ENTANGLEMENT IN AN OPTIMAL QUANTUM MEMORY UNIT

ENTRELAZAMIENTO EN UNA UNIDAD DE MEMORIA CUÁNTICA ÓPTIMA

Fabián Soberanes Martín[1*], Manuel Ávila Aoki[2]. Giselle N. Morales Rosales[1]

[1] TecNM: Tecnológico de Estudios Superiores de Chalco La Candelaria Tlapala, C.P. 56641, Chalco, Edo. Méx., México.
[2] Centro Universitario UAEM Valle de Chalco, María Isabel, 56615 Valle de Chalco, Edo. Méx., México.

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Abstract

It is modeled a quantum memory unit as a two qubits interacting with an external bath. The transfer of information from one qubit into the other can be optimally achieved if there exist a good entanglement between them. The two qubits are allocated in a spatial separation d. It is proposed a phenomenological ansatz where their entanglement in presence of noise depends on d. The above opens new possibilities for further operative technologies of quantum memories.

Keywords: entanglement, qubit, quantum memory, decoherence.

Resumen

Se modela una unidad de memoria cuántica como dos qubits interactuando con un baño externo. La transferencia de información de un qubit a otro puede lograrse de manera óptima si existe un buen entrelazamiento entre ellos. Los dos qubits están ubicados a una separación espacial d. Se propone un ansatz fenomenológico en el que su entrelazamiento en presencia de ruido depende de d. Lo anterior abre nuevas posibilidades para futuras tecnologías operativas de memorias cuánticas.

* fabian_sm@tesch.edu.mx
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Palabras clave: entrelazamiento, qubit, memoria cuántica, decoherencia.

Introduction

Entanglement is responsible for the power of quantum computers [1]. Paradoxically, it is also the source of one of their major obstacles. The reason for this is quantum decoherence, a process by which the richness of information contained in an entangled quantum state is gradually lost [2–5]. For instance, the two-qubit state, also called the Bell state, is given by

$$| \varphi\rangle = \frac{1}{\sqrt{2}}(| 01\rangle + | 10\rangle) \tag{1}$$

is maximally entangled with a value of entanglement 1 [6]. In absence of decoherence, the Bell state of Eq. (2) executes efficiently quantum gates [2]. In the presence of decoherence such a state loses its power for performing efficiently quantum tasks. The irruption of the unwanted decoherence arises from the unavoidable interaction of a quantum system with its environment. Understanding decoherence is essential for measurement, quantum information processing, and, more fundamentally, for the study of the transition from the quantum to the classical world [7]. When the advantages of quantum information processing are lost, the quantum computer loses its full power.

Although decoherence is a serious obstacle to efficient quantum information processing, it is also necessary to enable the measurement of a system, as demonstrated by Zurek (1982). Certainly, without measurements the technological possibilities of Quantum Mechanics would be lost. That is the case of a quantum memory [8, 9]. However, an uncontrolled noise would be pernicious for the development of an efficient quantum memory.

As mentioned above, decoherence arises as a consequence of the unavoidable system–environment interaction. Thus, to study decoherence in a correct way requires a good modeling of the quantum environment. This has led to simple models of quantum environments. Among others are baths of harmonic oscillators [10, 11] or the simplest model of a system-environment interaction, consisting of a central qubit coupled to a bath of spin-1 particles [10]. Other approaches to modeling the environment consist of a reservoir of quantum oscillators with infinite degrees of freedom [12]. For readers interested in quantum noise they may refer to a detailed review done by Schlosshauer (2019).

In what follows, we consider a two-qubit quantum memory in the presence of external noise. It is worth noting that entanglement between the two qubits of the quantum memory is a key ingredient for an optimal exchange of signals. Consequently, noise is an obstacle for a good operation of the quantum memory. On the other hand, we may note that if the two qubits are very closed together they feel the same noise. By assuming that the two qubits of the quantum memory are each other separated by a finite distance d, it is calculated the respective entanglement as a function of d all of this in presence of noise. In order to achieve this, a phenomenological ansatz is introduced to model the strength of the interaction between such qubits. It is argued on physical grounds for the phenomenological ansatz. The present paper is organized as follows: in Section 1, the model is introduced, while in Section 2, we calculate the two-qubit entanglement as a function of d. We conclude by offering some remarks.

Thermal bath model

Depending on different coupling mechanisms between the environment and quantum systems in different models, the environment has a dual nature that influences the entanglement between the qubits. Within our approach, the environment considered here is a reservoir of quantum oscillators with infinite degrees of freedom [12]. We model a quantum memory as a two qubits with mutual dipole-dipole interaction. We shall assume that the system–environment interaction is very weak and that the characteristic correlation time of the environment is very short compared to the time scale of system evolution. With such an approximation the master equation governing the state of the quantum memory is

$$\frac{d\rho(t)}{dt} = -i[H_I, \rho(t)] + \frac{\Delta}{2}\sum_{i,j=1}^{2}(2\sigma_-^i \rho(t)\sigma_+^j - \sigma_+^j \sigma_-^i \rho(t) - \rho(t)\sigma_+^j \sigma_-^i), \tag{2}$$

where Δ is the coupling constant of the system-environment interaction and the qubit-qubit interaction Hamiltonian is

$$H_I = \mathrm{g}(J_+ J_- - J_z - 1), \tag{3}$$

being g the strength (intensity) of the coupling qubit-qubit and

$$J_\pm = \sum_{a=1}^{2} \sigma_\pm^a, \qquad J_z = \sum_{a=1}^{2} \frac{\sigma_z^a}{2} \tag{4}$$

whit $\sigma_\pm^a = (\sigma_x^a \pm \sigma_y^a), \sigma_x = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right), \sigma_y = \left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right)$, and $\sigma_z = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$. It is assumed that the two qubits of the quantum memory are separated by a distance d. Inspired by Yukawa-like interactions [13], a phenomenological ansatz is proposed in which

$$\mathrm{g} = \mathrm{g}_0 e^{-d}, \tag{5}$$

where g_0 is the strength of the coupling in the limit of null separation that is

$$\lim_{d \to 0} \mathrm{g} = \mathrm{g}_0. \tag{6}$$

It can be observed from Eq. 5 that if the qubits are separated by a large distance, the qubit–qubit interaction vanishes, that is,

$$\lim_{d \to \infty} \mathrm{g} = 0. \tag{7}$$

If it is employed the computational basis $\{|\,0\rangle, |\,1\rangle\}$ for each of the two qubits of the quantum memory then the Hilbert space for a two-qubit state is $\{|\,00\rangle, |\,01\rangle, |\,10\rangle, |\,11\rangle\} \equiv \{|\,0\rangle, |\,1\rangle, |\,2\rangle, |\,3\rangle\}$

where we have used the binary-decimal number notation. In order to solve Eq. 1 we need to fix the initial conditions. By considering the two-qubit state as a two atoms of spin-1/2 initially in a triplet state, the initial state can be written as

$$\mid \varphi(t=0)\rangle = \frac{1}{\sqrt{2}}(\mid 1\rangle + \mid 2\rangle) \tag{8}$$

with which the initial density matrix should be

$$\rho(t=0) = \mid \varphi(t=0)\rangle\langle\varphi(t=0) \mid = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

By employing the quantum jump approach for solving Eqs. (2) and (6) it is obtained [2]

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{e^{-igt-\Delta t}}{2} & 0 \\ 0 & \frac{e^{-igt-\Delta t}}{2} & \frac{e^{-2\Delta t}}{2} & 0 \\ 0 & 0 & 0 & \frac{1-e^{-2\Delta t}}{2} \end{pmatrix}. \tag{10}$$

The above is describing the state of the two qubits of the quantum memory after an elapsed time t. Now we proceed to calculate the entanglement of the two qubits.

Results

The entanglement of formation of the two qubits is given by the concurrence [14], which is given by

$$\mathtt{C} = \{0, \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4\}, \tag{11}$$

where $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ are the eigenvalues of the matrix $[\rho^{\frac{1}{2}}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho^{\frac{1}{2}}]^{\frac{1}{2}}$. The concurrence varies from $\mathtt{C} = 0$ for a disentangled state to $\mathtt{C} = 1$ for a maximally entangled state. Through the use of Eq. 10 the concurrence of the two qubits is

$$\mathtt{C}(t) = \frac{1}{2}\sqrt{(e^{-2\Delta t} - 1)^2 + 4e^{-2\Delta t}(\sin 2\mathrm{g}t)}, \tag{12}$$

where g is given by Eq. 5.

Due that $-1 \leq \sin 2\mathrm{g}t \leq 1$ then the concurrence of Eq. 12 satisfies that

$$(1 - e^{-2\Delta t})/2 \leq \mathsf{C}(t) \leq (1 + e^{-2\Delta t})/2. \tag{13}$$

From the above equation, we can see that for a very large lapses of time $t \to \infty$ the respective stationary (s) concurrence satisfies the bounds

$$\mathsf{C}_s(t \to \infty) = \frac{1}{2}. \tag{14}$$

Likewise from Eq. 13 we obtain that in the limit of very penetrating noise (vpn) $\Delta \to \infty$ the respective bounds for the concurrence are

$$\mathsf{C}_{vpn}(\Delta \to \infty) = \frac{1}{2}. \tag{15}$$

According to Eq. 13, the concurrence reaches its minimum values C_{min} when $\sin 2\mathrm{g}t = -1$, that is, for $2\mathrm{g}t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$. Considering that g depends on the spatial separation d between the qubits as described by Eq. 5, the following condition is obtained:

$$\mathsf{C} = \mathsf{C}_{min} \;\; for \;\; 2\mathrm{g}_0 e^{-d} t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots \tag{16}$$

Similarly, maximum concurrence values C_{max} are obtained for $\sin 2\mathrm{g}t = 1$ that is for $2\mathrm{g}t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ or using Eq. 5 it is obtained that

$$\mathsf{C} = \mathsf{C}_{max} \;\; for \;\; 2\mathrm{g}_0 e^{-d} t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots \tag{17}$$

Conclusion

From Eqs. 16 and 17, it can be observed that the times at which the entanglement of the two qubits in the quantum memory reaches its extreme values depend on the spatial distance between them. This result opens new and promising technological perspectives. In order for the information signals to flow effectively from one qubit to the other, a sufficient amount of entanglement between the two qubits is required. Therefore, it is possible to experimentally position the qubits at a specific spatial separation d such that a maximal degree of entanglement can be achieved.

Recent experimental and theoretical investigations continue to explore optimal conditions for stable quantum memories and long-lived entanglement transfer [7–9]. The pursuit of scalable and technologically feasible quantum memory units remains an active line of research. The present study contributes to this ongoing effort, highlighting the importance of spatial configuration and decoherence control for future implementations of quantum memory systems.

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