ESTIMATION OF NUCLEAR STRUCTURE OF ^{186}Hg NUCLEUS BY THE IBM-1 AND THE IBM-2 MODELS

ESTIMACIÓN DE LA ESTRUCTURA NUCLEAR DEL NÚCLEO ^{186}Hg MEDIANTE LOS MODELOS IBM-1 E IBM-2

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(Received: jan./2024. Accepted: may./2024)

Abstract

This paper presents an analysis of the Interacting Boson Model (IBM-1) and IBM-2, which are considered representative models for the translation limit of SU(3)-O(6). An appropriate method for fitting is expected to develop the optimum parameters for the calculated energy level of ^{186}Hg nucleus. The intended energy states for the various bands, for example, ground, γ, and β bands of mutually exclusive models, are associated with the prevailing measured data. The strengths of quadruple electromagnetic transitions in this nucleus, established by the IBM-1 and IBM-2 models, are conveyed and compared with reasonable prior measured data. The potential energy

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surfaces (PES) of this nucleus for the distortion parameter in the SU(3)-O(6) symmetry in IBM-1 are determined and analyzed.

Keywords: IBM-1, IBM-2, energy level, B(E2), potential energy, ^{186}Hg isotope.

Resumen

Este artículo presenta un análisis del Modelo de Bosones Interactuantes (IBM-1) y el IBM-2, los cuales se consideran modelos representativos para el límite de transición de SU(3)-O(6). Se espera que un método adecuado de ajuste desarrolle los parámetros óptimos para el nivel de energía calculado del núcleo de ^{186}Hg. Los estados de energía previstos para las diversas bandas, como la banda fundamental, la banda γ y la banda β de modelos mutuamente excluyentes, se asocian con los datos medidos predominantes. Las intensidades de las transiciones electromagnéticas cuadrupolares en este núcleo, determinadas mediante los modelos IBM-1 e IBM-2, se presentan y se comparan con datos medidos previamente y considerados razonables. Se determinan y analizan las superficies de energía potencial (PES) de este núcleo para el parámetro de distorsión en la simetría SU(3)-O(6) en IBM-1.

Palabras clave: IBM-1, IBM-2, nivel de energía, B(E2), energía potencial, isótopo ^{186}Hg.

1. Introduction

Iachello and Arima used the Interacting Boson Model (IBM-1) to examine the unified assets of nuclear matter in medium-mass nuclei [1]. The nucleus comprises neutrons and protons, known as nucleons, and they were not distinguished in IBM-1. In the other class of model known as IBM-2 [2, 3], it is reputable that the nucleons are distinguished outside the closed shells. Based

on the angular momentum value L (0 or 2), each boson can lodge in one of two levels: the s-boson ($L = 0$) or the d-boson ($L = 2$). The use of IBM-1 for the even-even low-lying nuclei is typical of a stationary number, which is identified as the boson number. Besides, the IBM-1 model formed γ-soft O(6), rotational SU(3), and vibrational U(5) symmetry from the mathematical U(6) group [4, 5]. However, many researchers recommend that nuclei have three intermediate structures that are made up of U(5)–O(6), U(5)–SU(3), and SU(3)–O(6) transitions [6, 7].

The shape coexistence and quadruple collectivity at low energies in neutron midshell nuclei near the Z = 82 and N = 126 shell closures are not entirely known yet. Significant attention has been devoted in recent decades to the analytical description of the nuclear structure at the critical point of phase transitions. The ^{186}Hg nucleus consists of 80 protons and 106 neutrons. The configurations of ^{186}Hg nuclei are $\pi(h_{11/2})^{-4}\nu(i_{13/2})^{-20}$, which indicates 4 proton holes and 20 neutron holes existed in the ^{186}Hg nucleus according to double magic shell closure ^{208}Pb. This nuclear structure is intricate, as every nucleon connects by means of other nucleons, therefore, diagonalizing the transitional Hamiltonian in such situations requires the use of complex numerical methods. These configurations permit E2 transitions in yrast states, from excited states to ground states. The improvement of the nuclear structure of the even neutron-deficient ^{186}Hg nucleus has hardly been observed in the literature. There is significant agreement in the investigation of various types of bands (ground state, γ, and β) and B(E2) strength in Mercury isotopes with even mass numbers ($A = 188 - 190$) [8–10] . The neutron deficiency co-existence in the ^{186}Hg nucleus was explored in lifetime measurements by Siciliano et al. [8].

It was discovered that the triaxial deformation energy surfaces coexist in a ^{190}Hg configuration [9]. Garcia-Ramos et al. [10] utilized IBM and configuration mixing to explain the even-even Hg isotopes in ^{188,190}Hg isotopes. Considerable emphasis was placed on the characterization of the shape of nuclei and its correlation

with shape synchronicity phenomena. Recently, the SU(3) limit has been applied to the IBM-1 and IBM-2 models with respect to the ^{158}Gd nucleus [11]. The IBM-1 model was applied to ^{108,110,112}Ru isotopes to examine O(6) symmetry [12]. Al-Jubbori et al. determined that special neutron valence nucleons of double magic nuclei ^{208}Pb comprise the nuclear structure of rare-earth Er-Os nuclei at N=100,102,104 [13–15]. Hussain et al. [16, 17] study the properties of nuclear structure, such as energy levels and electric reduced transition probabilities for $^{190-198}$Hg isotopes. They were used in the interacting boson model-1 and the interacting boson fermion model-1 for even-even and even-odd nuclei, respectively. A. Salam et al. [18] applied the interacting boson model-1 to compute the electric reduced transition probabilities for ^{186}W and ^{186}Os isobars.

Currently, we have opted to investigate the ^{186}Hg nucleus due to its location between shells Z = 82 and N = 126. The objective of this article is to manipulate the different types of bands (g, γ, and β bands), reduced transition strength of B(E2), and potential energy surfaces for the ^{186}Hg nucleus using IBM-1 and IBM-2. The systematic motivation for accountability of the current exertion is needed to explore the phenomenological interacting boson model, IBM-1, and IBM-2 to explain previous measured data for the nature of three types of energy bands, B(E2) and PES for the ^{186}Hg nucleus. The investigation of the ^{186}Hg nucleus using the IBM-1 and IBM-2 models has been described for the first time.

2. Theoretical Calculations

2.1. IBM-1 Model

In the IBM1 model, possession is assigned to a reduced model space. This one provides a mathematical explanation for particles that are indistinguishable through $L = 0$ or 2. Hamiltonian is specified according with IBM-1 [1, 19, 20]:

$$\begin{aligned}\hat{H} &= \varepsilon_s(s^\dagger \cdot \tilde{s}) + \varepsilon_d(d^\dagger \cdot \tilde{d}) \\ &+ \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L \left[\left[d^\dagger \times d^\dagger\right]^{(L)} \times \left[\tilde{d} \times \tilde{d}\right]^{(L)} \right]^{(0)} \\ &+ \frac{1}{\sqrt{2}}\upsilon_2 \left[\left[d^\dagger \times d^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{s}\right]^{(2)} + \left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{d}\right]^{(2)} \right]^{(0)} \\ &+ \frac{1}{2}\upsilon_0 \left[\left[d^\dagger \times d^\dagger\right]^{(0)} \times \left[\tilde{s} \times \tilde{s}\right]^{(0)} + \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{d} \times \tilde{d}\right]^{(0)} \right]^{(0)} \\ &+ \frac{1}{2}u_0 \left[\left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{s} \times \tilde{s}\right]^{(0)} \right]^{(0)} + u_2 \left[\left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{s}\right]^{(2)} \right]^{(0)},\end{aligned} \tag{1}$$

The IBM-1 Hamiltonian can be represented by nine terms, of which two represent the impression in one-body terms ($L = 0$ and $L = 2$), while ε_s and ε_d represent the energies of the s and d bosons. The remaining two-body terms consist of (C_0, C_2, C_4, υ_0, υ_2, u_0, u_2). The number of conserved bosons is marked by N_b. The IBM-1 for the Hamiltonian is specified as [21, 22]:

$$\hat{H} = \epsilon\hat{n}_d + a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} + a_3\hat{T}_3\cdot\hat{T}_3 + a_4\hat{T}_4\cdot\hat{T}_4, \tag{2}$$

boson energy: $\varepsilon = \varepsilon_d - \varepsilon_s$. The operators are given as:

$$\left.\begin{aligned}&\hat{n}_d = d^\dagger \cdot \tilde{d} \\ &\hat{P} = 0.5[(\tilde{d}\cdot\tilde{d}) - (\tilde{s}\cdot\tilde{s})] \\ &\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^{(1)} \\ &\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)} \\ &\hat{T}_r = [d^\dagger \times \tilde{d}]^{(r)}\end{aligned}\right], \tag{3}$$

where $\hat{n}_d$ is the d-boson operator, $\hat{P}$ is the operator for pairing, $\hat{L}$ is the operator for angular momentum, $\hat{Q}$ is the operator for quadrupole, and $\hat{T}_r$ denotes the hexadecapole and octupole operators for $r = 4$ and 3. The representation χ denotes the

quadrupole creation limits 0 and $\pm\frac{\sqrt{7}}{2}$ [23].The symbols a_0, a_1, a_2, a_3, and a_4 designate the limits for the operators $\hat{P}$, $\hat{L}$, $\hat{Q}$, and $\hat{T}_r$ connections between the bosons. The PHINT [24] program's interaction parameters are specified: $\epsilon = EPS$, $a_0 = 2PAIR$, $a_1 = \frac{ELL}{2}$ *and* $a_3 = 5OCT$, $CHI = 0$.

There are three different types of active symmetry in the IBM-1, and its eigenvalues are as follows [19, 25]:

$$\left.\begin{aligned} E(n_d,\nu,L) &= \varepsilon n_d + \frac{a_1}{12}n_d(n_d+4) + \left(\frac{a_3}{7} - \frac{a_1}{10} - \frac{3a_4}{70}\right)\nu(\nu+3) + \frac{1}{14}(a_4-a_3)L(L+1), && \mathrm{U}(5)\\ E(\lambda,\mu,L) &= \frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)) + \left(a_1 - \frac{2a_2}{8}\right)L(L+1), && \mathrm{SU}(3)\\ E(\sigma,\tau,L) &= \frac{a_0}{4}(N-\sigma)(N+\sigma+4) + \frac{a_3}{2}\tau(\tau+3) + \left(a_1 - \frac{a_3}{10}\right)(L(L+1)), && \mathrm{O}(6) \end{aligned}\right] \tag{4}$$

the symbols ε, a_0, and a_2 indicate the limits for U(5), O(6), and SU(3), respectively. The Hamiltonian [1, 21, 26] is familiar for the designs that break rendering as

$$\hat{H} = a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} \tag{5}$$

2.2. IBM-2 Model

The alternative model referred to as IBM-2 [2, 3] consists of protons and neutrons as nuclei. The differentiation and boson number of these constituents have been accounted for in relation to the nucleons that exist beyond the main closed shells. Bosons can exist in one of two levels, denoted as s or d, depending on the angular momentum L $(0, 2)$. States with $L = 0$ and $L = 2$ are established by pairing the applied constructions of identical particles. $L = 0$ is represented by the symbol s, while $L = 2$ is $L = \mathrm{d}$.

The fundamental components of the IBM-2 model consist of the proton and neutron bosons. The neutron and proton are designated by ν and π, respectively. The symbols s_π, s_ν, d_π, and d_ν specify proton and neutron bosons with $L = 0$ and $L = 2$, respectively.

As stated in IBM-2, the Hamiltonian is [2, 3]:

$$H = H_\nu + H_\pi + V_{\pi\nu} \tag{6}$$

where H_π and H_ν correspond to the Hamiltonian of the proton and neutron bosons, respectively, and $V_{\pi\nu}$ symbolizes the proton-neutron interaction. A basic Hamiltonian in IBM-2 is given as [27]:

$$H = \varepsilon_\pi \hat{n}_{d_\pi} + \epsilon_\nu \hat{n}_{d_\nu} + \kappa Q_\pi \cdot Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}, \tag{7}$$

where ε_ν and ε_π are the energies of neutrons and protons, respectively. To simplify computation, assume that ε is equivalent for protons and neutrons ($\varepsilon_\nu = \varepsilon_\pi = \varepsilon$). The quadrupole operator is given as:

$$Q_\rho = \left(d^\dagger \times s + s^\dagger \times \tilde{d}\right)^{(2)}_\rho + \chi_\rho \left(d^\dagger \times \tilde{d}\right)^{(2)}_\rho, \quad \rho = \pi, \nu, \tag{8}$$

χ_ρ is a variable used to compute the configuration of the boson quadrupole operators.

The combination of $V_{\pi\pi} + V_{\nu\nu}$ implies the existence of d-bosons that preserve the residual interactions between n-n as well as p-p. Therefore, Eq. 9 may be expressed as:

$$\hat{V}_{\rho\rho} = \sum_{k=1,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C^\rho_L \left[\left(d^\dagger_\rho \times d^\dagger_\rho\right)^{(L)} \cdot \left(\tilde{d}_\rho \times \tilde{d}_\rho\right)^{(L)}\right]^{(0)}, \quad \rho = \pi, \nu, \tag{9}$$

The last term, Majorana interactions ($M_{\pi\nu}$) in Eq. 7, can be written as:

$$\begin{aligned} M_{\pi\nu} = {} & \xi_2 \left(s^\dagger_\nu \times \tilde{d}_\pi - \tilde{d}_\nu \times s^\dagger_\pi\right)^{(2)} \cdot \left(s_\nu \times d^\dagger_\pi - d^\dagger_\nu \times s_\pi\right)^{(2)} \\ & - 2 \sum_{k=1,3} \xi_k \left(d^\dagger_\nu \times d^\dagger_\pi\right)^{(k)} \cdot \left(\tilde{d}_\nu \times \tilde{d}_\pi\right)^{(k)}. \end{aligned} \tag{10}$$

The position of these states in the spectrum could be determined due to the sufficient variation of the Majorana parameters. The energy levels are determined using NPBOS code [28]. We perform diagonalization of the Hamiltonian described in Eq. 7 and reach an agreement on the values of parameters ε, κ, χ_π, χ_ν, and C_L that yield the most accurate and satisfactory measured data.

3. Consequences and argument

3.1. $R_{4/2}$ categories

Based on the ratio $R_{4/2}$, even-even collective energies of nuclei are divided into three groups:

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)} \tag{11}$$

The energy level at 4_1^+ is denoted as $E(4_1^+)$, while the energy level at 2_1^+ is denoted as $E(2_1^+)$ [29, 30]. The value $R \approx 3.33$ represents the empirical data for deformed nuclei SU(3), $R \approx 2.50$ corresponds to γ-unstable nuclei O(6), and $R \approx 2.00$ is observed in vibrational nuclei U(5) [1]. The SU(3)-O(6) transition symmetry was identified in the ^{186}Hg nucleus based on the $R_{4/2}$ values of 2.66 [29, 30] as shown in Table 1. Figure 1 displays the ratio of $E(4_1^+)/E(2_1^+)$ values for the ^{186}Hg nucleus. The solid line represents the presence of U(5), O(6), and SU(3) symmetry [1]. The data ratios of $R_{4/2}$ for this nucleus, which were observed within the boundaries defined by the solid lines SU(3) and O(6) [1, 29, 30], are displayed in Figure 1.

Nucleus	^{186}Hg [29, 30]
$R_{4/2}$	2.66

TABLE 1. *$R_{4/2}$ ratio for ^{186}Hg nucleus*

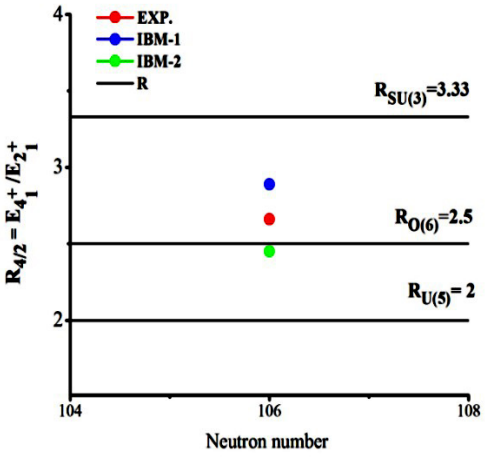

FIGURE 1. *The $R_{4/2}$ ratio for ^{186}Hg nucleus. Black soild line indicate* U(5), O(6), *and* SU(3) *limits*

3.2. Number of bosons

The boson number is determined by the total number of paired valence nucleons. The valence nucleons are determined through the calculation of double-magic nuclei. The present study involves the calculation of valence nucleons derived from the double magic nucleus ^{208}Pb (Z = 82, N = 126). The total number of protons N_π and neutrons N_ν is equal to $(N_\pi + N_\nu)$ bosons. This study used ^{208}Pb as an inert core to determine the number of bosons within ^{186}Hg nucleus. The ^{186}Hg nucleus, which sits close to the shell closure of ^{208}Pb, has 2 vacancies resulting from a deficiency of valence protons in the shell closure with a magic number Z = 82. Additionally, there are 20 vacancies due to a shortage of valence neutrons in the shell closure with a magic number N = 126. The total number of bosons in ^{186}Hg is calculated as (2/2+20/2) = 11, as shown in Table 2.

Nucleus	$N_\pi + N_\nu = N$	ε	a_0	a_1	a_2	a_3	a_4	CHQ (χ)
^{186}Hg	1 + 10 = 11	0.000	0.0435	0.0042	0.000	0.0106	0.000	0.000

TABLE 2. *The parameters used for IBM-1 calculations all parameters in MeV*

3.3. Energy levels in ^{186}Hg

We have provided a detailed analysis of the excitation spectra for the g states, γ, and β bands of ^{186}Hg isotopes. The energy levels of the g-band, γ-band, and β-band of ^{186}Hg are calculated for both models of IBM in SU(3)-O(6) transition limits. The energy ratio $R = E4_1^+/E2_1^+$ represents the degree of symmetry exhibited by a nucleus. The patterns $E4_1^+$ and $E2_1^+$ represent the recorded energy levels in ^{186}Hg [30] at 4_1^+ (1.080 MeV) and 2_1^+ (0.405 MeV), respectively.

The parameters used for IBM-1 and IBM-2 calculations of the ^{186}Hg nucleus are presented in Table 2 and Table 3, respectively. In Table 2, all parameters are given in MeV, with the exception of N and CHQ(x) for IBM-1. In Table 3, all parameters are given in MeV for IBM-2, with the exception of N. The comparative study of the ground state (g), γ, and β-bands of the ^{186}Hg nucleus are presented in Table 4. The calculated data from IBM-1 and IBM-2 are consistent with established experimental results. The deviation of data in IBM-1 is more favorable compared to IBM-2 in the β band for the ^{186}Hg nucleus. Furthermore, our calculations were compared with previous studies such as i. an interacting boson model with configuration mixing (IBM-CM) [31], ii. a beyond mean-field model (BMF)[32] and iii. the General Bohr Hamiltonian (GBH) [33] and presented in Table 4. From this comparison, the present results of IBM-1 are better than those of previous studies in the ground states and β-band.

Nucl.	$N_\pi + N_\nu = N$	ε_d	$\kappa_{\pi\upsilon}$	χ_π	χ_υ	ξ_2	C_0L_π 2 4	C_0L_υ 2 4
^{186}HG	1 + 10 = 11	0.570	– 0.160	-0.080	0.200	0.010	0.000 0.000 0.000	0.250 -0.070 0.110

TABLE 3. *The parameters used for IBM-2 calculations. All parameters in MeV*

^{186}HG						
g-band						
J^π	IBM-1	IBM-2	EXP	IBM-CM	BMF	GBH
0_1^+	0.000	0.000	0.000	0.00	0.00	0.00
2_1^+	0.403	0.370	0.405	0.404	0.404	0.280
4_1^+	1.168	0.908	1.080	0.799	0.766	0.700
6_1^+	2.293	1.596	1.677	1.137	1.335	1.285
8_1^+	3.780	2.424	1.588	1.549	2.068	1.978
10_1^+	5.6290	2.692	2.077	2.035	2.983	2.810
γ-band						
2_2^+	0.721	0.762	0.621	–––	–––	–––
3_1^+	1.336	1.270	1.433*	–––	–––	–––
4_2^+	1.592	1.358	1.228*	–––	–––	–––
5_1^+	2.441	1.958	1.868*	–––	–––	–––
6_2^+	2.814	2.093	–––	–––	–––	–––
β-band						
0_2^+	0.520	1.130	0.522	0.519	0.280	0.725
2_3^+	0.924	1.474	1.096	0.634	0.684	0.824
4_3^+	1.688	1.850	1.966	1.137	1.401	1.392

TABLE 4. *Comparative studies of three types band in ^{186}Hg nucleus by IBM-1, IBM-2, and previous exp. [29, 30], IBM-CM [31], BMF [32], and GBH [33]. (*) represents unknown spin and angular momentum. All energy levels in MeV*

3.4. Electric Reduced Transition Probabilities B(E2)

The energy levels of excited states in even-even nuclei ($L_i^+ = 2_1^+, 4_1^+, 6_1^+, 8_1^+ ...$) typically decay to the lower-lying state with an angular momentum difference of $L_f^+ = L_i^+ - 2$ through the E2 transition. An essential parameter for

comprehending the decay characteristics of a nucleus is the investigation of the strength of the reduced transition probability $B(\mathrm{E2})$. Currently, the $B(\mathrm{E2})$ data are computed using the computer code PHINT [26], which necessitates the input of effective charge values (e_B) obtained in Eq. 12.

$$B(\mathrm{E2}: 2_1^+ \rightarrow 0_1^+) = \frac{\alpha_2^2}{5} N(N+4) = \frac{e_B^2}{5} N(N+4) \tag{12}$$

To achieve reduced transition probability values, we have precisely correlated the calculated absolute strengths $B(\mathrm{E2})$ of transitions within the ground state band to experimental values. The Eq. 12 was used to normalize the experimental data $B(\mathrm{E2}; 2_1^+ \rightarrow 0_1^+)$ for each isotope and determine the value of the effective charge (α_2) in IBM-I. The parameter α_2 for each isotope has been computed based on the provided measured value of transitions $2_1^+ \rightarrow 0_1^+$, and subsequently utilized to determine the $B(\mathrm{E2})$ values for these transitions $4_1^+ \rightarrow 2_1^+$, $6_1^+ \rightarrow 4_1^+$, and $8_1^+ \rightarrow 6_1^+$.

The model wave functions were obtained through the process of diagonalizing the IBM-2 Hamiltonian. To estimate the electromagnetic transition, the program NPBEM [28] was utilized.

The E2 transition operator is given as [34]:

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu, \tag{13}$$

hence, the effective charges of bosons designated as e_π and e_ν, which depend on the boson number, can be estimated through the fitting of $B(\mathrm{E2}: 2_1^+ \rightarrow 0_1^+)$ to the measured data. The effective charges e_π and e_ν are in unit e_b. The values of e_π and e_ν used in the present work are 0.167 and 0.184 eb, respectively. The $B(\mathrm{E2})$ values in unit e^2b^2 and previous experimental data are presented in Table 5. The measured data and the $B(\mathrm{E2})$ data of IBM-1 and IBM-2 are comparable. Otherwise, present calculations are compared with previous studies [31–33]. This comparison shows that the calculated electric reduced transition probabilities by IBM-2 are better than those of previous studies.

3.5. Potential Energy Surface (PES)

The symmetrical forms and microscopic structure of atomic nuclei can be identified using the potential energy surface (PES). These PES plots were generated by the Skyrme mean field method with the IBM Hamiltonian [35–38]. The IBM-1 energy surface is formed by merging the expectation value of the IBM-1 Hamiltonian (Eq. 1) with the coherent state ($|N, \beta, \gamma\rangle$) [1]. The coherent state is generated by applying the creation operators (b_c^+) to the boson vacuum state in the following manner [39]:

$$|N, \beta, \gamma\rangle = \frac{1}{\sqrt{N!}} (b_c^\dagger)^N |0\rangle, \tag{14}$$

^{186}Hg Isotope	B(E2)↓ in e^2b^2					
$J_i \to J_f$	Exp.	IBM-1	IBM-2	IBM-CM	BMF	GBH
$2_1^+ \to 0_1^+$	0.449 ± 0.082	0.4440	0.5012	0.28	0.29	0.97
$4_1^+ \to 2_1^+$	1.26 ± 0.07	0.6151	0.7755	0.40	2.00	1.68
$6_1^+ \to 4_1^+$	1.45 ± 0.069	0.6863	0.9587	1.84	2.37	2.10
$8_1^+ \to 6_1^+$	1.263 ± 0.088	0.7046	1.0737	1.97	2.62	2.42
$10_1^+ \to 8_1^+$	1.5 ± 0.059	0.6880	1.1603	1.99	2.88	2.70
$2_2^+ \to 0_1^+$	$0.5 \times 10^{-3} \pm 2 \times 10^{-3}$	0.0617	0.0915	0.004	0.23	0.03
$2_2^+ \to 0_2^+$	≥ 1.7	0.0093	0.1910	0.97	0.22	0.21
$4_1^+ \to 2_2^+$	$3.1 + 1.5 - 0.6$	0.0214	0.8620	1.22	0.04	0.02

TABLE 5. *Comparative studies of reduced transition probability B(*E*2)↓ in e^2b^2 of the ^{186}Hg nucleus, including data from previous experiments [29, 30] IBM-CM [31], BMF [32], GBH [33] and the present IBM-1 and IBM-2 models*

were

$$b_c^\dagger = \frac{1}{\sqrt{1+\beta^2}} \left(s^\dagger + \beta \left[\cos\gamma(d_0^\dagger) + \sqrt{\frac{1}{2}} \sin\gamma(d_2^\dagger + d_{-2}^\dagger) \right] \right), \tag{15}$$

thus, EPS could be stated as a function of β and γ as

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2} \left[\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4 \right] \tag{16}$$

The parameters of α_1, α_2, α_3, and α_4 are linked to the coefficients of C_L, v_2, v_0, and u_0. The β factor denotes the overall deformation of a nucleus. The shape of a nucleus can vary between a sphere and a distorted shape, depending on whether the value of β is equal to zero or not. In addition, the difference in symmetry of the nucleus is defined by the γ-parameter. A nucleus is considered prolate when γ is equal to 0, and oblate when γ is equal to 60°. Figure 2 displays the contour plot of the PES for ^{186}Hg isotope. The color box reflects the PES values in MeV units.

4. Conclusions

The IBM-1 and IBM-2 models were used to calculate the g-band, γ-band, and β-band, as well as the reduced transition strength B(E2) and potential energy surface (PES) of the ^{186}Hg nucleus. The determined results for the energy states of this nucleus match with the previously experimental data. The

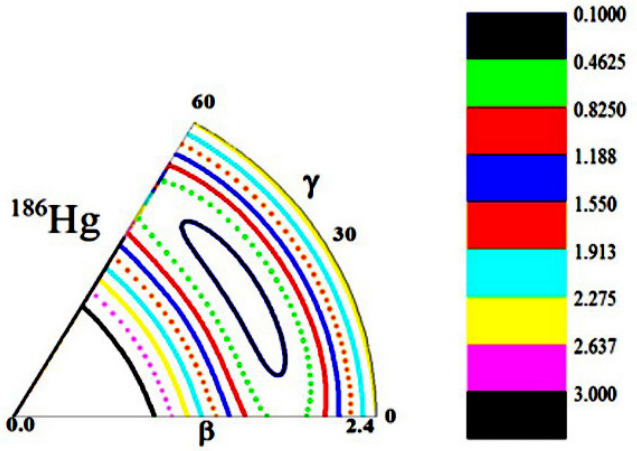

FIGURE 2. *The PES contour plot for ^{186}Hg nucleus*

B(E2) reduced transition probabilities obtained from the IBM-1 and IBM-2 calculations are consistent with the available experimental data. The potential energy surfaces of ^{186}Hg nucleus are discussed in terms of IBM-1. All the calculations by IBM-1 and IBM-2 under discussion of ^{186}Hg nucleus have a transition limit of SU(3)-O(6).

Acknowledgments

We acknowledge our thanks to the University of Alkafeel and University of Mosul for their invaluable assistance throughout this research. The authors express their gratitude to Prof. Dr. Hayder Al Mousawy, Ph.D., in English for editing the English language of this work.

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