CHARGED HIGGS IN 3-3-1 MODEL THROUGH e^-e^+ COLLISIONS

HIGGS CARGADOS DEL MODELO 3-3-1 MEDIANTE LA COLISIÓN e^-e^+

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Abstract

In this work we present an analysis of production and signature of charged Higgs bosons $H_2^{\pm}$ in the version of the 3-3-1 model containing heavy leptons at the CLIC (Cern Linear Collider). The production rate is found to be significant for the direct production of $e^-e^+ \rightarrow H_2^+ H_2^-$.

Keywords: charged Higgs, ILC, CLIC, 3-3-1 model, branching ratio.

Resumen

En este trabajo presentamos un análisis de la producción y señales de los bosones de Higgs $H_2^{\pm}$ cargados en la versión del modelo 3-3-1 con leptones pesados en el CLIC (Cern Linear Collider). Se ha encontrado que la tasa de producción es significativa para la producción directa de $e^-e^+ \rightarrow H_2^+ H_2^-$.

Palabras clave: Higgs cargados, ILC, CLIC, modelo 3-3-1, tasa de decaimiento.

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1 Introduction

The Higgs sector remains one of the most dense part of the standard model (SM) [1–3], but it still represents a fundamental rule by explaining how particles gain masses through an isodoublet scalar field, which is responsible for the spontaneous breakdown of the gauge symmetry, the process by which the spectrum of all particles is generated. This process of mass production is the so-called *Higgs mechanism*, which plays a central role in gauge theories.

The SM provides a very good description of all phenomena related to hadron and lepton colliders. This includes the Higgs boson, which appears as an elementary scalar and is created by the electroweak symmetry breaking. Thus, on July 4, 2012, the discovery of a new particle with a mass measured by the CMS Collaboration to be 125.35 GeV was announced; physicists suspected that it was the Higgs boson. Since then, the particle has been shown to behave, interact, and decay in many of the ways predicted for Higgs particles by the Standard Model, as well as having even parity and zero spin, two fundamental properties of a Higgs boson. This also means that it is the first elementary scalar particle discovered in nature[4, 5].

Different types of Higgs bosons, if they exist, may lead us into new areas of physics beyond the SM. Since the SM leaves many questions open, there are several extensions. For example, if the Grand Unified Theory (GUT) includes the SM at high energies, then the Higgs bosons associated with the GUT symmetry breaking must have masses of the order of $M_X \sim \mathcal{O}(10^{15})$ GeV. Supersymmetry [6] provides a solution to the hierarchy problem by eliminating the quadratic divergences by fermionic and bosonic loop contributions [7–10]. Furthermore, the Minimal Supersymmetric extension of the SM can be derived as an effective theory of supersymmetric GUT [11–13].

There are also other classes of models based on $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge symmetry (3-3-1 model) [14–16], where the anomaly cancellation mechanisms occur when the three basic fermion families are considered and not family by family as in the SM. This mechanism is peculiar because it requires that the number of families is an integer multiple of the number of colors. This property, together with the asymptotic freedom, which is a property of quantum chromodynamics, requires that the number of families be three. Moreover, according to these models, the Weinberg angle is restricted to the value $s_W^2 = \sin^2\theta_W < 1/4$ in the heavy-leptons version [14]. Thus, if it evolves to higher values, it shows that the model loses its perturbative character when it reaches a mass scale of about 4 TeV [17]. Therefore, the 3-3-1 model is one of the most interesting extensions of the SM and is phenomenologically well motivated to be studied at the CLIC and other accelerators.

Among the most studied candidate theories for electroweak symmetry breaking (EWSB) in the literature are the Higgs mechanism within the Standard Model (SM) [18–20], the Minimal Supersymmetric Standard Model (MSSM), [21–23] and the 3-3-1 Model [24]. Contrary to the SM and MSSM, three Higgs scalar triplets are required in the 3-3-1 Model. This leads to seven physical Higgs bosons instead of the single physical Higgs boson in the SM and five in the MSSM, these states are: three scalar (H_1^0, H_2^0, h^0), one pseudoscalar neutral (H_3^0) and three charged Higgs bosons ($H_1^\pm$, $H_2^\pm$, $H^{\pm\pm}$).

The outline of this work is as follows. In Sec. 2 we present the relevant features of the model. In Sec. 3 we compute the total cross sections of the process and in Sec. 4 we summarize our results and conclusions.

2 Relevant Features of the Model

If there exist any model beyond the standard model (BSM), then it is not surprising that it is still hidden in the scalar sector. As one of the simplest BSM models, is the $3-3-1$ model, which has been extensively studied in the literature.

These is a promissory model which is based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3-3-1 for short) semi-simple symmetry group [25], which contains both charged Higgs bosons. In this model the three Higgs scalar triplets

$$\eta=\left(\begin{array}{c}\eta^0\\ \eta_1^-\\ \eta_2^+\end{array}\right)\sim(\mathbf{3},0)\,,\quad \rho=\left(\begin{array}{c}\rho^+\\ \rho^0\\ \rho^{++}\end{array}\right)\sim(\mathbf{3},1)\,,$$

$$\chi=\left(\begin{array}{c}\chi^-\\ \chi^{--}\\ \chi^0\end{array}\right)\sim(\mathbf{3},-1) \qquad (1)$$

generate the fermion and gauge boson masses in the model. The neutral scalar fields develop the vacuum expectation values (VEVs) $\langle\eta^0\rangle = v_\eta$, $\langle\rho^0\rangle = v_\rho$ and $\langle\chi^0\rangle = v_\chi$, with $v_\eta^2+v_\rho^2 = v_W^2 = (246\text{ GeV})^2$. The pattern of symmetry breaking is

$$\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_N \xmapsto{\langle\chi\rangle} \mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y \xmapsto{\langle\eta,\rho\rangle} \mathrm{U}(1)_{\mathrm{em}}$$

and so we can expect $v_\chi \gg v_\eta, v_\rho$. The η and ρ scalar triplets give masses to the ordinary fermions and gauge bosons, while the χ scalar triplet gives masses to the new fermions and new gauge bosons. The most general, gauge invariant and renormalizable Higgs potential is

$$\begin{aligned}V(\eta,\rho,\chi) = {}& \mu_1^2\eta^\dagger\eta+\mu_2^2\rho^\dagger\rho+\mu_3^2\chi^\dagger\chi+\lambda_1\left(\eta^\dagger\eta\right)^2+\lambda_2\left(\rho^\dagger\rho\right)^2+\\ & \lambda_3\left(\chi^\dagger\chi\right)^2+\left(\eta^\dagger\eta\right)\left[\lambda_4\left(\rho^\dagger\rho\right)+\lambda_5\left(\chi^\dagger\chi\right)\right]+\\ & \lambda_6\left(\rho^\dagger\rho\right)\left(\chi^\dagger\chi\right)+\lambda_7\left(\rho^\dagger\eta\right)\left(\eta^\dagger\rho\right)+\lambda_8\left(\chi^\dagger\eta\right)\left(\eta^\dagger\chi\right)+\\ & \lambda_9\left(\rho^\dagger\chi\right)\left(\chi^\dagger\rho\right)+\lambda_{10}\left(\eta^\dagger\rho\right)\left(\eta^\dagger\chi\right)+\\ & \frac{1}{2}\left(f\epsilon^{ijk}\eta_i\rho_j\chi_k+\text{H. c.}\right). \end{aligned} \qquad (2)$$

Where f is a constant with dimensions of mass and the λ_i, $(i=1,\ldots,10)$ are adimensional constants with $\lambda_3<0$ from the positivity of the scalar masses. The term proportional to λ_{10} violates the lepto-barionic number so that, it was not considered in the analysis of the Ref. [24] (another analysis of the 3-3-1 scalar sector are given in ref. [26] and references cited therein). We can see that this term contributes to the mass matrices of the charged scalar fields, but not to the neutral ones. However, it can be checked that in the approximation $v_\chi \gg v_\eta, v_\rho$ we can still work with the masses and eigenstates given in Ref. [24]. Here this term is important for the decay of the lightest exotic fermion. Therefore, we keep it in the Higgs potential.

The symmetry breaking is initiated when the scalar neutral fields are shifted as $\varphi = v_\varphi + \xi_\varphi + i\zeta_\varphi$, with $\varphi = \eta^0$, ρ^0, χ^0. Thus, the physical neutral scalar eigenstates H_1^0, H_2^0, H_3^0 and h^0 are related to the shifted fields as

$$\begin{pmatrix} \xi_\eta \\ \xi_\rho \end{pmatrix} \approx \frac{1}{v_W} \begin{pmatrix} v_\eta & v_\rho \\ v_\rho & -v_\eta \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}, \quad \xi_\chi \approx H_3^0, \quad \zeta_\chi \approx h^0, \quad (3)$$

and in the charge scalar sector we have

$$\eta_1^+ \approx \frac{v_\rho}{v_W} H_1^+, \qquad \rho^+ \approx \frac{v_\eta}{v_W} H_2^+, \qquad \chi^{++} \approx \frac{v_\rho}{v_\chi} H^{++}, \qquad (4)$$

with the condition that $v_\chi \gg v_\eta, v_\rho$ [24].

In this work, we will study the production mechanism for charged Higgs particles ($H^- H^+$) and their signatures in e^+e^- colliders such as the CERN Linear Collider (CLIC) ($\sqrt{s} = 3.0$ and 5.0) TeV.

The CLIC will have three stages. The CLIC-1, where the collision energy is of 380 GeV and will serve as a factory for Higgs and quarks top. Neither of these particles has yet been studied in electron-positron collisions. The CLIC-2 is an intermediate high-energy stage with 1.5 TeV and CLIC-3 is the highest energy level with 3 TeV and would accumulate 5 ab^{-1} of integrated luminosity.

The advantage of CLIC as the next-generation collider after the High-Luminosity LHC (HL-LHC) is that it gives access to two complementary search paths for new physics. The first path focuses on the study of known Standard Model processes with unprecedented precision to search for deviations from the predicted behaviour. Such deviations would provide indirect evidence for BSM physics. The second path involves direct searches for new physics phenomena such as the production of new particles [27, 28].

3 Cross section production

The process $e^- e^+ \to H_2^- H_2^+$, takes place through the exchange of the bosons γ, Z and Z' in the s channel. The model does not allow any interaction between the neutral and charged Higgs. The differential cross section for the $H_2^\pm$ was given by [29].

The total width of the Higgs H_2^- into quarks, leptons, gauge bosons $V^-\gamma$, ZV^-, $Z'V^-$, gauge bosons and Higgs bosons W^+H^{--}, $V^-H_1^0$, $V^-H_2^0$, $V^-H_3^0$, V^-h^0 are respectively given by, [30].

$$\begin{aligned}\Gamma\left(H_2^\pm \to \text{all}\right) &= \Gamma_{H_2^\pm \to q\bar{q}} + \Gamma_{H_2^\pm \to E_a^\pm \nu(\bar{\nu})} + \Gamma_{H_2^\pm \to V^\pm \gamma} + \Gamma_{H_2^\pm \to ZV^\pm} + \\ &\quad \Gamma_{H_2^\pm \to Z'V^\pm} + \Gamma_{H_2^\pm \to W^\mp H^{\pm\pm}} + \Gamma_{H_2^\pm \to H_1^0 V^\pm} + \\ &\quad \Gamma_{H_2^\pm \to H_2^0 V^\pm} + \Gamma_{H_2^\pm \to H_3^0 V^\pm} + \Gamma_{H_2^\pm \to h^0 V^\pm} \end{aligned} \tag{5}$$

4 Results and Conclusions

Here we present the cross section for the process $e^- e^+ \to H_2^- H_2^+$ for the CLIC (3 and 5 TeV). All calculations have been done according to [24] where we obtain the following representative values for the parameters and the VEV: $\lambda_1 = 1.74756 \times 10^{-1}$, $\lambda_2 = 1.0$, $\lambda_3 = -2.5 \times 10^{-2}$, $\lambda_4 = 2.14$, $\lambda_5 = -1.57$, $\lambda_6 = 1.0$, $\lambda_7 = -2.0$, $\lambda_8 = -5.0 \times 10^{-1}$, $v_\eta = 195$ GeV, and $\lambda_9 = 0.0(0.0, 0.0)$. We take the following values for the VEV $v_\chi = 3.5(4.0, 5.0)$ TeV, these parameters and VEV are used to estimate the values for the particle masses which are given in table 1. In particular, the value of $v_\eta = 195$ GeV has been used to estimate the mass of H_1^0 [31, 32].

Differently from what we did in other's papers [33–35], where arbitrary parameters were taken, in this work we consider the approximation $-f \simeq v_\chi$, which also serves to fix the values of parameters and VEV [31, 32]. Note that the branching ratios of $H_2^\pm$ depend on the parameters of the 3-3-1, which determines the size of several decay modes.

The Higgs $H_2^\pm$ in 3-3-1 model is coupled to quarks, $q\bar{q}$; heavy leptons and neutrinos, $E_i^\pm \nu(\bar{\nu})$; charged bosons and photons, $V^\pm\gamma$;

f	v_χ, m_{J_1}	m_E	m_M	$m_{H_3^0}$	m_{h^0}	$m_{H_1^0}$	$m_{H_2^0}$	m_V	m_U	$m_{Z'}$	$m_{J_{2,3}}$
-3500	3500	521.15	3062.60	1106.80	5037.72	125.51	3560.15	1608.59	1607.57	5976.43	4949.70
-4000	4000	595.60	3500.02	1264.91	5756.99	125.51	4068.75	1837.72	1836.83	6830.21	5656.80
-5004.3	5001.0	744.50	4375.00	1581.46	7198.47	125.51	5086.95	2296.62	2295.91	8539.47	7071.00

TABLE 1. *Values for the particles masses used in this work. All the values in this Table are given in GeV. Here,* $m_{H^{\pm\pm}} = 2825.0(3227.7; 4035.5)$ *GeV for* $v_\chi = 3500.0(4000.0; 5001.0)$ *GeV and* $m_T = 2v_\chi$.

gauge bosons and charged bosons, $ZV^\pm$; charged gauge bosons and double-charged Higgs bosons, $W^\pm H^{\mp\mp}$; charged bosons and neutral Higgs, $V^\pm H_i^0$ where $i = 1, 2, 3$. The Higgs $H_2^\pm$ can be heavier than 1190.65 GeV for $v_\chi = 3.5$ TeV; 1360.51 GeV for $v_\chi = 4.0$ TeV and 1698.72 GeV for $v_\chi = 5.0$ TeV, so the Higgs $H_2^\pm$ is a heavy particle for the v_χ taken above.

For several models, ATLAS excludes a Z' with mass up to 2.8 TeV. Table 1 shows the masses of the exotic boson Z', in agreement with the CMS and ATLAS estimates [36–39].

4.1 CLIC at 3 TeV

We will use CLIC instead of the International Linear Collider (ILC) because the masses generated by the model are not supported by the ILC.

If we consider that the expected integrated luminosity for the CLIC collider will be of the order of 5 ab^{-1}/yr, then we obtain a total of $\simeq 2.04 \times 10^4(0.00; 0.00)$ events per year if we take the mass of the Higgs boson $m_{H_2^\pm} = 1.3$ TeV and $v_\chi = 3.5(4.0; 5.0)$ TeV, and $1.31 \times 10^3(8.35 \times 10^2; 0.00)$ events per year if we take the mass of the Higgs boson $m_{H_2^\pm} = 1.47$ TeV. Please refer to Fig. 1 for the same vacuum expectation values v_χ, taken above. The value of 0.0 is for $v_\chi = 4.0(5.0)$ TeV and mass $m_{H_2^\pm} = 1.3$ TeV. These values of v_χ restrict Higgs mass to values up to 1360.51(1698.72) GeV. The total widths for these vacuum expectation values are, $\Gamma_{H_2^\pm} = 8.08 \times 10^{-4}(0.00; 0.00)$ for $m_{H_2^\pm} = 1.3$ TeV and $\Gamma_{H_2^\pm} = 9.71 \times 10^{-4}(6.96 \times 10^{-4}; 0.00)$ for $m_{H_2^\pm} = 1.47$ TeV, (see Fig. 2).

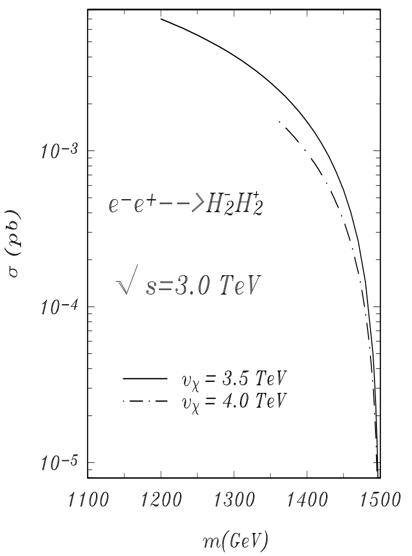


FIGURE 1. *Total cross section for the process* $e^- e^+ \to H_2^- H_2^+$ *as a function of* $m_{H_2^\pm}$ *at* $\sqrt{s} = 3.0$ *TeV for a)* $v_\chi = 3.5$ *TeV (solid line) and b)* $v_\chi = 4.0$ *TeV (point dashed line)*

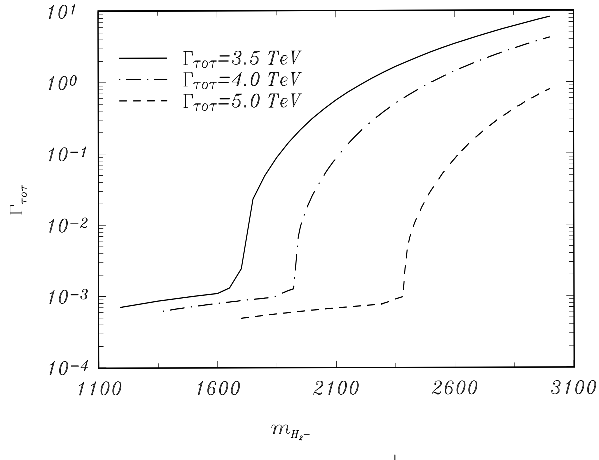
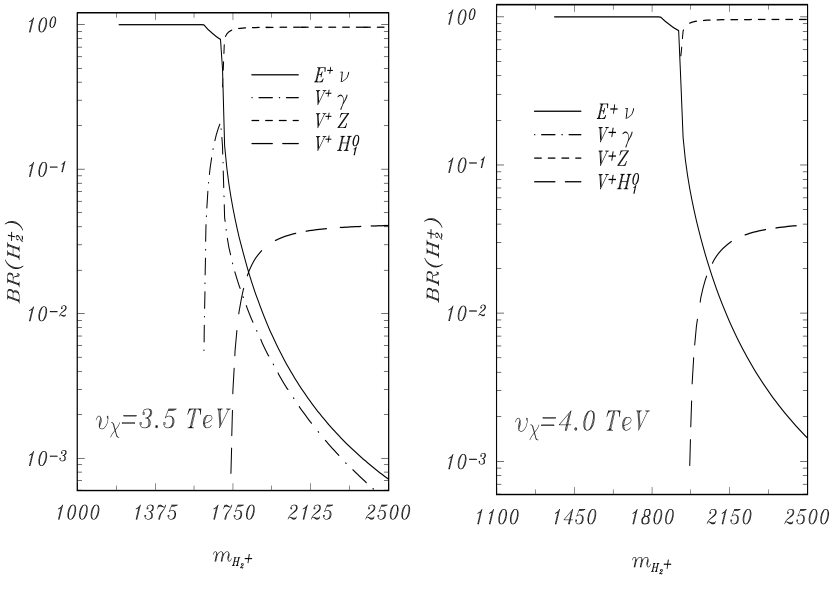


FIGURE 2. *The charged Higgs boson* $H_2^\pm$ *decay versus its mass for (a)* $v_\chi = 3.5$ *TeV (solid line), b)* $v_\chi = 4.0$ *TeV (point dashed line) and* $v_\chi = 5.0$ *TeV (short dashed line)*

To obtain event rates, we multiply the production cross sections by the respective branching ratios. Taking into account that the branching ratio for $v_\chi = 3.5$ TeV and $m_{H^\pm} = 1.3$ TeV will be $BR(H_2^\pm \to E^\pm \nu(\bar{\nu})) = 100\%$, then we would have about $\simeq 2.04 \times 10^4$ events per year, for $m_{H^\pm} = 1.47$ TeV the number of events will be $\simeq 1.31 \times 10^3$, and for $v_\chi = 4.0$ TeV and $m_{H^\pm} = 1.47$ TeV, the number of events will be $\simeq 8.35 \times 10^2$, respectively, remember that for $v_\chi = 4.0$ the corresponding masses start at $m_{H^\pm} = 1.36$ TeV(see Fig. 3a and Fig. 3b).



(a) (b)

FIGURE 3. *Branching ratios for the charged Higgs decays as a function of* $m_{H_2^\pm}$ *for (a)* v_χ *=3.5 TeV and (b)* v_χ *=4.0 TeV*

Due to our ignorance about the decay of exotic particles, we assumed that the exotic leptons would be measured by the electromagnetic calorimeter in the same way as the SM leptons, i.e. $E^\pm$ produces tracks in the inner detectors and deposits its energy in the electromagnetic calorimeter.

Furthermore, the background events can also be easily reduced by imposing the Z window cut $|m_{\ell\ell} - m_Z| > 15$ GeV, this removes events where the leptons come from the Z decay [40, 41]. With respect to the $\bar{\nu}(\nu)$ we set the cut to the missing transverse momentum $\not{p}_T > 20$ GeV, both of which allow a very strong reduction of the backgrounds.

The processes that have the same signature of the signal $E^+E^-\nu\bar{\nu}$ are $e^+e^- \to \tau^+\tau^-, W^+W^-, Z^0Z^0$ and W^+W^-Z, the last process is suppressed at least by $\alpha/sin^2\theta_W$ relative to the process involving a double gauge boson, so using the COMPHEP [42], the total cross section for these processes at 3.0 TeV will be equal to 5.33×10^{-1} pb, hence the number of background events for the signal $e^+e^-\nu\bar{\nu}$ will be approximately 3.11×10^4.

Considering the signal events, which is equal to $\simeq 2.04 \times 10^4$ for $m_{H_2^\pm} = 1.3$ TeV and $v_\chi = 3.5$ TeV, then the statistical significance is $> 5.0\sigma$, and for $m_{H_2^\pm} = 1.47$ TeV and same $v_\chi = 3.5$ TeV the signal events is $\simeq 1.31 \times 10^3$, hence the statistical significance is $> 5.0\sigma$. Taking into account that for mass $m_{H_2^\pm} = 1.47$ and same $v_\chi = 4.0$ TeV the signal events is 8.35×10^2, it follows that the statistical significance is $\simeq 4.7\sigma$.

So, we have evidence for $> 5.0\ \sigma$ discovery in the $E^+E^-\nu\bar{\nu}$ final state in almost all scenarios.

It may also be that these $E^\pm$ particles, being exotic, produce exotic signals that we are not accustomed to, i.e. signals, that would be more difficult to detect experimentally, probably requiring new detection tools.

4.2 CLIC at 5 TeV

For a charged Higgs mass greater than 1.5 TeV, it will be necessary to have a CLIC with energy greater than 3 TeV, so we will eventually do the phenomenology with a CLIC equal to 5 TeV [43, 44]. Let us assume that the luminosity will be about 15 ab^{-1}/yr, this is a conservative approach, i.e. 3 times greater than for the 3 TeV CLIC, considering that for the 1.5 TeV ILC, the 3 TeV CLIC is 13 times greater.

Then the statistics we expect for this collider for $v_\chi = 3.5(4.0, 5.0)$ TeV and $m_{H_2^\pm} = 2.0$ TeV are $6.98 \times 10^5 (2.46 \times 10^5; 7.16 \times 10^4)$ of $H_2^\pm$ particles produced per year. In respect to the $m_{H_2^\pm} = 2.46$ TeV and the same parameters used above, this gives a total of $1.86 \times 10^4 (6.60 \times 10^3; 1.91 \times 10^3)$ of $H_2^\pm$, see

Fig. 4. The total widths given by [29], for the vacuum expectation values given above are, $\Gamma_{H_2^\pm} = 3.09 \times 10^{-1}(2.56 \times 10^{-2}; 6.33 \times 10^{-4})$ for $m_{H_2^\pm} = 2.0$ TeV and $\Gamma_{H_2^\pm} = 2.34(8.37 \times 10^{-1}; 1.74 \times 10^{-2})$ for $m_{H_2^\pm} = 2.46$ TeV, see Fig. 2.

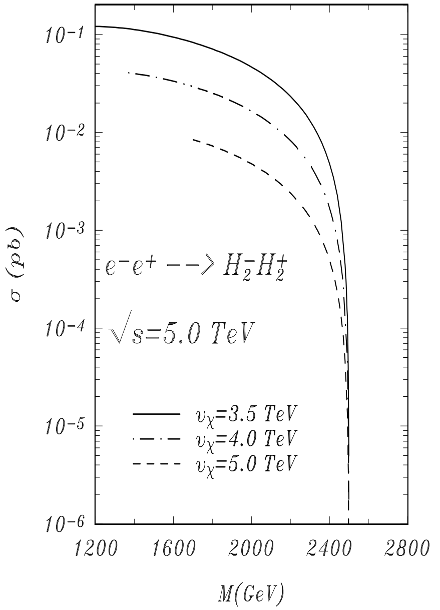


FIGURE 4. *Total cross section for the process $e^- e^+ \to H_2^- H_2^+$ as a function of $m_{H_2^\pm}$ at $\sqrt{s} = 5.0$ TeV for a) $v_\chi = 3.5$ TeV (solid line), b) $v_\chi = 4.0$ TeV (point dashed line) and $v_\chi = 5.0$ TeV (short dashed line)*

Taking into account that the signal for $H_2^\pm$ are $E^\pm \nu(\bar{\nu})$ and taking into consideration, that the BRs for these particles would be $BR(H_2^\pm \to E^\pm \nu(\bar{\nu})) = 0.47\%(7.90 \times 10^{-2})$ %, for the mass of the charged Higgs boson $m_{H^\pm} = 2.0(2.46)$ TeV and $v_\chi = 3.5$ TeV, then we would have approximately $\simeq 3.28 \times 10^3(15)$ events per year for CLIC with 5 TeV, for the same signal as above, that is $E^+ E^- \nu \bar{\nu}$ (see Fig. 3*a*).

Considering that the second signal for $H_2^\pm$ are $ZV^\pm$ and considering that the BRs for these particles would be $BR(H_2^\pm \to ZV^\pm) = 95.79$ %(95.81 %), for the mass of the charged Higgs boson $m_{H^\pm} = 2.0(2.46)$ TeV and $v_\chi = 3.5$ TeV, and that $V^\pm$ decays into $W^\pm Z$, whose $BR(V^\pm \to W^\pm Z) = 99.20$ %(99.35 %), see Fig. 5 and [30], followed by leptonic decay of the boson $W^{\pm}$ into $\ell^{\pm}\nu(\bar{\nu})$, whose branching ratios for these particles would be $BR(W \to \ell^{\pm}\nu(\bar{\nu})) = 10.8\%$ and hadronic Z decays into $b\bar{b}$, whose branching ratios for these particles would be $BR(Z \to b\bar{b}) = 15.2\%$, then we would have approximately $1.66 \times 10^{3}(44)$ events per year for $b\bar{b}b\bar{b}b\bar{b}b\bar{b}\ell\ell\nu\bar{\nu}$.

For the signal $H_2^{\pm} \to E^{\pm}\nu(\bar{\nu})$, taking into account that $v_{\chi} = 4.0$ TeV, $m_{H_2^{\pm}} = 2.0(2.46)$ TeV and taking into consideration that the BRs for these particles are $BR(H_2^{\pm} \to E^{\pm}\nu(\bar{\nu})) = 4.21\%(1.66 \times 10^{-1}\%)$, then we would have about $1.04 \times 10^{4}(11)$ events per year for the signal $E^{+}E^{-}\nu\bar{\nu}$, as shown in Fig. 3b.

Considering that the signal for $H_2^{\pm}$ being $ZV^{\pm}$, whose BRs for these particles would be $BR(H_2^{\pm} \to ZV^{\pm}) = 93.26\ \%(95.81\ \%)$, for the mass of the charged Higgs boson $m_{H_2^{\pm}} = 2.0(2.46)$ TeV, and that $V^{\pm}$ decays into $W^{\pm}Z$, whose $BR(V^{\pm} \to W^{\pm}Z) = 99.36\ \%(99.48\ \%)$, see Fig. 5 and [30], followed by leptonic decay of the boson $W^{\pm}$ into $\ell^{\pm}\nu(\bar{\nu})$, whose branching ratios for these particles would be $BR(W \to \ell^{\pm}\nu(\bar{\nu})) = 10.8\%$ and hadronic Z decay into $b\bar{b}$, whose branching ratios for these particles would be $BR(Z \to b\bar{b}) = 15.2\%$, then we would have about $5.69 \times 10^{2}(16)$ events per year for $b\bar{b}b\bar{b}b\bar{b}b\bar{b}\ell\ell\nu\bar{\nu}$ for, $v_{\chi} = 4.0\ TeV$.
With respect to the vacuum expectation value $v_{\chi} = 5.0$ TeV, for the masses of $m_{H_2^{\pm}} = 2.0(2.46)$ TeV. Considering the same signal as above, that is, $E^{+}E^{-}\nu\bar{\nu}$ and considering that the branching ratios for $H_2^{\pm}$ would be $BR(H_2^{\pm} \to E^{\pm}\nu(\bar{\nu}) = 100.00\ \%(4.86\ \%)$, then we would have $7.16 \times 10^{4}(93)$ events per year for the signal $E^{+}E^{-}\nu\bar{\nu}$, as shown by Fig. 5.

Considering that the second signal for $H_2^{\pm}$ are $ZV^{\pm}$ and considering that the BRs for these particles would be $BR(H_2^{\pm} \to ZV^{\pm}) = 0.0\ \%(92.62\ \%)$, for the mass of the charged Higgs boson $m_{H^{\pm}} = 2.0(2.46)$ TeV, and that $V^{\pm}$ decays into $W^{\pm}Z$, whose $BR(V^{\pm} \to W^{\pm}Z) = 0.0\ \%(99.57\ \%)$, see Fig. 5 and [30], followed by leptonic decay of the boson $W^{\pm}$ into $\ell^{\pm}\nu(\bar{\nu})$, whose branching ratios for these particles would be $BR(W \to \ell^{\pm}\nu(\bar{\nu})) = 10.8\%$ and

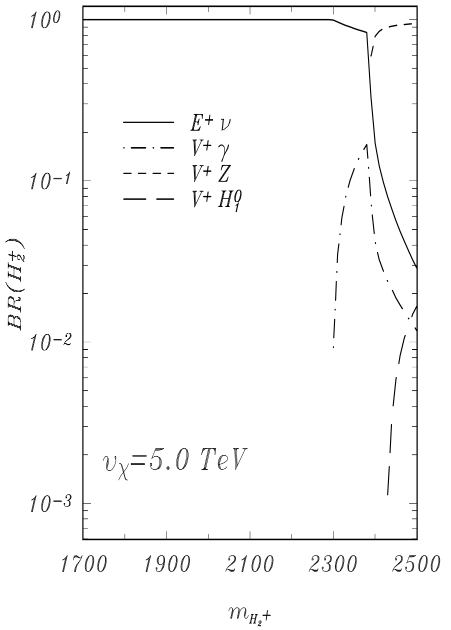


FIGURE 5. *Branching ratios for the charged Higgs decays as a function of* $m_{H_2^\pm}$ *for* $v_\chi = 5.0$ *TeV.*

hadronic Z decays into $b\bar{b}$ whose branching ratios for these particles would be $BR(Z \to b\bar{b}) = 15.2\%$, then we would have about 0.0(4) events per year for the signal $b\bar{b}b\bar{b}b\bar{b}b\bar{b}\ell\ell\nu\bar{\nu}$.

We obtain the value of $BR(H_2^\pm \to ZV^\pm) = 0.00$, because the $v_\chi = 5.0$ TeV limits the mass of $m_{V^\pm}$ up to the value of 2380 GeV.

Taking into account the same backgrounds mentioned above and using the COMPHEP [42], the total cross section for these processes for CLIC (5.0 TeV) will be equal to 2.31×10^{-1} pb, hence the number of background events for the same signal $b\bar{b}b\bar{b}b\bar{b}b\bar{b}\ell\ell\nu\bar{\nu}$ will be equal to 21.60.

Considering the signals events, that is E^+E^-X, which is equal to $\simeq 3.28 \times 10^3(15)$ for $m_{H_2^\pm} = 2.0(2.46)$ TeV and $v_\chi = 3.5$ TeV, then the statistical significances are $\gg 5.0(3.23)\sigma$, for $v_\chi = 4.0$ TeV, the signal events are $1.04 \times 10^4(11)$, whose significances are $\ggg 5.0(2.37)\sigma$ and for $v_\chi = 5.0$ the number of signal events are $\simeq 7.16 \times 10^4(93)$, so the statistical significances are $\ggg 5.0(> 5.0)\sigma$.

Therefore, we have evidence for the $> 5.0\ \sigma$ discovery in the $E^+E^-\nu\bar{\nu}$ final state, for the case of $m_{H_2^\pm} \leq 2.0$ TeV and $v_\chi = 3.5(4.0; 5.0)$ TeV, but in all scenarios careful Monte Carlo work must be done to determine the signal.

Taking into account the second signal for $H_2^\pm$, that is $b\bar{b}b\bar{b}b\bar{b}b\bar{b}\ell\ell\nu\bar{\nu}$, we would have about $1.66 \times 10^3(44)$ events per year for $m_{H_2^\pm} = 2.0(2.46)$ TeV and $v_\chi = 3.5$ TeV, then the statistical significance in this case are $\ggg 5.0(> 5.0)\sigma$, for $v_\chi = 4.0$ TeV, the signal events are about $5.69 \times 10^2(16)$ events per year, then the statistical significances are $\gg 5.0(3.45)\sigma$, and for $v_\chi = 5.0$ TeV, we have approximately 0.00(29) events for these signals given above, whose statistical significances are $0,00(> 5.0)\sigma$.

In summary, we have shown in this paper that in the context of the $3-3-1$ model in some scenarios the signatures for the charged Higgs boson $H^\pm$ can be significant at both 3.0 TeV and 5.0 TeV. In other scenarios, the signal events will be too small to be observed, so the above cuts must be applied, and in other situations there are no signal events. If this model is realisable in nature, new particles will certainly appear, such as $H_2^\pm, E^\pm, V^\pm$ in the context of this study.

It is worth mentioning that as the mass increases, the statistical significance decreases in all cases, which allows us to think that if the mass of the charged Higgs is large, we should think about more powerful accelerators.

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