CROSS SECTION ESTIMATION FOR HEAVY ION NUCLEAR REACTIONS WITH A CASCADE CODE OF FUSION EVAPORATION

ESTIMACIÓN DE SECCIONES EFICACES PARA REACCIONES NUCLEARES DE IONES PESADOS CON UN CÓDIGO DE CASCADAS DE FUSIÓN EVAPORACIÓN

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Abstract

This article presents a study of fusion-evaporation nuclear reactions. Starting from a detailed description of the semi-classical theoretical framework behind this nuclear reaction, quantities such as the cross section of compound nucleus formation and various evaporation residues after its formation, as well as their cross sections (proportional to the events number), were estimated by means of a `Python` code. The code splits the compound nucleus formation process and its subsequent decay into several residual nuclei, which occurs as a sequential particle emission. In order to prioritize a first approximation theory, different nuclear models, with semi-classical and statistical origin, related to projectile-target fusion, light particle evaporation (n, p, α) and fission, were described in detail.

The values obtained with the computational routine developed were compared with experimental values and results from the `PACE` code. Cross sections were calculated for about 90 proposed reactions that produce residues with excess protons. In general, the results obtained show significant discrepancies, especially in heavy nuclei

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reactions, although some agreements are found even taking into account the limitations of the code. The main reason for this discrepancy may be associated with the lack or overestimation of some channels which may affect the proportion of events. This motivates a more sophisticated analysis in the future that could allow a wider range of channels.

Keywords: nuclear reaction, fusion-evaporation, decay cascade, `PACE`.

Resumen

En este artículo se presenta un estudio de las reacciones nucleares de fusión-evaporación. Partiendo de una descripción detallada de la teoría semi-clásica detrás de la reacción, se estimaron cantidades como la sección eficaz de formación del núcleo compuesto y diferentes residuos de la evaporación después de su formación, así como sus secciones eficaces (proporcionales al número de eventos), por medio de un código de `Python`. El código divide el proceso de formación del núcleo compuesto y su posterior decaimiento en varios núcleos residuales, lo que ocurre como una emisión secuencial de partículas. Para priorizar una descripción de primera aproximación, la teoría recopilada aborda diferentes modelos nucleares, de origen semi-clásico y estadístico, relacionados a los procesos de fusión proyectil-blanco, evaporación de partículas ligeras (n, p, α) y fisión.
Los valores obtenidos con la rutina computacional desarrollada se compararon con valores experimentales y resultados provenientes del código `PACE`. Se calcularon las secciones eficaces para alrededor de 90 reacciones propuestas que producen residuos con exceso de protones. Los resultados obtenidos, en general, muestran notables discrepancias, sobre todo en reacciones de núcleos pesados, aunque se encuentran diversas coincidencias incluso teniendo en cuenta las limitaciones del código. La razón principal para esta discrepancia puede estar asociada a la falta o sobreestimación de ciertos canales que puedan alterar la proporción de los eventos. Esto motiva una análisis

más sofisticado en el futuro que pueda permitir una mayor variedad de canales.

Palabras clave: reacción nuclear, fusión-evaporación, cascada de decaimientos, `PACE`.

Introduction

The fusion-evaporation nuclear reaction has been key in the discovery of many isotopes, which have been discovered in this kind of reactions for heavy ions [1], especially in neutron deficient nuclei [2], and has increasingly become in a promising route for the synthesis of superheavy elements [3]. This reaction is also used to populate high spin states of many nuclei [2].

In this reaction, the collision of a projectile nucleus P with a target nucleus T forms an intermediate nucleus (which is relatively long lived, $\sim 10^{-16}$ s [4]), the *compound nucleus* CN, which is unstable due to a high excitation energy. The compound nucleus is allowed to decay, usually, emitting a light particle or *ejectile* x and transforming in a residue R. This is summarized as follows:

$$P + T \rightarrow \mathrm{CN}^* \rightarrow x + R, \tag{1}$$

where the asterisk indicates an excitation state of the compound nucleus. This reaction is usually denoted as $T(P, x)R$.

Due to its high excitation energy, after one decay, the compound nucleus is still excited enough to emit more particles, in a process called *evaporation*. This means that the compound nucleus can go through a series of *sequential decays* or a *cascade*, until, the final residue has an energy insufficient for any type of decay [5]. These evaporated particles are mostly neutrons, protons, alpha particles, etc. For heavy nuclei, fission is a very important decay mode that has to be taken into account. The probabilistic nature of nuclear decays implies that this process can occur in different ways, resulting in various types of residues, or *exit channels*.

In a nuclear reaction like this, one of the most important physical quantities of interest is the cross section of the reaction. Especially,

the evaporation cross section of each exit channel is the main objective of the calculations.

In this article, those calculations are performed by means of the development of a computational code in `Python` which simulates evaporation cross sections for this reaction for a list of physically reasonable residues. This code uses semi-classical arguments for those calculations: (1) a fusion cross section by means of the partial waves expansion, Hill-Wheeler transmission coefficients and Bass potential [6], and (2) a statistical analysis for the decay widths, given by the Weisskopf-Ewing evaporation formalism (and Bohr-Wheeler formalism for fission).

The main strategy of the code is applying Monte Carlo algorithm to select random decay modes in each step of the cascade. With the residues of all the cascades, evaporation cross sections of all channels are calculated by means of the proportion of events found.

The aim of this document is to provide a detailed description of how this code works, the theory behind it, and to evaluate some reactions compared to experimental values. Almost 90 fusion-evaporation reactions are tested in this code with their cross sections shown later, compared with experimental values from a list of proton rich reactions [7] and values from a more sophisticated code like PACE [8–10].

This article is based on the first author's B.Sc. dissertation under the supervision of D. A. Torres [11].

Theoretical framework

This reaction can be analyzed by means of the Bohr independence hypothesis, dividing the reaction in two *independent* stages [12]: (1) the formation of the compound nucleus (by fusion of projectile and target nuclei), and (2) the decay of the compound nucleus (by evaporation or fission). Then, the cross section of the total reaction can be expressed as follows:

$$\sigma_{P+T\rightarrow x+R} = \sigma_{P+T\rightarrow \mathrm{CN}} P_{\mathrm{CN}\rightarrow x+R}, \tag{2}$$

where $\sigma_{P+T\to \mathrm{CN}}$ is the compound formation cross section (or fusion cross section), and $P_{\mathrm{CN}\to x+R}$ is the probability of decay of the compound nucleus (e.g., by evaporation). Both terms can be estimated with a semiclassical analysis as follows.

Consider a collision between target and projectile nuclei with energy in the laboratory frame $E^{(\mathrm{lab})}$, is such that the center of mass energy of the system is $E^{(\mathrm{CM})} = (m_{\mathrm{t}}/M)E^{(\mathrm{lab})}$ where $M = m_{\mathrm{p}} + m_{\mathrm{t}}$. All masses are calculated through the mass excess from the NuDat 3.0 database [13].

The well-known result of the the absorption cross section of the interaction target-projectile from the partial wave analysis allows to find an expression for the fusion cross section, in terms of the transmission coefficients T_l:

$$\sigma_{\mathrm{fus}} = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l, \tag{3}$$

where $k^2 = 2\mu E^{(\mathrm{CM})}/\hbar^2$ and $\mu = m_{\mathrm{targ}}m_{\mathrm{proj}}/M$.

An expression for this coefficients can be easily approximated by means of the Hill-Wheeler coefficients, a semiclassical result associated with transmission through a parabolic barrier, an approximation of the barrier of a Coulomb + centrifugal + nuclear potential. If R_{B} is the position of the barrier of the Coulomb + nuclear $V(r)$ (where $V'(r) = 0$), and $V_{\mathrm{B}} = V(R_{\mathrm{B}})$ is the height of the barrier, let $V_{\mathrm{B},l} = V_{\mathrm{B}} + \hbar^2 l^2/2\mu R_{\mathrm{B}}^2$ be the barrier of the total potential. Then, the coefficients take the form [5]:

$$T_l = \frac{1}{1+\exp[2\pi(V_{\mathrm{B},l} - E^{(\mathrm{CM})})/\hbar\omega_{\mathrm{B}}]}, \tag{4}$$

where $\omega_{\mathrm{B}}^2 = -V''(R_{\mathrm{B}})/\mu$ (here, it is assumed that ω_{B} does not depend on l). R_{B} y ω_{B} can be estimated for a suited nuclear potential; a simple and useful choice is the Bass potential, suggested by R. Bass [6]. Using (3) and (4), an explicit expression for the cross

section can be written by some approximations, as C. Y. Wong [14] does in the Wong formula:

$$\sigma_{\text{fus}} = \frac{\hbar\omega_{\text{B}}^2 R_{\text{B}}^2}{2E^{(\text{CM})}} \ln\left\{1 + \exp[2\pi(E^{(\text{CM})} - V_{\text{B}})/\hbar\omega_{\text{B}}]\right\}. \tag{5}$$

This expression allows transmission below the barrier due to quantum tunneling, in contrast of the classical result. At energies greater than the barrier, Wong formula approximates to the classical result:

$$\sigma_{\text{fus}}(E^{(\text{CM})} \gg V_{\text{B}}) = \pi R_{\text{B}}^2 \left(1 - \frac{V_{\text{B}}}{E^{(\text{CM})}}\right). \tag{6}$$

Equation (5) is used to estimate fusion cross section, but, expression (6) is a result easier to compute in the cascade calculations, as will be shown subsequently. However, at high energies, cross section tends to decrease due to instability of the nucleus for spontaneous fission. To account for this, beyond a critical angular momentum l_{crit}, the cross section can be expressed as [5]:

$$\sigma_{\text{fus}} = \frac{\pi\hbar^2 l_{\text{crit}}^2}{2\mu E^{(\text{CM})}}, \tag{7}$$

for $E^{(\text{CM})} > E_{\text{crit}} = V_{\text{B}} + \hbar^2 l_{\text{crit}}^2/2\mu R_{\text{B}}^2$. Critical angular momentum and fission barriers can be extracted from the A. Sierk routine [15, 16].

After fusion and the formation of the compound nucleus, this nucleus has an excitation energy given by $E^* = Q_{P+T\to\text{CN}} + E^{(\text{CM})}$, where $Q_{P+T\to\text{CN}} = (m_{\text{p}} + m_{\text{t}} - m_{\text{cn}})c^2$ is the Q *value* of the reaction. On the other side, decay widths of each decay mode are necessary to compute the cascade simulation. A statistical analysis, the Weisskopf-Ewing formalism, which was one of the first quantitative analysis of the compound nuclear reaction [12], is a viable choice, without adding high complexity into the calculations. This

statistical analysis is motivated by comparing the emission of particles from an excited compound nucleus with the *evaporation* of molecules from a fluid [12].

If $\Gamma_x/\hbar$ is the probability of decay per unit of time, the Weisskopf-Ewing formula writes the decay width for the decay mode x as [12]:

$$\Gamma_x = \int_{V_{\mathrm{CB}}}^{E^*+Q_{\mathrm{CN}\to x+R}} \frac{g_x m_x \sigma_{\mathrm{inv}}}{\pi^2\hbar^2} \frac{\rho_R(E^* + Q_{\mathrm{CN}\to x+R} - \epsilon_x)}{\rho_{\mathrm{CN}}(E^*)} \mathrm{d}\epsilon_x, \quad (8)$$

where σ_{inv} is the cross section of the inverse reaction (i.e., fusion of the residue and the emitted particle), $g_x = 2s_x + 1$, s_x, m_x, ϵ_x is the spin, mass and energy of the evaporated particle, V_{CB} is the Coulomb barrier (which is the threshold of the classical cross section, without taking into account the nuclear interaction, for simplicity), and $\rho(E^*)$ is the density of states of each nuclei.

The cross section of the inverse process is taken as the classical result (6) (valid only above the barrier):

$$\sigma_{\mathrm{inv}}(\epsilon_x) = \pi R_{\mathrm{CB}}^2 \left(1 - \frac{V_{\mathrm{CB}}}{\epsilon_x}\right), \quad (9)$$

where $R_{\mathrm{CB}} = 1.2(A_x^{3/2} + A_R^{3/2})$ fm is the threshold distance between both nuclei (in contact), and $V_{\mathrm{CB}} = 1.44 Z_x Z_R / R_{\mathrm{CB}}$ MeV.

The density of states of a given nuclei is taken as the total or intrinsic level density of the Gilbert-Cameron model (without taking into account pairing effects, for simplicity) [17, 18], with a level density parameter $a = A/8$ MeV^{-1} [5].

A probability distribution for the energy of the ejectile nucleus energy ϵ_x can be defined from the Weisskopf-Ewing formula, normalized with (8), and can be used to sample random energies of emitted particles:

$$P(\epsilon_x) = \frac{1}{\Gamma_x} \frac{g_x m_x \sigma_{\mathrm{inv}}}{\pi^2\hbar^2} \frac{\rho_R(E^* + Q_{\mathrm{CN}\to x+R} - \epsilon_x)}{\rho_{\mathrm{CN}}(E^*)}. \quad (10)$$

Fission can be described with the Bohr-Wheeler formalism, which introduces a statistical analysis at the saddle point (sd) of deformation, which is represented by the fission barrier B_{f}. Then, the width for fission of the compound nucleus can be written as [5]:

$$\Gamma_{\mathrm{f}} = \int_0^{E^*-B_{\mathrm{f}}} \frac{\rho_{\mathrm{sd}}(E^* - B_{\mathrm{f}} - \epsilon_{\mathrm{f}})}{2\pi\rho_{\mathrm{CN}}(E^*)} \mathrm{d}\epsilon_{\mathrm{f}}, \tag{11}$$

where ϵ_{f} is the energy associated with the fission parameter at the saddle point. Here, we are only interested in the occurrence of fission events, not on the dynamics of posterior fission fragments.

With the decay widths for each cascade step, the proportion of all kind of available residues can be found with all cascades, and therefore, each evaporation cross section of each exit channel.

A more detailed description of the theory explained can be found in the thesis in which is based this article [11].

The computational code

The main focus of the computational code is to calculate cross sections of evaporation and fission events from a compound nucleus reaction, with a semi-classical theoretical frame, aiming for simplicity. This code is centered on the evaporation cascade routine, in which a decay mode can be selected randomly by a Monte Carlo routine, with the following discrete probability distribution:

$$P_x = \frac{\Gamma_x}{\Gamma}, \qquad \text{where,} \qquad \Gamma = \Gamma_{\mathrm{p}} + \Gamma_{\mathrm{n}} + \Gamma_{\alpha} + \Gamma_{\mathrm{f}}, \tag{12}$$

where x = n, p, α or f. Every step of the cascade updates the residue nuclei taking into account the particle evaporated (or stops if fission occurs). Also, the energy of the residue is updated only

for evaporation taking into account the energy of the evaporated particle ϵ_x from the distribution (10): $E_R^* = E_{\rm CN}^* + Q_{{\rm CN}\to x+R} - \epsilon_x$. The cascade stops if all modes are unavailable for decay, i.e., $\Gamma = 0$. The final residue (or fission event) is saved in memory. This process is explained in detail in the flowchart (1).

With all the residues of a big number of cascades $N_{\rm casc}$, the proportion of final events can be calculated, and therefore the evaporation cross section of each exit channel. By means of equation (2), the cross section of each channel can be expressed as:

$$\sigma_{\rm channel} = \sigma_{\rm fus} P_{{\rm CN}\to{\rm channel}} = \sigma_{\rm fus} \frac{n_{\rm event}}{N_{\rm casc}}, \tag{13}$$

where $\sigma_{\rm fus}$ can be calculated with (5) or (7), and the proportion of events is $n_{\rm event}/N_{\rm casc}$ using the number of events found $n_{\rm event}$.

The structure of the whole routine is described as follows. The code receives as entries the projectile and target nuclei (mass number and atomic number), the projectile energy in the laboratory frame and the number of cascades desired.

Then, (1) some physical quantities of the initial fusion reaction are calculated, e.g., the center of mass energy, excitation energy or fusion cross section, (2) all $N_{\rm casc}$ cascades are simulated with routine (1) for each cascade, saving all final events in memory, (3) all types of events are counted, listing all exit channels, (4) with the number of events of each channel, all cross sections are calculated with equation (13).

The output of the code are the list of all evaporation (and fission) channels found in this process with their cross sections. This is summarized in the flowchart (2).

This code is developed in `Python` language, a relatively new language in nuclear reaction calculations, and it is available for download in the repository [19].

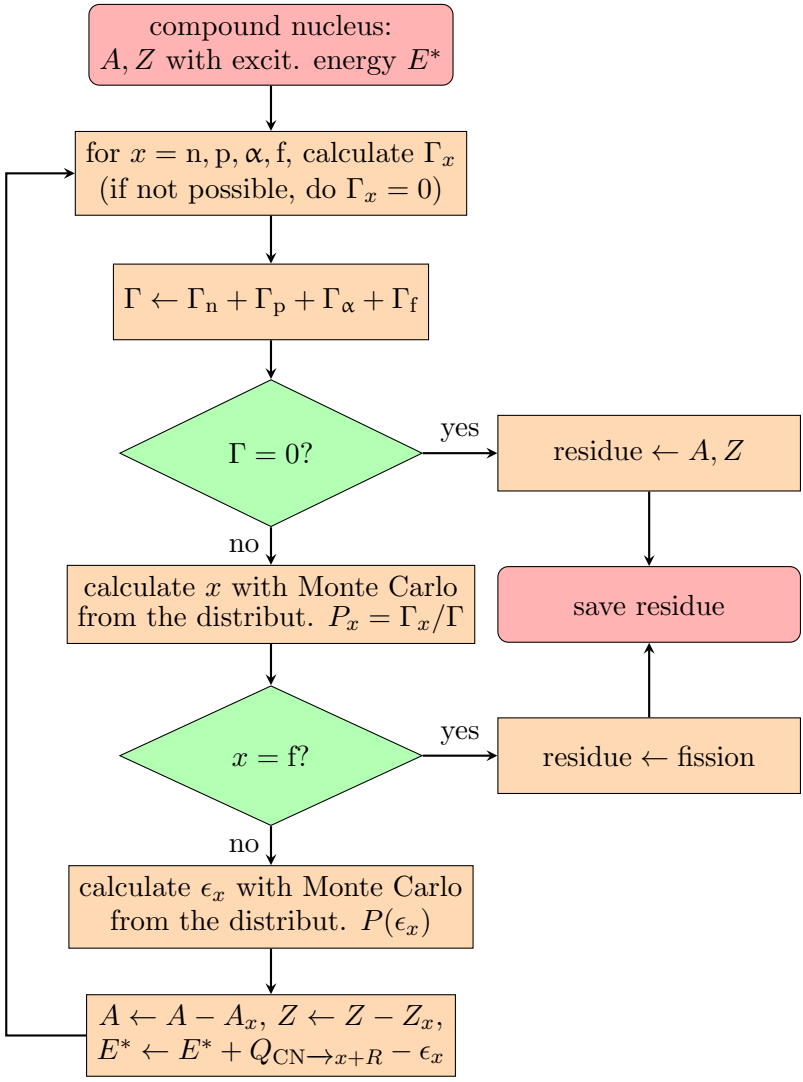

FIGURE 1: *Flowchart of a decay cascade of a compound nucleus*

Results and discussion

For a given projectile and target nuclei at a certain incident energy and number of cascades, the code returns a list of final residues, found after all cascades. The cross section of a certain exit channel can be compared with reference values, if it is found.

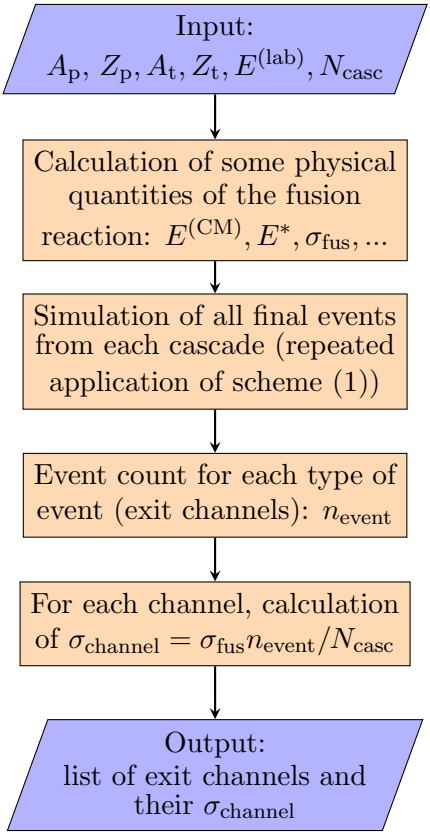

FIGURE 2: *Global scheme of the implementation of the code*

With this in mind, a list of almost 120 reactions with experimental evaporation cross sections from the article [7] is chosen to compare with values simulated with the code described in this article. These reactions, with proton rich residues, are a compilation of results from almost 45 experimental studies. Additionally, these values are compared with cross sections calculated with the `PACE` code, originally developed by A. Gavron [9], and it is now in the version `PACE4`, present in the package `LISE++` [8, 10].

The simulated, experimental and `PACE` values for approximately 90 of these reactions are shown in table (1). In most reactions, a

thousand cascades were used with both codes; for very rare events, 10 thousand cascades maximum were used for the code of this article and 100 thousand for `PACE` code. Some experimental values do not address uncertainty values due to inaccuracy of experimental techniques [7].

It is important to note that the randomness behind the Monte Carlo process implies that the list of residues (and its cross sections) obtained are not always the same each time the calculations are made, especially in results with few events, which tend to fluctuate more. Furthermore, the variety of final residues increases as excitation energy and number of cascades rises, which means that competition between decay modes is favored because more decays are available by the high excitation energy.

Some reactions show the limitations of both codes: some of the listed reactions have very small cross sections, and therefore, are very rare to find in a cascade simulation, requiring an excessive number of cascades ($\sim 10^6$, to find a cross section of $\sigma_{\text{channel}} \sim 10^{-4}$ mb for $\sigma_{\text{fus}} \sim 10^2$ mb). This can be an explanation of some residues not being found, but also, this can be addressed simply to the physical assumptions taken in the calculations, which are not enough to account for a broader variety of decays.

When the results of the code explained in this article and from `PACE` are compared, we see that `PACE` gives a broader variety of residues; this is clearly due to a more complete formalism, which allows access to more channels or a different distribution of events. Comparing more closely the data from table (1), the cross section values from both codes have substantial differences with experimental values in several reactions, especially in very low cross sections (less than $\sim 10^{-3}$ mb), though there are several coincidences in order of magnitude. In light nuclei, there is an overestimation of the cross section, probably attributed to a fusion cross section larger than expected, or missing channels not found, which could decrease the excess of events.

TABLE 1: *Cross sections of the fusion-evaporation reaction, (1) calculated with the code developed in this article, (2) consulted in the* ***PACE*** *code, and (3) experimental consulted in [7]. For each one of the chosen reactions, mass and atomic numbers of residue, projectile, target and incident projectile energy ($E = E^{(lab)}$), are shown. Experimental errors are included (not all error values are shown in the reference)*

A	Z	A_p	Z_p	A_t	Z_t	E (MeV)	σ_{calc} (mb)	σ_{PACE} (mb)	σ_{exp} (mb)
Light nuclei ($N \approx Z$)									
64	30	12	6	54	26	37	1.73E+02	5.59E+01	(1.60 ± 0.07)E+02
64	31	54	26	12	6	150	5.61E+01	2.98E+00	7.90E+01
64	32	40	20	27	13	102	1.48E+02	2.55E+00	(4.00 ± 0.60)E-01
64	32	54	26	12	6	150	1.86E+02	1.70E+00	(3.40 ± 0.90)E-01
64	32	54	26	12	6	165	2.72E+02	1.21E+00	(5.00 ± 3.00)E-01
68	34	58	28	12	6	175	2.28E+02	1.48E+00	(3.80 ± 1.60)E-02
68	34	58	28	12	6	220	3.52E+02	4.36E+00	(2.00 ± 0.50)E-01
72	36	16	8	58	28	55	2.75E+02	2.75E+00	(1.00 ± 0.30)E-01
72	36	58	28	16	8	170	1.69E+02	3.69E-01	(6.00 ± 2.50)E-02
76	38	54	26	24	12	175	4.51E+01	2.40E+01	(1.00 ± 0.50)E-02
80	40	58	28	24	12	190	5.73E+01	8.14E-01	(1.00 ± 0.50)E-02
80	39	58	28	24	12	190	3.09E+01	2.06E+01	(2.00 ± 1.00)E+00
80	38	58	28	24	12	190	6.87E+01	3.86E+01	(4.40 ± 0.40)E+01
^{100}Sn *region*									
98	46	58	28	50	24	250	8.59E-02	9.48E+01	(2.20 ± 0.20)E+01
98	47	58	28	50	24	250	2.57E-01	5.85E+01	(3.00 ± 0.60)E-01
99	47	58	28	50	24	250	8.59E-02	8.50E+01	(3.60 ± 0.40)E+00
99	48	58	28	50	24	249	1.68E+00	1.59E+00	(3.20 ± 2.00)E-02
99	48	50	24	58	28	225	9.01E-02	5.64E-01	(2.50 ± 0.80)E-02
99	48	58	28	58	28	394	2.98E-01	1.29E+01	(3.10 ± 2.00)E-02
100	47	50	24	58	28	225	9.01E-02	1.42E+02	3.90E+00
100	48	50	24	58	28	225	3.55E+01	6.07E+01	1.00E+00
100	49	50	24	58	28	225	6.49E+00	2.51E+00	1.00E-03
100	49	58	28	50	24	319	7.37E-01	1.13E+00	2.60E-03
100	49	58	28	58	28	371	5.27E-02	1.79E+00	1.70E-03
100	49	58	28	58	28	394	4.96E-01	1.29E+00	1.60E-03
100	50	50	24	58	28	225	7.01E+01	3.56E-03	4.00E-05
101	48	58	28	50	24	250	1.95E+01	6.05E+01	(1.80 ± 0.20)E+01
101	50	58	28	50	24	249	1.08E+02	7.93E-01	(1.60 ± 0.40)E-05
101	50	58	28	50	24	250	9.51E+01	3.21E-01	1.00E-05
101	50	58	28	58	28	325	1.20E-01	5.50E-02	(9.00 ± 4.00)E-06
101	50	58	28	58	28	348	1.35E+00	1.40E-01	(1.30 ± 0.30)E-05
101	50	58	28	58	28	371	9.48E-01	3.69E-02	(2.80 ± 1.00)E-05
101	50	58	28	58	28	394	3.97E-01	5.14E-02	(7.00 ± 4.00)E-06
102	48	58	28	50	24	250	9.45E-01	5.29E+00	(6.30 ± 1.90)E+01
102	49	58	28	50	24	249	8.49E-01	3.17E+00	(9.00 ± 5.00)E-01
102	49	58	28	50	24	348	6.76E+00	2.42E+00	(1.10 ± 0.60)E+00
102	49	58	28	58	28	325	6.01E-02	8.17E+00	(1.20 ± 0.60)E+00
102	49	58	28	50	24	348	5.62E-02	4.68E+00	(1.20 ± 0.60)E+00
102	49	58	28	58	28	348	5.62E-02	5.03E+00	(7.00 ± 3.00)E-01
102	49	58	28	58	28	371	1.58E-01	1.26E+01	(1.00 ± 0.50)E+00
102	49	58	28	58	28	394	1.84E+00	1.44E+01	(9.00 ± 4.00)E-01
103	48	58	28	50	24	250	8.59E-01	6.86E+01	(2.70 ± 0.20)E+01
103	49	58	28	50	24	250	1.05E+01	3.18E+01	(6.40 ± 0.80)E+00
104	48	58	28	50	24	250	2.58E+00	6.50E+00	(1.79 ± 0.07)E+02
104	49	58	28	50	24	250	1.19E+01	2.69E+01	(5.80 ± 1.60)E+01
104	50	58	28	50	24	250	2.93E+02	6.74E+00	(1.80 ± 0.20)E+00
105	49	58	28	50	24	250	3.44E-01	1.60E-01	(1.16 ± 0.06)E+02
105	50	58	28	50	24	250	6.01E-01	2.41E-01	(1.00 ± 0.20)E+01
Heavier nuclei									
171	79	78	36	96	44	361	1.16E+01	1.39E+00	1.10E-03
171	79	78	36	96	44	359	1.49E+01	2.12E+00	2.00E-03
171	79	78	36	96	44	363	2.13E+01	1.03E+00	6.00E-04
170	79	78	36	96	44	386	1.17E+02	5.52E-01	9.00E-05
173	80	78	36	102	46	384	5.32E-01	5.11E-02	4.00E-06
172	80	78	36	96	44	361	8.91E-03	2.58E-03	4.00E-06
171	80	78	36	96	44	361	2.00E+02	9.03E-03	2.00E-06
176	81	78	36	102	46	384	6.79E+01	2.04E-02	3.00E-06

(continued on next page)

Table 1: *(continued)*

A	Z	A_p	Z_p	A_t	Z_t	E (MeV)	σ_{calc} (mb)	σ_{PACE} (mb)	σ_{exp} (mb)
173	80	80	36	96	44	400	5.68E-02	1.30E-02	1.50E-05
Proton emitter: pn *channel*									
185	83	92	42	95	42	410	3.37E-01	3.32E-01	1.00E-04
185	83	92	42	95	42	420	7.90E-01	2.62E-02	6.00E-05
Proton emitter: p2n *channel*									
109	53	58	28	54	26	195	9.31E-02	-	1.00E-02
109	53	58	28	54	26	195	1.40E+02	-	(1.60 ± 0.40)E-02
109	53	58	28	54	26	240	2.71E+01	-	3.00E-03
109	53	58	28	54	26	229	9.99E+01	-	5.00E-02
109	53	58	28	54	26	250	5.51E+00	-	4.00^{+4}_{-2}E+01
109	53	58	28	58	28	250	6.43E+00	-	$3.00^{+3}_{-1.5}$E+01
113	55	58	28	58	28	250	4.43E+01	1.35E-01	3.00E+01
147	69	58	28	92	42	260	3.61E+02	3.53E+00	1.80E-02
151	71	58	28	96	44	266	3.00E+02	2.54E-01	(7.00 ± 1.00)E-02
161	75	58	28	106	48	270	3.53E+01	4.68E+00	(6.30 ± 1.80)E-03
167	77	78	36	92	42	357	1.77E+01	7.15E+00	1.10E-01
171	79	78	36	96	44	389	2.76E-01	1.73E-01	2.00E-03
171	79	78	36	96	44	370	1.45E+01	9.02E-01	6.00E-04
171	79	78	36	96	44	361	1.28E+01	1.00E+00	1.10E-03
171	79	78	36	96	44	359	1.52E+01	1.09E+00	2.00E-03
171	79	78	36	96	44	363	2.01E+01	1.10E+00	6.00E-04
171	81	78	36	102	46	370	2.02E+01	4.20E-01	3.00E-05
Proton emitter: p3n *channel*									
112	55	58	28	58	28	259	5.06E+01	2.28E-02	5.00E-04
146	55	58	28	92	42	287	5.21E+01	5.76E-02	1.00E-03
150	71	58	28	96	44	287	2.41E+01	7.47E-03	2.56E-03
150	71	58	28	96	44	292	2.21E+01	7.02E-03	3.05E-03
160	75	58	28	106	48	300	2.26E+02	7.60E-01	1.00E-03
166	77	78	36	92	42	384	2.99E+01	2.24E+00	6.30E-03
176	81	78	36	102	46	384	5.99E+01	3.07E-02	3.00E-06

In heavy nuclei, where fission is important, experimental cross sections are much lower than expected. In these reactions, because these cross sections are very small, evaporation is not a common decay mode for the compound nucleus, and therefore, evaporation events are rare to find. Then, both codes overestimate the events of evaporation in heavy nuclei. Indeed, the number of fission events is similar in both codes and the difference can be attributed to the way the fusion cross section is calculated.

But even with this difficulties, the code of this article achieves a reasonable simulation of the decay channels given a projectile and target, taking into account the rather modest theoretical framework, which makes much easier for new students to understand how a evaporation code works. We see that the results of the cross sections are acceptable when compared with a more sophisticated code like `PACE` or experimental values, for cross sections not very low and nuclei not too heavy. These cases need special treatment, and motivate a discussion on modifications of the code described in this article, to those who are interested.

Conclusions

This article provides a guide on the semi-classical theory behind the fusion-evaporation reaction, based in the division in the *independent* mechanisms of formation (fusion) and decay (evaporation by n, p, α or fission) of the compound nucleus. This theoretical framework is based in a rather modest description of the reaction, which is much easier to understand for new students, and can produce reasonable results of the decay channels.

The computational code explained in this article, which applies the theory described, returns a list of possible residues of the reaction (for different possibilities of evaporation or fission for heavy nuclei), selecting sequential decay modes with Monte Carlo. The proportion of the different type of events is a measure of the reaction cross section. Increasing the number of cascades makes more likely to find rare events, and that is the main limitation of the code: a high number of cascades implies an excessive calculation time.

This article compiles the results of about 90 proton-rich reactions, which were compared with experimental values and values from a more sophisticated code like `PACE`. Both codes are limited by a high number of cascades ($\sim$ 10 thousand), which makes difficult to find rare events in some heavy nuclei reactions, for example. The results show substantial differences in several reactions, especially in very low cross sections. About of 20 reactions are accurate enough in order of magnitude with `PACE` or experimental values. In heavy nuclei, the results are much bigger in both codes, which indicates a overestimation of evaporation in these reactions, showing a possibly deficient treatment of fission.

The overestimation of cross sections in several reactions can be attributed to channels that were not found and would decrease excess events. This suggests a formalism, which may allow a more diverse list of exit channels that would result in a different proportion of events (for example, allowing evaporation of heavier ejectiles, gammas or fission fragments). This motivates for a deeper analysis of the limitations of this code and possible modifications, for new students interested in this topic.

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