MCDA method develops an integral preference method as an aggregation procedure. The method generates a recommendation in an alternative ranking format in descending order from best to worst.

The first stage of the portfolio selection problem consists of generating a stock evaluation ranking. The classic MCDA method will analyze NYSE stocks at the same level, evaluating all criteria simultaneously. In this way, one can find out which shares are the best and which are the worst. It is well known that evaluating stock selection requires various information commonly addressed from the Dow Jones indices.

The analysis at a single level of shares is limited to evaluating the performance of these shares, and it is not allowed to understand how some subcriteria (indices) interact with each other to show the performance of a category at a higher level (e.g., market ratio). However, it is possible to analyze the problem of ordering shares as a multiple criteria hierarchy process (MCHP) to analyze the problem of financial ratios (evaluation of shares) in subgroups of criteria and evaluate the interaction among a subset of criteria concerning the category to which they belong.

A practical application often imposes a hierarchical structure of criteria (Corrente *et al*., 2012). A hierarchical structure addresses the decision-making problem and is used to organize them into one part of the problem. The basic idea of MCHP is based on considering the preference relationships at each node of the hierarchical criteria tree. These preference relationships refer to obtaining information on preferences and the phase of analysis of a final recommendation by the decision-maker (Corrente *et al*., 2012).

A hierarchical criteria structure can be viewed as a criteria tree. A classical multicriteria approach deals with a multicriteria decision aid problem that evaluates criteria at the same level. However, the same problem can be analyzed in more minor problems, such as a hierarchy problem. Figure 2 illustrates a tree criteria structure; as observed, some nodes contain more nodes, making a tree of secondary problems. The MCHP was integrated with the ELECTRE III method (Corrente *et al*., 2017). Some important notations are explained as follows:

G is a comprehensive set of all criteria at all levels considered in the hierarchy;

G_0 is the root of the criteria;

l_G is the set of indices of the criteria in G;

$E_G \subseteq l_G$ is the set of indices of the elementary criteria;

g_r is the generic criterion other than the root (where r is a vector with length equal to the criterion level);

$g_{(r,1)}, \dots, g_{(r,n(r))}$ are the immediate subcriteria of the criterion g_r (located at the level below g_r);

$E(g_r)$ is the subset of indices of all elementary criteria descending from g_r;

$E(F)$ is the set of indices of an elementary criterion that descend from at least one criterion of the subfamily $F \subseteq G$ (that is, $E(F) = \cup_{gr \in F} E(gr)$);

G_r^l is the set of subcriteria of g_r located at level l in the hierarchy (under g_r).

To better understand the above notation, figure 2 presents a depiction of the hierarchical structure, where Level 1 contains the macrocriteria and the elementary criteria that descend from these are decomposing the subproblem. The entire set of elementary criteria is contained in the last level of the hierarchy. As shown in figure 2, a different approach to the multicriteria decision aid problem can be implemented when generating a hierarchical structure concerning the criteria of interest at a particular hierarchy level.

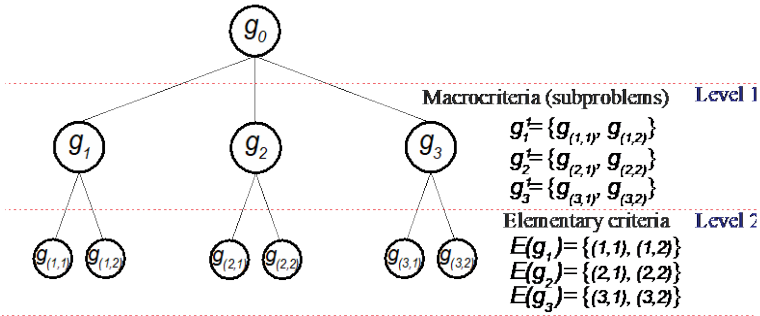


Figure 2. Structure of the problem in the multiple criteria hierarchy process. **Source:** authors.

Figure 3 shows a structure with two macrocriteria of the entire hierarchical problem of stock selection for the Dow Jones Index. The macro criterion Market ratio ($g1$) integrates six elementary criteria, the ratio of results (g_2) integrates eight elementary criteria, and so on until the dividend ratio macrocriterion (g_7) integrates six elementary criteria. The evaluation of the Dow Jones index shares includes 47 elementary criteria in two-level hierarchy structure. Seven macrocriteria are defined at level 1 and 47 elementary criteria at level 2.

For the problem of analyzing the performance of NYSE shares, it is easy to observe the hierarchical structure of the decision criteria (indices) and its seven categories. These can be considered as groups of subcriteria (figure 3). The current situation is frequent because in the case that a practical application imposes a hierarchical structure (Corrente *et al.*, 2012). In this sense, a different method for evaluating shares by a subset of criteria, such as the MCHP, would be valuable to solve the stock selection problem. For this reason, the analysis of the hierarchy of criteria is proposed to generate a ranking of NYSE stocks.

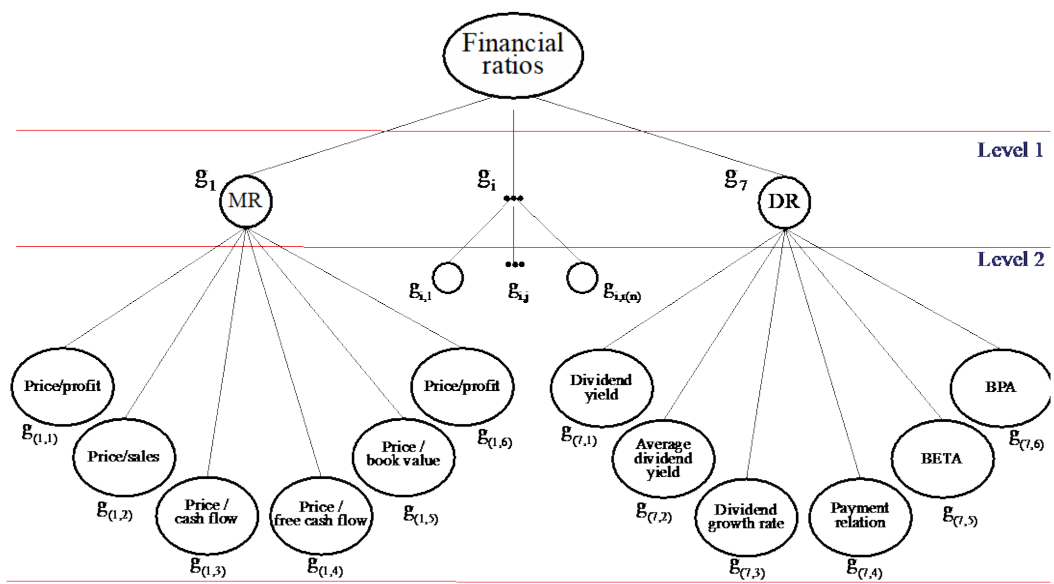


Figure 3. A simplified structure for the MCHP for NYSE stock selection problem. **Source:** authors.

ELECTRE III *hierarchical method*

In the work by Roy (1990), the ELECTRE III method was proposed to help the decision of overcoming a relationship by including the decision-maker's preferences within the projected solution. This method is pertinent to the proposal presented here since it is considered that the evaluation of the shares must consider the investor's preferences. Therefore, the decision-maker must define specific parameters, including thresholds and importance values of the criteria. These are included in a non-compensatory relational function that generates fuzzy values to evaluate the shares. Precisely, this fuzzy valuation allows evaluating a particular subjectivity of the investor to evaluate if one action is superior to another.

The use of ELECTRE III, a non-compensatory method, is particularly pertinent in contexts where at least one of the following features is present (Figueira *et al.*, 2013):

the presence of qualitative scales for some criteria;

the presence of heterogeneous scales;

the need to avoid systematic compensatory effects;

the need to consider the imperfect knowledge of data and some arbitrariness when building criteria; and

the need to take into account the reasons for and the reasons against an outranking.

For the investment portfolio selection problem, at least features of ii) and iv) are present.

The adapted version of the hierarchical ELECTRE III method was first introduced by Corrente *et al.* (2017) and implemented in a computational tool developed by Alvarez *et al.* (2020). The ELECTRE method is developed in two steps: the first, corresponds to the aggregation of preferences, where information is developed by building a model in the valued improvement relationship; in the second step, the distillation process exploits the valued outranking relation, thus generating a partial or complete ranking of alternatives explained as follows for each elementary criterion $g_r \in E_g$.

The elementary concordance index for each elementary criterion g_r is:

$$\varphi_t(a,b) = \begin{cases} 1 & if \quad g_r(b) - g_r(a) \leq q_t, (aS_t b) \\ \dfrac{p_t - [g_r(b) - g_r(a)]}{p_t - q_t} & if \quad q_t < g_r(b) - g_r(a) < p_t, (bQ_t a) \\ 0 & if \quad g_r(b) - g_r(a) \geq p_t, (bP_t a) \end{cases} \quad . \qquad (1)$$

The elemental discordant index, for each elemental criterion g_r:

$$d_t(a,b) = \begin{cases} 1, & if \quad g_r(b) - g_r(a) \geq v_t, \\ \dfrac{[g_r(b) - g_r(a)] - p_t}{v_t - p_t} & if \quad p_t < g_r(b) - g_r(a) < v_t, \\ 0, & if \quad g_r(b) - g_r(a) \leq p_t \end{cases} \quad . \qquad (2)$$

The partial concordance index for each non-elementary criterion g_r:

$$C_r(a,b) = \frac{\sum_{t \in E(g_r)} w_t \varphi_t(a,b)}{\sum_{t \in E(g_r)} w_t} \quad . \qquad (3)$$

Partial credibility index:

$$\sigma_r(a,b)=\begin{cases} C(a,b)\,x \prod_{g_t \in E(g_r)} \dfrac{1-d_t(a,b)}{1-C_r(a,b)} & \text{if} \quad d_t(a,b)>C_r(a,b) \\ C(a,b) & \end{cases} \quad (4)$$

Exploitation of the valued outranking relation

The valued improvement ratio generated in the previous step corresponds to the decision maker's preferential model. The distillation method is used to exploit the preferential model and proceeds in descending and ascending order. Therefore, the final pre-order is obtained as the intersection of the two distillations. A general description of the distillation method was described in the work by Giannoulis and Ishizaka (2010). For the pair a, $b \in A$ in the hierarchical process alternatives are ordered in a partial or complete pre-order for each non-elementary criterion g_r.

aP_rb: a is strictly preferred to b in the macro criterion g_r if in at least one of the rankings a is placed before b, and if in the other ranking a is at least as good as b.

aI_rb: a is indifferent to b in the macro criterion g_r if the two shares belong to the same position in the two pre-orders.

aR_rb: a is incomparable to b in the macro criterion g_r if a is ordered better than b in ascending distillation, and b is better ordered than a in descending distillation or vice versa.

Capital asset pricing model (CAMP)

The expected return is the profit an investor expects to obtain from a share in a determined period. The actual return can be higher, lower, or equal. The variance and standard deviation correspond to the volatility of the profitability of security. It is calculated according to the deviation from the average profitability. Covariance and correlation assume that the returns of individual securities are related to each other.

In contrast, covariance is a statistical measure of the interaction of two securities; the interaction can also be expressed in terms of the correlation between them. Covariance and correlation are two ways to measure whether two variables (two assets) are related. Diversifying allows investors to cushion their risk if they invest all their available money in one company. When one has more than two investments, the return and the portfolio risk are based on the same concepts as the two assets, whose objective is to reduce the unsystematic risk through diversification to minimize the standard deviation of the set of investments. This calculation is done through following equations 5-11 to below.

Expected return:

$$R_1 = \sum_{i=1}^{n} P_i \,.\, R_i \,. \quad (5)$$

Standard deviation:

$$\sigma_1 = \sqrt{\sigma_1^2} \,. \quad (6)$$

Variance:

$$Var(P) = \sigma_p^2 = \sum_{j=1}^{n} P_y \left(R_y - \overline{R}_1\right)^2 . \quad (7)$$

Covariance:

$$\sigma_{AB} = \mathrm{E}\left[\left(R_{Aj} - \overline{R_A}\right)\left(R_{Bj} - \overline{R_\mathrm{B}}\right)\right] . \quad (8)$$

Variance of a portfolio of 2 assets:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_{AB} . \quad (9)$$

Correlation:

$$P_{A,B} = \frac{\sigma_{A,B}}{\sigma_A . \sigma_B} . \quad (10)$$

Minimum variance:

$$w_A^* = \frac{\sigma_B^2 - \sigma_{A,B}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{A,B}} . \quad (11)$$

The importance of the decision-maker will be, according to the theory, with a capital *m* invested in the investment portfolio. To find the desired portfolio, one must realize that the slope of the line through and by is the maximum possible and that it corresponds to another optimization problem. According to the CAPM, the investor will prefer a position in the "market portfolio" with or without debt. Then, the optimal portfolio is given by the solution to a problem of optimization (Velez-Pareja, 2003). In this case, it is about maximizing, according to Franco-Arbeláez *et al.* (2011).

$$Max\ E(Rp) = \sum_{i=1}^{n} w_i \cdot E(R_i) \quad (12)$$

Subject to:

$$\sigma^2(Rp) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j \cdot \sigma_{ij} \leq \sigma_0^2$$

$$\sum_{i=1}^{n} w_1 = 1;\ w_i \geq 0\ (\ i = 1\ , \dots , n) \quad (13)$$

where n is the number of assets in the portfolio; R_i the random variable asset yield i; $E(R_i)$ the expected return on an asset i; R_p the random variable portfolio performance; $E(R_p)$ is the expected return of the portfolio; w_i is the proportion of the investor's budget allocated to asset i; $\sigma^2(R_p)$ is the variance of portfolio performance; σ_{ij} is the covariance between the returns on assets i; and j. σ^2 the maximum variance.

RESULTS

Analysis of Dow Jones stocks with the multicriteria hierarchical process

The analysis is performed based on the financial statements for the first quarter of this year, obtained from the Investing financial portal compiled from the NYSE that generates a performance index that informs companies in the Dow Jones Index and shows the existing capacities for investors. Table 1 shows the 30 companies analyzed in the current study.

Table 1.

Dow Jones Index companies.

Label	Company	Label	Company
A1	Merck	A16	Johnson & Johnson
A2	3M	A17	JPMorgan Chase
A3	American Express	A18	McDonald's
A4	Apple	A19	Microsoft
A5	Boeing	A20	Nike
A6	Caterpillar, Inc.	A21	Pfizer
A7	Chevron corporation	A22	Procter & Gamble
A8	Cisco	A23	The Travelers Companies
A9	Coca-Cola	A24	United Technologies
A10	Dupont	A25	United Health Group
A11	ExxonMobil	A26	Verizon Communications
A12	Goldman Sachs	A27	Visa
A13	Home Depot	A28	Wal-Mart
A14	IBM	A29	Walt-Disney
A15	Intel	A30	Twitter

Source: authors.

The financial ratios are elementary criteria to evaluate each company's performance. The interaction between rations provides evidence on the financial situation and prospects and assists in the evaluation of the position of a company compared to others. Data obtained from the NYSE is grouped into seven dimensions to evaluate stocks traded on the Dow Jones. Each dimension is constituted by a subgroup of indicators (elementary criteria). Table 2 shows the 47 indicators used to evaluate shares by the 30 companies in the Dow Jones Index.

The MCHP is applied in the NYSE data to analyze the stock's performance and the interaction of criteria in subgroups of different levels of the hierarchy through the ranking of the Dow Jones companies. The weights for each elementary criterion and macrocriteria are defined in table 2 based on the decision-maker's preferences. As observed, investors' preferences related to the relative importance of the elementary criteria and macrocriteria was obtained through an interactive elicitation process. It used a hierarchical version of the deck card method described in Alvarez *et al.* (2022) to generate the weights of the elementary criteria and macrociteria.

The indifference (q) and preference (p) thresholds are shown in Table 2. The veto threshold was not used because the investor was unwilling to express that information as part of the preference information.

Table 2.

Macrocriteria and elementary criteria for the selection of shares.

Index	Macrocriteria	Index	Elementary criteria	Weight	q	p
g1	Market ratio	g1,1	Price / earnings ratio (12 months)	0,0558	5	10
		g1,2	Price / sales (12 months)	0,0372	1	2
		g1,3	Price / cash flow (most recent quarter)	0,0090	10	20
		g1,4	Price / free cash flow (12 months)	0,0186	20	35
		g1,5	Price / book value (most recent quarter)	0,0465	5	10
		g1,6	Tangible price / book value (most recent quarter)	0,0279	0,01	2
g2	Results ratio	g2,1	Gross margin (12 months)	0,0302	0,01	0,05
		g2,2	Gross margin (5-year average)	0,0194	0,01	0,05
		g2,3	Operating margin (12 months)	0,0121	0,015	0,2
		g2,4	Operating margin (5-year average)	0,0157	0,015	0,2
		g2,5	Margin before tax (12 months)	0,0229	0,05	0,2
		g2,6	Margin before tax (5-year average)	0,0266	0,05	0,2
		g2,7	Net margin (12 months)	0,0339	0,01	0,03
		g2,8	Net margin (5-year average)	0,0084	0,05	0,2
g3	Market value ratio	g3,1	Benefits / share (12 months)	0,0434	15	40
		g3,2	Basic earn per share (Annual)	0,0493	3	9
		g3,3	Diluted earn per share (Annual)	0,0376	3	8
		g3,4	Book value / most recent quarter share	0,0258	10	20
		g3,5	Tangible Book Value / most recent quarter share	0,0141	1	5
		g3,6	Cash / share (most recent quarter)	0,0200	2	6
		g3,7	Cash flow / share (12 months)	0,0318	3	8
g4	Profitability ratio	g4,1	Financial profitability (12 months)	0,0222	0,05	0,25
		g4,2	Financial profitability (5-year average)	0,0190	0,05	0,2
		g4,3	Economic profitability (12 months)	0,0031	0,05	0,08
		g4,4	Economic profitability (5-year average)	0,0095	0,02	0,07
		g4,5	Return on equity (12 months)	0,0063	0,03	0,08
		g4,6	Return on investment (5-year average)	0,0286	0,02	0,07
		g4,7	Earn per share vs. previous year	0,0158	0,02	0,2
		g4,8	Earn per share (12 months) vs previous year	0,0254	0,05	0,15
		g4,9	Sales (12 months) vs. previous year	0,0127	0,05	0,1
g5	Liquidity ratio	g5,1	Sales growth (5-year average)	0,0151	0,01	0,05
		g5,2	Capital expenditure growth (5-year average)	0,0030	0,01	0,03
		g5,3	Acid test (most recent quarter)	0,0090	0,7	1,2
		g5,4	Solvency ratio (most recent quarter)	0,0181	1	1,5
		g5,5	Long-term debt to equity (most recent quarter)	0,0120	0,01	0,5
		g5,6	Total debt to equity (most recent quarter)	0,0060	0,5	1,5
g6	Effectiveness ratio	g6,1	Asset turnover (12 months)	0,0240	0,5	1
		g6,2	Inventory turnover (12 months)	0,0180	2	5
		g6,3	Employee / benefit (12 months)	0,0120	100000	350000
		g6,4	Net income / employee (12 months)	0,0060	30000	70000
		g6,5	Turnover of accounts receivable (12 months)	0,0300	5	8
g7		g7,1	Annual dividend yield	0,0221	0,01	0,03

Index	Macrocriteria	Index	Elementary criteria	Weight	q	p
	Dividend ratio	g7,2	Average dividend yield (5-year average)	0,0166	0,01	0,025
		g7,3	Annual dividend growth rate	0,0110	0,01	0,05
		g7,4	Payment ratio (12 months)	0,0055	0,05	0,4
		g7,5	BETA	0,0277	0	0,8
		g7,6	Earn per share	0,0331	1,5	3

Source: authors.

Concerning the methodology proposed in the section "A multicriteria hierarchy process for share evaluation," the MCHP is applied to selecting stocks in order to integrate an investment portfolio. MCHP helps analyze the stock's performance and the interaction of indicators from each subgroup and supports decision-making.

The problem is structured in a hierarchy of multiple criteria in the first step, decomposing the problem into 7 macrocriteria. Here each macrocriteria is seen as an individual problem. As shown in the hierarchical structure in figure 3, NYSE's stocks are structured in a hierarchy concerning the 7 macrocriteria and the 47 elementary criteria.

The hierarchical ELECTRE III with the distillation method described in the section "A multicriteria hierarchy process for share evaluation" is a computational tool available on GitHub5 for practitioners dealing with MCHP (Alvarez *et al.*, 2022). The method is applied to solve each subproblem g_i (macrocriterion) at each level. Table 3 illustrates the comprehensive ranking g_0 that generates 24 positions of the shares of the companies analyzed.

Table 3.

Comprehensive ranking (g_0) of the Dow Jones index.

Rank	Company	Rank	Company
1	A4, A19	13	A17
2	A12	14	A26
3	A13	15	A30
4	A18	16	A22
5	A15	17	A7
6	A6	18	A23
7	A21, A28	19	A29
8	A14	20	A25
9	A16	21	A11
10	A1	22	A5
11	A2, A3, A8, A20	23	A10
12	A9, A24	24	A27

Source: authors.

Table 4 lists the ranking of company shares from each macrocriterion. Analyzing each macrocriterion shows that different companies appear in the first place on each macrocriterion. It means that one company that is the best in one macrocriterion is not as good as in another, e.g., Microsoft (A19) is the best in the macrocriterion Market ratio (g1). However, this company is not that good in others macrocriteria; this situation is the same for companies in the first place in other macrocriteria. These variations are essential

[5] https://github.com/paac80/hierarchical-ELECTREIII

because they can identify how much the ordering can change if different parameters are used with the same information. In this sense, the rankings are not absolute, and they can change based on the decision-maker's preferences. Therefore, it is essential to use methodologies to model decision-maker preferences based on different expression formats to integrate financial indicators and investor profiles into an investment portfolio.

Table 4.

Individual ranking of company shares.

Ranking	g1	g2	g3	g4	g5	g6	g7
1	A19	A21	A12	A20	A30	A18	A24
2	A22	A19	A23	A9	A20	A28	A26
3	A4	A1	A24	A4	A13	A4	A15
4	A9	A14	A17	A26	A19	A26	A13
5	A20	A8	A4	A1	A14	A29	A12
6	A16	A9, A16	A28	A21	A29	A13	A16
7	A30	A18	A6	A19	A24	A15	A18
8	A18	A3	A25	A14	A5	A20	A17
9	A1	A12	A3	A16	A4	A19	A2
10	A7	A26	A7	A28	A8	A8	A4
11	A28	A2	A15	A13	A16, A23	A7, A11	A28
12	A8	A15	A13	A15	A10	A22	A23
13	A21	A4	A11	A2	A25	A9, A14	A6
14	A14	A30	A19	A18	A17	A25	A1
15	A2	A22	A29	A8	A1	A2	A7
16	A6, A13	A20, A29	A2	A3, A6	A21	A24	A21, A22
17	A29	A13	A14	A29	A28	A6	A9
18	A24	A6	A16	A30	A18	A19, A16	A3
19	A3, A26	A7	A30	A24	A27	A5	A11
20	A25	A25	A5	A17	A22	A21	A8
21	A17	A11	A1, A10, A20, A26	A23	A12	A30	A29
22	A15	A17	A18	A25	A2	A3	A14
23	A11	A23	A8	A22	A3	A12	A25
24	A23	A10	A22	A12	A7	A17, A23, A27	A19
25	A12	A28	A21	A5	A6		A27
26	A5, A10, A27	A5, A24	A27	A27	A26		A10
27		A27	A9	A11	A11		A20
28				A7, A10	A9		A5
29					A15		A30

Source: authors.

Investment portfolio integration

Expected returns and individual risks

The variance of the returns of a financial asset is an expected value that weights the differences of each possible return concerning the expected, previously squared. Therefore, it allows determining the risk of investment, where the higher the standard deviation, the greater the investment risk. In the case of the analysis of the eight assets selected from the ranking generated by the models, based on the advice of the brokerage firms established, there is no maximum number of shares or assets to develop a portfolio, but it will rather depend on the profile of the investor and the level of risk accepted. Therefore, the decision-maker selected eight assets to evaluate, which would make up the portfolio in their case.

In order to establish the ideal investment portfolio and optimize investment decisions, three assets with higher returns and different indices of variance were considered, in doing so, different combinations were based on the Markowitz model.

In the stock analysis, Apple, with an expected return of 2.96%, shows a higher return, comparatively not significant to that of Microsoft (2.95%), which is practically the same in expected return (figure 4). Although Apple and Microsoft generate very similar expected returns on the face of the expected risk, it can be said that either of them offers the same profitability but with different risks. Apple's asset has a higher risk than Microsoft's (figure 5), hence investor's preference will depend on their perception of risk. In United Technology (A24) and Twitter (A30), higher returns are obtained concerning other companies that reflect a negative return. Still, the asset of Twitter reflects a higher level of risk and a negative return (14.45 and -0.23%).

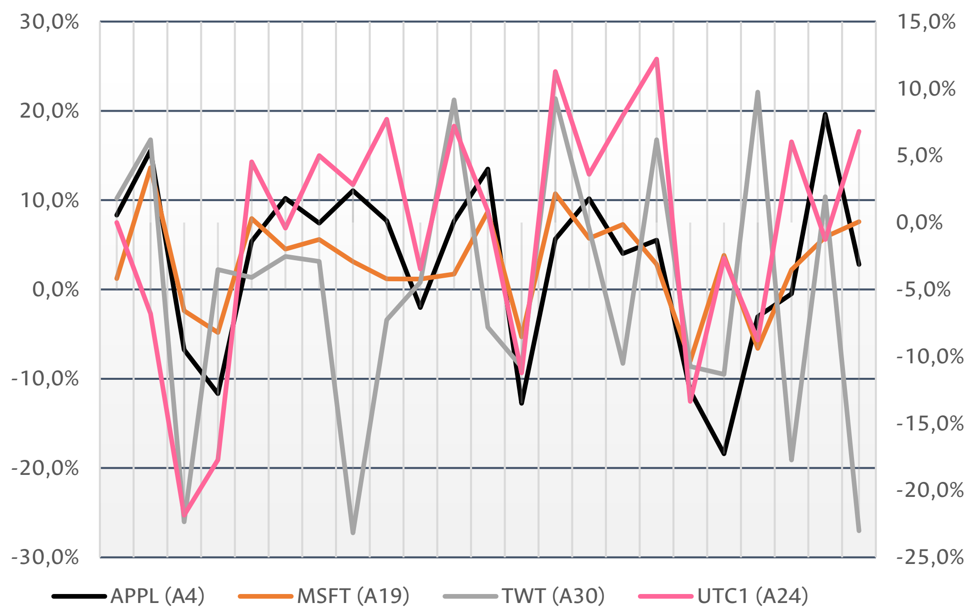
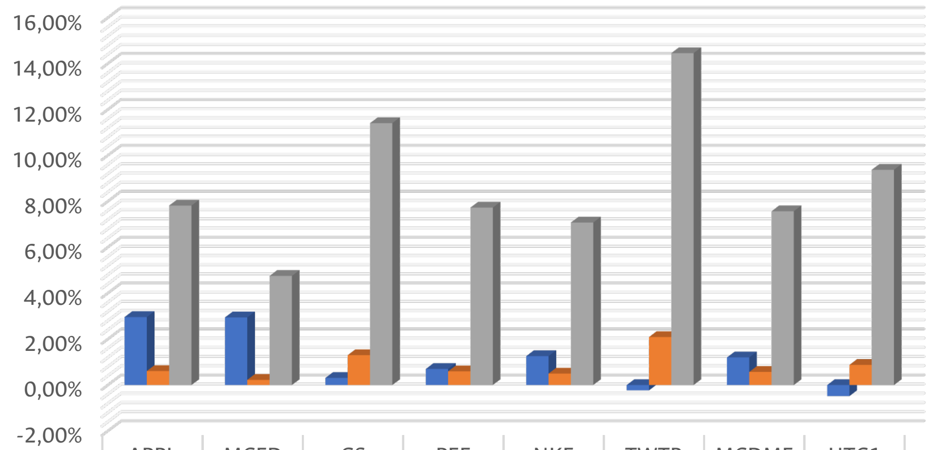


Figure 4. Expected returns per share. **Source:** authors.

For selecting the shares, leading companies with the highest expected return were selected based on the process of the MCHP multicriteria model, as observed in figures 4 and 5. This information will allow decision-making to integrate investment portfolios based on the Markowitz model.



	APPL (A4)	MCFD (A19)	GS (A12)	PFE (A21)	NKE (A20)	TWTR (A30)	MCDMF (A18)	UTC1 (A24)
Retorno Esperado	2,96%	2,95%	0,31%	0,70%	1,26%	-0,23%	1,20%	-0,48%
varianza	0,61%	0,23%	1,30%	0,60%	0,50%	2,09%	0,57%	0,88%
desv. Stand.	7,81%	4,75%	11,41%	7,73%	7,08%	14,45%	7,57%	9,37%

Figure 5. Distribution of returns and individual risks. **Source:** authors.

Covariance and Correlation Matrix

The covariance matrix is applied to integrate portfolios of different assets (Brealey *et al.*, 2014). Based on the experience of brokers, a maximum of eight assets is considered in the portfolio; these are selected by position in the ranking (table 4). From the analysis of the covariance, the investment strategies for integrating the investment portfolio are established. According to Markowitz's theory, in terms of risks, individual returns of companies are not substantial. Instead, the portfolios are integrated to combine assets to minimize the portfolio risk and determine the covariance. For example, successively, the covariance between A19 and A4 is 0.00366 (table 5). Based on this information, the investment strategies in diversification are established to reduce risk due to incorporating a more significant number of assets into a portfolio and the potential to diversify.

Table 5.

The covariance matrix of eight assets.

	A4	A19	A12	A21	A20	A30	A18	A24
A4	0.00964	0.00366	0.00798	0.00183	0.00413	0.00479	0.00194	0.00485
A19	0.00366	0.00299	0.00345	0.00161	0.00201	0.00180	0.00293	0.00293
A12	0.00798	0.00345	0.01116	0.00149	0.00467	0.00926	0.00337	0.00638
A21	0.00183	0.00161	0.00149	0.00400	0.00107	-0.0022	0.00144	0.00039
A20	0.00413	0.00201	0.00467	0.00107	0.00495	0.00290	0.00114	0.00391
A30	0.00479	0.00180	0.00926	-0.0022	0.00290	0.01977	0.00447	0.00371
A18	0.00194	0.00194	0.00337	0.00144	0.00114	0.00290	0.00361	0.00235
A24	0.00485	0.00293	0.00638	0.00039	0.00391	0.00371	0.00235	0.00774

Source: authors.

The correlation coefficient allows diversification; the higher the coefficient between two assets, the smaller the potential to diversify. Different combinations between assets (two, three, or more) were elaborated when integrating the portfolio, reducing risk, and constructing the portfolios based on the correlation criteria. Consequently, Microsoft's concerning McDonald's is negative, which would be in a combination to choose. Given that the gain of the other asset would offset the losses of one asset, the effect would consequently be null to the portfolio. On the contrary, both have the same correlation trend. Therefore, if they participate in the same market or macroeconomic variables, the result will affect the price of their shares and expected returns similarly (table 6).

Table 6.

Correlation matrix.

	A4	A19	A12	A21	A20	A30	A18	A24
A4	1.00							
A19	0.79	1.00						
A12	0.92	0.82	1.00					
A21	0.60	0.36	0.22	1.00				
A20	0.56	0.51	0.52	0.14	1.00			
A30	0.22	- 0.01	0.38	- 0.70	0.21	1.00		
A18	- 0.07	0.29	- 0.04	- 0.01	0.63	0.40	1.00	
A24	0.56	0.75	0.72	0.07	0.06	0.11	0.07	1.00

Source: authors.

Investment portfolios and top performance

The hierarchical ELECTRE III method evaluated the asset in the first phase, as it helps to construct four different portfolios to evaluate performance and risk diversification. Table 7 shows the portfolio risk and performance indices, which present some expected returns with specific diversification.

Table 7.

Portfolio risk and performance indices.

Indicators	P_1	P_2	P_3	P_4
Expected Return	2.07%	2.37%	2.39%	2.20%
Standard deviation	7.62%	5.78%	6.29%	5.81%
Portfolio performance	27.19%	41.00%	37.99%	37.93%

Source: authors.

The Solver tool was used to find the combination of shares necessary to integrate the portfolio with poor high performance, that is, the best relationship between risk and return for each of the four portfolios. The returns for each of the selected portfolios are shown in figure 6.

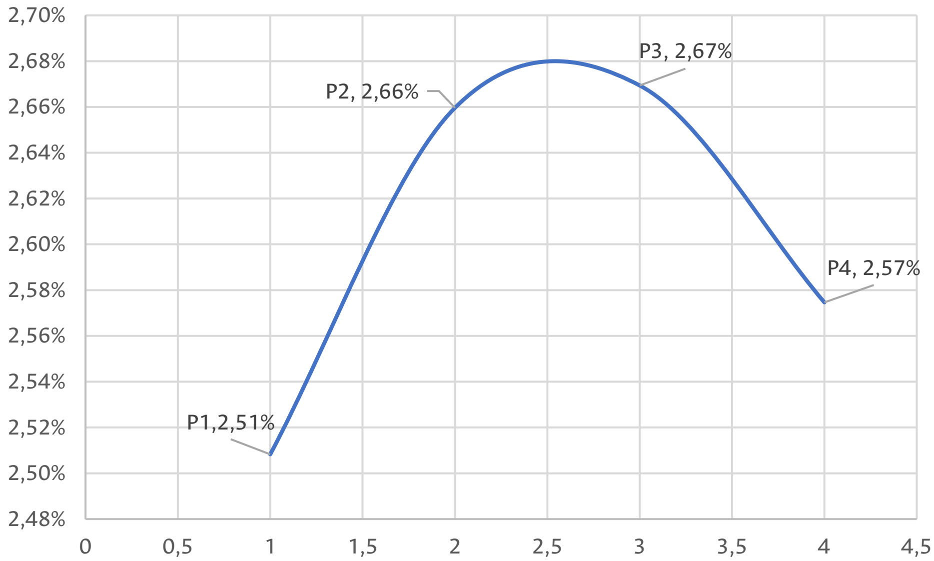
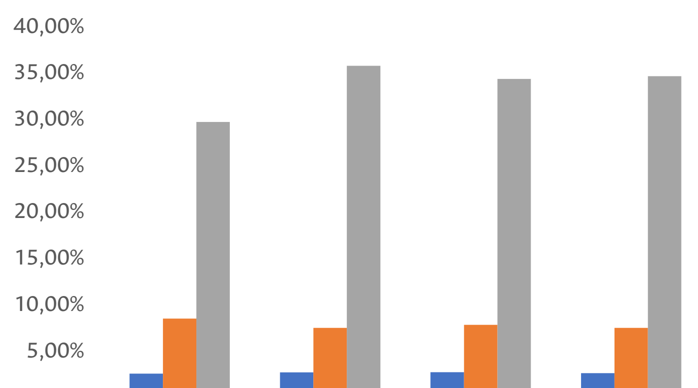


Figure 6. Maximum return on performance. **Source:** authors.

The indicator and maximum performance and risk are shown in figure 7. For (P1 = 2.51%) with an investment distribution WA = 73.1%, WB = 26.9%, WC = 0%, and a measured risk of 8.46%, discarding the investment of the company's asset Goldman Sachs (A12). Therefore, it would be the highest risk and lowest return (figure 7). In contrast, portfolios P2 and P3 show no significant difference concerning the performance of 0.01%, with risk P2 > P3 0.33%. Since the combination is different in portfolio P2 with the McDonald's company and in P3 with the Nike Company, and with different market sectors. As a result, the decision concerning risk and higher return the portfolio option P3 and the distribution of the investment generated by Solver would be the following: WA = 66.5%, WB = 16.6%, WA = 17%.



	P1	P2	P3	P4
Maximum performance return	2,51%	2,66%	2,67%	2,57%
Standard deviation	8,46%	7,45%	7,78%	7,44%
Maximum performance of the portfolio	0,30	0,36	0,34	0,35

Figure 7. Indicators of maximum performance and risk. **Source:** Authors.

The synergy between shares is not analyzed. The current portfolio models in stock markets analyze the risk and return objectives. In means, the shares in a portfolio will be diversified. In synergy terms, it will be expected to have negative effects because it is expected that one share does not influence other shares to reduce the risk exposure.

DISCUSSION OF RESULTS

In the context of analyzing the stock exchange, diversification, assurance, and coverage are aspects considered to transfer risk (Buchner *et al*., 2017). Applying the hierarchical ELECTRE III method identifies the best investment due to integration with the Markowitz model. Based on the portfolios shown in table 7, it is possible to compare options P1 and P2.

Apple, Microsoft, and Goldman constitute P1. The second obtained 21.79% per year. Apple, Microsoft, and McDonald's constitute P2; they performed 41.0% per year. However, concerning the level of risk (standard deviation), P1 is better than P2 (P1 > P2), which means that the investment has a higher risk, and the portfolio has a lower yield P1. The portfolio option is the best investment expectation in obtaining better yield performance and lower risk (standard deviation) (see table 7). The analysis is made of the main assets that obtained the highest performance, Microsoft and Apple, which are part of each portfolio. The companies with positive returns were included (see figure 5).

The present study shows how shares perform when observing them for each category. A variation of the ordering can be reflected when it is analyzed in each of its categories. For example, among the best companies, we can count Apple (A4), Microsoft (A19), Goldman Sarchs (A12), and Home Depot (A13). However, we can also see (table 4) that Apple is shown in the first places only in the macrocriteria Market ratios (g1), Market value ratios (g3), Profitability ratios (g4), and Effectiveness ratio (g6), while Microsoft is in first place in the macrocriteria Market ratios (g1), Profit ratio (g2), and Liquidity ratios (g5). Apple and Microsoft show less performance on the macro-criterion Dividend ratio (R7). In this last macro criterion, Goldman Sarchs, and Home Depot perform better than Apple and Microsoft. Grounded on this finding, the main aspects of a company's performance can be analyzed through profitability, asset turnover, financial leverage, liquidity, and market value ratios (Besley & Brigham, 2013).

The decision of which portfolio to select by the decision-maker will depend on the investor's profile regarding the risk they are willing to assume. The maxim of Markowitz's portfolio theory mentions that the best portfolio is the one that minimizes risk, diversification in investment reduces risk, hence the best combination to add to our portfolio. Therefore, there is no significant difference in performance in forming four different portfolios to maximize performance. Finally, the decision of the risk managers is willing to accept the degree of portfolio risk. For case P1, the risk-return ratio is higher risk and the lowest return. For P2, P3, and P4, the differences are not significant between a standard deviation range from 7.78% to 7.44%.

The generated portfolios correspond to the analysis of the risk and return criteria, that is, the portfolio theory of weighted shares with an acceptable risk level and expected return.

It is known that investors, in some cases, prefer portfolios behind the non-dominated frontier of the Markowitz model; even other portfolios dominate them (Ehrgott *et al*., 2004). This situation is presented because investors have preferences not considered in the current non-dominated portfolios. One explanation is that the return and risk objectives are not the complete information that investor considers in portfolio selection.

On the other hand, considering investors' preferences allows the generation of a portfolio that includes their preferred shares, even if other portfolios dominate the scene. In this sense, this subject constitutes a line of further research.

The multicriteria analysis approach evaluates the decision alternatives considering multiple factors or attributes. The main characteristic corresponds to the fact that it considers the decision-maker's preferences. In this approach, evaluating all the decision criteria is common, and although it corresponds to a single level, it is common to find groups of criteria that evaluate specific concepts of the problem in problems with several criteria.

CONCLUSIONS

This research proposes a new approach to portfolio generation using two different methodologies: the multiple criteria decision-making and the mean-variance model. It is a hybrid methodology adopted to analyze assets and generate portfolios based on investors' preferences, whose main contribution happens to be precisely the application of hybrid approaches grounded on decision-making and financial insights.

Applying the multiple criteria hierarchy process (MCHP) allows the evaluation of shares in the Dow Jones index. Forty-seven subcriteria were evaluated at two levels of hierarchy. The problem of stock selection and preparation of investment portfolios shows the opportunities and weaknesses of companies and allows more robust and reliable decision-making scenarios. The integration of the stock ordering and the construction of portfolios of the NYSE companies could be applied as an instrument to formulate more assertive policies and decisions within the organizations. Consequently, it would achieve favorable conditions to boost the investor.

The managerial implication in the current investment problem corresponds to a practical recommendation to the investor. The outcomes and analysis regarding the invertor's profile suggest that portfolio number two (P2), listed in table 7, is the recommended investment solution, as it reports a 41% performance and a low risk (5.78%), compared to other portfolios.

The methodology used in evaluating shares can be replicated in different stock markets. A social implication is related to economic benefits for public or private organizations, that could even be helpful for individual investors and whose effect may help at generating social benefits for a region. In the same vein, support for investments granted to organizations or practitioners could generate social impacts if organized in terms of sustainability or social benefits.

One limitation of the current approach is related to the final portfolios. The final portfolios generated correspond only with the best companies (shares) ranked by investor preferences. The worst companies (bottom of the ranking) will not be considered in the final portfolios. This situation is not necessarily wrong.

However, some companies at the bottom of the ranking can have exciting characteristics related to the portfolio's diversification.

A second limitation is related to the analyzed market, since our study only includes the New York Stock Exchange, which means that findings cannot be generalized for other markets, not only because of the use of different shares but also given the behavior of markets. In this sense, a future line of research could be focused on estimating the model in different stock markets.

DISCLOSURES

The authors declare no institutional or personal conflicts of interest.

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