SURVIVAL OF *Bacillus cereus* LSPQ 2872 AND *Escherichia coli* O157:H7 UNDER GAMMA RADIATION MODELED WITH TSALLIS ENTROPY

SUPERVIVENCIA DE *Bacillus cereus* LSPQ 2872 Y *Escherichia coli* O157:H7 BAJO RADIACIÓN GAMMA MODELADA CON ENTROPÍA DE TSALLIS

Arnold J. Quispe[1,2*], Mayomi Mestas[3], Danny G. Apaza[1,4,5]

[1] Departamento Académico de Física, Universidad Nacional de San Agustín de Arequipa, Arequipa, Perú.
[2] Facultad de Ingeniería y Arquitectura, Universidad Peruana Unión, Juliaca, Perú.
[3] Facultad de Ingeniería de Procesos, Universidad Nacional de San Antonio Abad del Cusco, Cusco, Perú.
[4] Departamento de Radioterapia, Instituto Regional de Enfermedades Neoplásicas del Sur, Arequipa, Perú.
[5] Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto, Departamento de Física, Universidade de São Paulo, Ribeirão Preto, Brasil.

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Abstract

The objective was to model the survival of *Bacillus cereus* LSPQ 2872 and *Escherichia coli* O157:H7 exposed to gamma radiation, using Tsallis entropy and the Monte Carlo simulation method. Monte Carlo simulations were performed with a noise level of 0.05 to assess the sensitivity of the Tsallis entropy-based models (dose-proportional effect, linear-quadratic dose-effect relationship, and Sotolongo *et al.*). Random values were generated for the parameters (α, β, γ, D_0, and D_{10}) of the gamma irradiation effect. The dose-proportional effect, linear-quadratic dose-effect relationship, and Sotolongo *et al.* models showed a good fit to the survival data of *Bacillus cereus* and *Escherichia coli* exposed to radiation. For *Bacillus cereus*, the dose-proportional effect and linear-quadratic dose-effect models showed similar lethal doses (0.935 and 0.844 kGy), while the Sotolongo *et al.* model showed a lower lethal dose (0.406 kGy), indicating greater radiation

* arquispequ@unsa.edu.pe doi: https://doi.org./10.15446/mo.n72.119468

efficacy. In *Escherichia coli*, the dose-proportional effect and linear-quadratic dose-effect models also showed similar lethal doses (0.716 and 0.745 kGy), and the Sotolongo *et al.* model showed a lower lethal dose (0.319 kGy), indicating greater efficacy. Tsallis entropy-based models are suitable for describing membrane behavior and biochemical changes in bacteria exposed to gamma radiation.

Keywords: pathogen bacteria, survival fraction curve, Tsallis entropy, gamma radiation.

Resumen

El objetivo fue modelar la supervivencia de *Bacillus cereus* LSPQ 2872 y *Escherichia coli* O157:H7 expuestas a radiación gamma, empleando la entropía de Tsallis y el método de simulación Monte Carlo. Se realizaron simulaciones de Monte Carlo con un nivel de ruido de 0,05 para evaluar la sensibilidad de los modelos basados en la entropía de Tsallis (efecto proporcional a la dosis, relación efecto-dosis de tipo lineal-cuadrático y Sotolongo *et al.*). Se generaron valores aleatorios para los parámetros (α, β, γ, D_0 y D_{10}) del efecto de la irradiación gamma. Los modelos de efecto proporcional a la dosis, relación efecto-dosis de tipo lineal-cuadrático y de Sotolongo *et al.* mostraron un buen ajuste a los datos de supervivencia de *Bacillus cereus* y *Escherichia coli* expuestos a radiación. Para *Bacillus cereus*, los modelos de efecto proporcional a la dosis y de relación dosis-efecto lineal-cuadrático mostraron dosis letales similares (0,935 y 0,844 kGy), mientras que el modelo de Sotolongo *et al.* mostró una dosis letal menor (0,406 kGy), lo que indica una mayor eficacia de la radiación. En *Escherichia coli*, los modelos de efecto proporcional a la dosis y de relación dosis-efecto lineal-cuadrático mostraron dosis letales también similares (0,716 y 0,745 kGy), y el modelo de Sotolongo *et al.* refleja una dosis letal menor (0,319 kGy), lo que indica una mayor eficacia. Los modelos basados en la entropía de Tsallis son adecuados para describir el comportamiento de la membrana y los cambios bioquímicos en bacterias expuestas a la radiación gamma.

Palabras clave: bacterias patógenas, curva de fracción de supervivencia, entropía de Tsallis, radiación gamma.

Introduction

Pathogenic bacteria, such as *Staphylococcus aureus*, *Bacillus cereus*, *Salmonella typhimurium*, and *Escherichia coli*, pose a risk to human health. These microorganisms are frequently found in food and various environments [1]. Contamination with these pathogens causes physiological disorders, hospitalization, and even death [2]. *Escherichia coli* O157:H7 is an enterohemorrhagic strain of the *Escherichia coli* bacterium that causes food poisoning due to the production of verotoxin [3–6]. Infection usually results in bloody diarrhea and, in some cases, kidney failure, mainly affecting young children and older adults [7].

Bacillus cereus LSPQ 2872 is a bacterium that produces toxins and heat-stable spores [8, 9]. This bacterium can be found in food, and its ingestion can cause food poisoning, manifesting in vomiting, diarrhea, and localized infections [10–12]. This pathogen constitutes a significant risk to human health [13].

Gamma radiation (Co-60) is an effective method for controlling pathogenic bacteria in food [4]. This irradiation breaks the DNA chains of microorganisms, generating their elimination or inhibition [10, 14]. The doses of gamma radiation required to eliminate pathogenic microorganisms present in food depend on the irradiation conditions and type of food [4, 15]. The dose to eliminate *Escherichia coli* O157:H7 is between 0.5 kGy and 5 kGy [2], and for *Bacillus cereus* LSPQ 2872 is between 4 kGy and 8 kGy [16, 17].

Computational and mathematical methods, such as Tsallis entropy, evaluate bacterial cell survival in response to gamma radiation (Co-60). This entropy, which generalizes the Boltzmann-Gibbs Entropy, describes complex biological phenomena with non-Gaussian distributions and nonlinear relationships, common in cellular responses to irradiation [18]. According to Tsallis (1988), this approach is useful for systems out

of thermal equilibrium, such as DNA damage and repair in bacteria exposed to radiation. Mathematical models do not replicate exact reality. However, they are important for understanding cellular mechanisms [19]. Construction of a model based on this entropy involves identifying key variables, such as radiation dose and the rate of cellular repair, and interrelating them through equations adjusted to experimental data. Additionally, this approach models cell survival curves more accurately by considering nonlinear interactions and bacterial heterogeneity [18].

The Monte Carlo method is a simulation technique that uses random numbers to solve complex problems, especially those involving uncertainty or variability. Through multiple simulations, it approximates numerical solutions to nondeterministic problems by calculating averages of the results [20]. This method simulates the variability of parameters such as α, β, γ, D_0, and D_{10}, generating simulations with probability distributions, allowing precise estimates of the effects and their confidence intervals for models based on Tsallis entropy such as the dose-proportional effect, the linear quadratic (LQ) model, and the one proposed by Sotolongo *et al.* [21].

Knowledge gap in the application of Tsallis entropy to model the nonlinear survival behavior of foodborne pathogens under gamma radiation. Existing models do not fully account for the complexity and variability of the responses of *Bacillus cereus* LSPQ 2872 and *Escherichia coli* O157:H7 [18]. This study addresses this limitation by integrating a non-extensive statistical framework with simulation techniques.

This work modeled the survival of *Bacillus cereus* LSPQ 2872 and *Escherichia coli* O157:H7 exposed to different doses of gamma radiation. These microorganisms represent complex biological systems whose response to ionizing radiation varies. Modeling these systems is challenging, requiring simplifications and assumptions to obtain a precise and manageable model [19, 22]. Tsallis entropy is ideal for modeling the survival of these pathogenic bacteria in response to irradiation, as it captures nonlinear and non-extensive behaviors in complex biological systems. Unlike Shannon or Boltzmann-Gibbs entropy, it describes systems where

the dose-effect relationship is not direct, as in DNA repair and bacterial resistance [23, 24].

Materials and Methods

Mathematical models based on Tsallis entropy

The system is described using a random variable, where the tissue effect $E(D)$ indicates the loss of bacterial vitality associated with a probability function $p(E)$ that calculates the probability of cell death within a range of effects E and $E + dE$. The fraction of bacteria killed at a dose that generates effects E is the cumulative probability, given by equation (1):

$$P(E) = \int_{\Omega_{\min}}^{E} p(E)\, dE, \tag{1}$$

Where $\Omega_{\min}$ is the lower bound of the range of the random variable E, and $\Omega_E = (\Omega_{\min}, \Omega_{\max})$. The survival fraction will be denoted by σ and is expressed as:

$$\sigma(E) = 1 - \int_{\Omega_{\min}}^{E} p(E)\, dE = \int_{E}^{\Omega_{\max}} p(E)\, dE. \tag{2}$$

This survival fraction is not a probability density function, as it is not normalized; it simply indicates the probability of survival in response to a specific tissue effect. To validate this approach, a classical approximation using Boltzmann-Gibbs entropy is used to recover the linear and quadratic formulations [25].

Classical Approximation

The classical approximation of bacterial survival begins with the Boltzmann-Gibbs entropy model:

$$S = -\int_{\Omega} p(E)\, \ln p(E)\, dE, \tag{3}$$

In this equation, the normalization conditions of the probability density function (PDF) are imposed, and the mean variable has a given value of $\langle E \rangle$.

$$\int_{\Omega} p(E)\, dE = 1, \tag{4}$$

$$\int_{\Omega} E\, p(E)\, dE = \langle E \rangle, \tag{5}$$

These conditions characterize the probability distribution of E through the following probability density function:

$$P(E) = \frac{1}{\langle E \rangle} e^{-\frac{E}{\langle E \rangle}}, \tag{6}$$

The probability that an irradiated bacterial cell dies after receiving a dose causing an effect in the interval $(E, E + dE)$ follows an exponential distribution. The survival fraction as a function of tissue effect will be calculated using equations (3) and (4), assuming $\Omega = (0, \infty)$.

$$\sigma(E) = \int_{E}^{\Omega_{\max}} \frac{1}{\langle E \rangle} e^{-\frac{E}{\langle E \rangle}}\, dE = e^{-\frac{E}{\langle E \rangle}}. \tag{7}$$

Dose-Proportional Effect Model

Assuming that the tissue effect is proportional to the absorbed dose, the dose causes first-order lesions in a single event, as occurs with high linear energy transfer (LET) radiations, being $E = \alpha D$ the proportionality constant. The PDF of the random variable depending on the dose is described according to equation 8.

$$\begin{aligned} p(E)dE = p(D)\frac{dD}{dE}dE = p(D)\left(\frac{dE}{dD}\right)^{-1} dE = \frac{1}{\alpha}p(D)dE \Rightarrow \\ p(D) = \frac{1}{\langle D \rangle} e^{-\frac{D}{\langle D \rangle}}, \end{aligned} \tag{8}$$

The mean of this random variable is $\langle D \rangle = 1/\alpha$, and by integrating, the survival fraction is obtained as follows:

$$\sigma(D) = \int_{D}^{\Omega_{\max}} \frac{1}{\langle D \rangle} e^{-\frac{D}{\langle D \rangle}}\, dD = e^{-\alpha D}, \tag{9}$$

Thus, the simple exponential model of cell survival is obtained under simple binding conditions and the assumption that the effect is proportional to the dose. By fitting the bacterial survival fraction $\log\left(\frac{N}{N_0}\right)$ with equation (9), the logarithm is applied, resulting in the following expression:

$$\log \sigma(D) = -\alpha D. \tag{10}$$

The parameter α represents the decay rate or the effect of the dose (D) on survival, being a positive value. Parameter D is the dose applied to bacteria cells [22].

Linear-Quadratic Dose-Effect Relationship Model

The linear-quadratic relationship is common between tissue effect and absorbed dose, as in the LQ model, $E = \tilde{\alpha}D + \tilde{\beta}D^2$. The parameter $\tilde{\alpha}$ represents the proportionality to first-order lesions, and $\tilde{\beta}$ represents second-order lesions. Substituting this relationship (LQ) into equation (7) and considering that $\langle E\rangle = \tilde{\alpha}\langle D\rangle + \tilde{\beta}\langle D\rangle^2$ is constant, we obtain the following equation:

$$\sigma(D) = e^{-\frac{\tilde{\alpha}D+\tilde{\beta}D^2}{\langle E\rangle}} = e^{-(\alpha D+\beta D^2)}, \tag{11}$$

With $\alpha = \frac{\tilde{\alpha}}{\langle E\rangle}$ and $\beta = \frac{\tilde{\beta}}{\langle E\rangle}$, this model is the linear-quadratic model. When fitting it to the bacterial survival fraction $\log(\frac{N}{N_0})$, equation (11) is transformed by applying the logarithm, resulting in the following expression:

$$\log \sigma(D) = -\alpha D - \beta D^2. \tag{12}$$

The parameter α represents the linear decay rate of survival for the dose (D), being a positive value. The parameter β represents the quadratic decay rate of survival for D, also a positive value [22].

Generalized Approximation

Sotolongo *et al.* propose a generalized approach using non-extensive statistical physics and applying Tsallis entropy to the maximum entropy principle. The Tsallis entropy for a continuous random variable is expressed as follows:

$$S_q(E) = \frac{1}{q-1}\left[1 - \int_\Omega p^q(E)\, dE\right], \tag{13}$$

The approach is generalized when $q = 1$; Tsallis entropy reduces to the Boltzmann-Gibbs entropy, recovering the classical approximation. Furthermore, Sotolongo *et al.* suggest that there is a maximum dose (D_0) beyond which the survival fraction is zero; this dose corresponds to a maximum tissue effect, $E_0 = E(D_0)$, related to $\Omega_{\max}$. Therefore, there is $\Omega = (0, E_0)$, with normalization conditions for the probability space, and specification of the mean value for the effect is given by the following equations:

$$\int_0^{E_0} p(E)\, dE = 1, \tag{14}$$

$$\int_0^{E_0} E\, p(E)\, dE = \langle E \rangle_q, \tag{15}$$

To maximize the entropy, the effect E_0 and the Lagrange multipliers must have the following values:

$$E_0 = \frac{2-q}{1-q}\left(\frac{\langle E\rangle_q}{2-q}\right)^{\frac{1}{2-q}}, \tag{16}$$

$$\lambda_0 = -\frac{q}{1-q}\left(\frac{\langle E\rangle_q}{2-q}\right)^{\frac{1-q}{2-q}}, \tag{17}$$

$$\lambda_1 = -\frac{q}{2-q}\left(\frac{\langle E\rangle_q}{2-q}\right)^{-\frac{1}{2-q}}, \tag{18}$$

The probability that a dying bacterial cell received a dose causing an effect $(E, E + dE)$ is given by:

$$p(E) = \left(\frac{2-q}{\langle E\rangle_q}\right)^{\frac{1}{2-q}} \left[1 - \frac{1-q}{2-q}\left(\frac{2-q}{\langle E\rangle_q}\right)^{\frac{1}{2-q}} E\right]^{\frac{1}{1-q}}, \tag{19}$$

Deriving the survival fraction from equations (2) and (19), the following equation is obtained:

$$\sigma(E) = \left[1 - \frac{1-q}{2-q}\left(\langle E\rangle_q\right)^{\frac{1}{2-q}} E\right]^{\frac{1}{1-q}}, \tag{20}$$

Taking into account the value of the effect E_0 in equation (16) and defining $\gamma = \frac{2-q}{1-q}$, it can be expressed as follows:

$$\sigma(E) = \begin{cases} \left(1 - \frac{E}{E_0}\right)^{\gamma} & \text{for } E \in \Omega \\ 0 & \text{for } E \notin \Omega \end{cases}, \tag{21}$$

In compact notation, we have:

$$\sigma(E) = \left(1 - \frac{E}{E_0}\right)^{\gamma} [H(E) - H(E - E_0)], \tag{22}$$

Given that $H(x - x_0)$ is a function associated with the Heaviside distribution or step function. When $q \to 1$, Tsallis entropy will converge to the Boltzmann-Gibbs entropy, and thus equation (7) is recovered. The general form of the survival curve for any value of parameter q is given by the expression:

$$\sigma(E) = \begin{cases} (1 - \frac{E}{E_0})^{\gamma}[H(E) - H(E - E_0)] & \text{if } q \neq 1 \Rightarrow \gamma < \infty \\ e^{-\frac{E}{\langle E\rangle_q}} & \text{if } q = 1 \Rightarrow \gamma \to \infty \end{cases}. \tag{23}$$

To complete the model, it is only necessary to assume a formal relationship between the tissue effect and the absorbed dose [22]. In this study, values $q = 1.6$ for *Bacillus cereus* LSPQ 2872 and $q = 1.5$ for *Escherichia coli* O157:H7 were used, generated from a Gaussian distribution $N(q_i, 0.1)$, where 0.1 corresponds to

the standard deviation. Since $q > 1$, non-extensive behavior is assumed according to the Tsallis statistic, which allows us to move away from the classical Boltzmann-Gibbs model and better fit the experimentally observed survival.

Sotolongo *et al.* Model

If the tissue effect is directly proportional to the dose, then the Sotolongo *et al.* model is obtained by taking $E = \alpha D$. It follows that $E_0 = \alpha D_0$, and equation (23) will take the form:

$$\sigma(D) = \begin{cases} \left(1 - \frac{D}{D_0}\right)^{\gamma} \left[H(D) - H(D - D_0)\right] & \text{if } \gamma < \infty \\ e^{-\frac{D}{\langle D \rangle}} & \text{if } \gamma \to \infty \end{cases}, \tag{24}$$

When $\gamma \to \infty$, the upper dose tends to infinity ($D_0 \to \infty$), and the convergence is absolute to the linear model. For $\gamma \to \infty$, that is, $\gamma \gg 1$, and at low absorbed doses, the previous expression can be approximated by a Taylor expansion:

$$-\ln \sigma(D) \approx \alpha_0 \left(\frac{2-q}{\langle E \rangle_q}\right)^{\frac{1}{2-q}} D + \frac{\alpha_0^2}{2} \frac{1-q}{2-q} \left(\frac{2-q}{\langle E \rangle_q}\right)^{\frac{1}{2-q}} D^2 + \vartheta(D^3), \tag{25}$$

This equation of Sotolongo *et al.* approximates the LQ model with parameters as follows:

$$\alpha_{\mathrm{LQ}} = \left(\frac{2-q}{\langle E \rangle_q}\right)^{\frac{1}{2-q}}, \tag{26}$$

$$\beta_{\mathrm{LQ}} = \frac{\alpha^2}{2} \frac{1-q}{2-q} \left(\frac{2-q}{\langle E \rangle_q}\right)^{\frac{1}{2-q}}. \tag{27}$$

The Sotolongo *et al.* model is a potential type, formally simple, and depends only on two parameters, D_0 and γ [22, 25].

Construction of mathematical models for lethal dose (D_{10})

The lethal dose (D_{10}) measures the dose necessary to kill 90% of the microbial population. D_{10} for the dose-proportional effect model can be expressed from equation (9), and its associated calculation can be presented as follows:

$$D_{10} = \frac{\log(10)}{\alpha}. \tag{28}$$

The D_{10} for the linear-quadratic dose-effect relationship model, applying the logarithm to both sides and substituting the value $\sigma(D_{10}) = 0.1$ in equation (11), we obtain:

$$\log \sigma(D) = -\alpha D - \beta D^2, \tag{29}$$

$$\log \sigma(D_{10}) = \log(0.1), \tag{30}$$

Since $\log(0.1) = -\log(10)$, the equation becomes:

$$-\alpha D_{10} - \beta D_{10}^2 = -\log(10), \tag{31}$$

The expression is simplified to consider only the linear term:

$$D_{10} = \frac{\log(10)}{\alpha}. \tag{32}$$

To obtain the D_{10} for the Sotolongo *et al.* model, substitute the value of $\sigma(D_{10}) = 0.1$ in equation (24), and we have:

$$\log(0.1) = \gamma \log\left(1 - \frac{D_{10}}{D_0}\right), \tag{33}$$

Solving, we have:

$$D_{10} = D_0\left(1 - e^{\left(\frac{\log(0.1)}{\gamma}\right)}\right). \tag{34}$$

Type and Design of the Research

This study is analytical, as it does not manipulate biological systems directly but rather analyzes and interprets existing data through mathematical modeling and simulation. The survival behavior of *Bacillus cereus* LSPQ 2872 [26] and *Escherichia coli* O157:H7 [5] under gamma radiation was examined by comparing different models based on Tsallis entropy. Parameters were estimated, hypotheses tested, and variability analyzed using Monte Carlo simulations to generate robust comparisons and conclusions.

Monte Carlo Method

The Monte Carlo method approximates solutions using random samples, making it useful for problems with probability distributions and event simulation [20]. This approach enables the estimation of the mean, standard deviation, confidence intervals, and the evaluation of uncertainty in predictions [21]. Mathematically, calculated as:

$$I = \frac{1}{N} \sum_{i=1}^{N} f(x_i). \tag{35}$$

where x_i represents random points from a known distribution and N is the number of simulations. In this study, 1000 Monte Carlo iterations were performed using a Gaussian ($\epsilon^{(i)} \sim N(0, \sigma^2)$) distribution for each parameter ($\alpha, \beta, \gamma, D_0,$ and D_{10}) with a noise level of 0.05, to assess the sensitivity and robustness of the Tsallis entropy-based models (dose-proportional effect, linear-quadratic dose-effect relationship, and Sotolongo *et al.*). Doses ranging from 0.00 to 2.50 kGy were simulated for *Bacillus cereus* LSPQ 2872 and from 0.00 to 1.00 kGy for *Escherichia coli* O157:H7. The simulation calculations were performed using MATLAB R2022a on a laptop with Windows 10 Pro 64-bit (AMD Ryzen 7, 2.60 GHz, 16 GB of RAM).

Data Analysis

Simulation data for the bacterial survival curve in each model (dose-proportional effect, linear-quadratic dose-effect relationship, and Sotolongo *et al.*) were analyzed using R^2 to evaluate model fit. Lethal dose (D_{10}) values were compared across models using Tukey's Honest Significant Difference (HSD) test with a significance level of 0.05, using the MATLAB R2022a program.

Results and Discussion

Figure 1a shows a good fit of the dose-proportional effect model with Tsallis entropy (Eq. 9) for the inactivation of *Bacillus cereus* LSPQ 2872 under gamma irradiation with an alpha coefficient (α) of 2.463 and an R^2 of 0.997, as shown in Table 1. The model indicates a high sensitivity of bacterial cells, where small irradiation doses are sufficient to reduce their viability. Furthermore, the R^2 indicates a good fit for the model.

Models	**Sensitivity Coefficients**				**R^2**
	Alpha (α)	**Beta (β)**	**Gamma (γ)**	**D_0 (Gy)**	
Dose-proportional effect model	2.463	-	-	-	0.997
Linear-quadratic dose-effect relationship model	2.728	0.129	-	-	0.999
Sotolongo et al. model	-	-	2281.322	927.154	0.997

TABLE 1. *Sensitivity coefficients of the models with Tsallis entropy in the survival fraction* $\log(\frac{N}{N_0})$ *of Bacillus cereus LSPQ 2872 exposed to gamma radiation (Co-60) and the coefficient of determination* R^2.

Figure 1b shows the fit of the linear-quadratic dose-effect relationship model with Tsallis entropy (Eq. 11) for the inactivation of *Bacillus cereus* LSPQ 2872 with an alpha coefficient (α) of 2.728 and a beta (β) of 0.129, as shown in Table 1. The model combines linear sensitivity and quadratic effects, showing rapid bacterial reduction at higher doses. The model presents the value of $R^2 = 0.999$, which indicates its accuracy and usefulness for optimizing irradiation treatments in food applications.

Figure 1c shows the fit of the Sotolongo *et al.* model with Tsallis entropy (Eq. 24) for the inactivation of *Bacillus cereus* LSPQ 2872 with a gamma coefficient (γ) of 2281.322 and a D_0 of 927.154 Gy, as shown in Table 1. The model indicates a complex relationship between dose and bacterial inactivation, including possible saturation effects. The high γ value indicates a high sensitivity of the bacteria, while the D_0 describes the dose required to reduce the bacterial population by 90%. Furthermore, the model presents an excellent fit ($R^2 = 0.997$) and is highly reliable for optimizing gamma irradiation in foods.

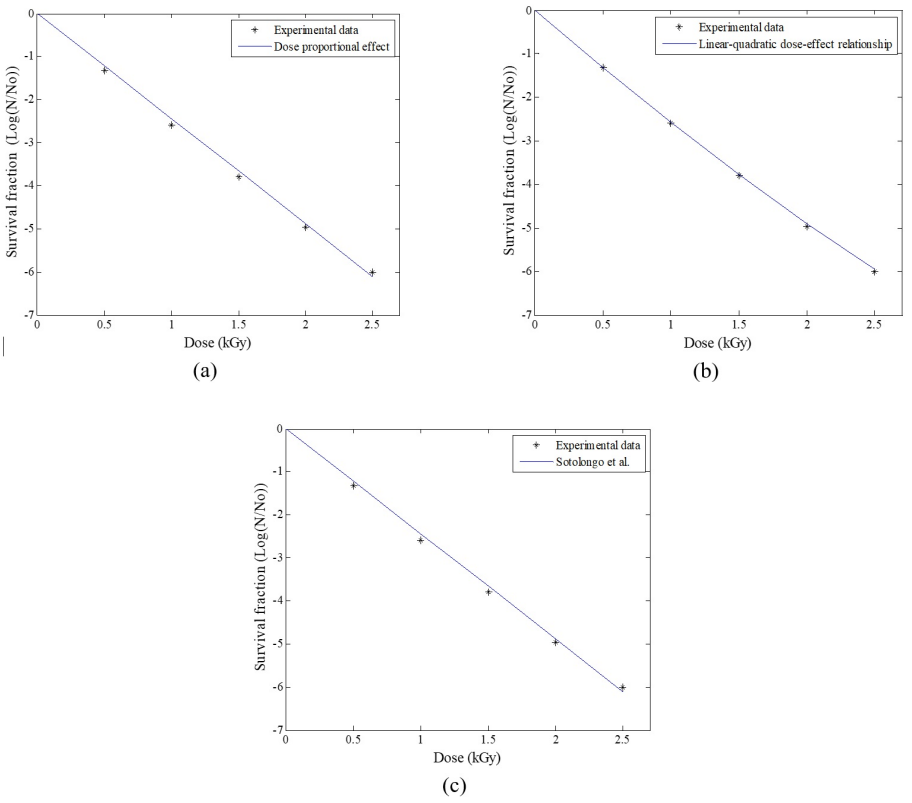

FIGURE 1. *Survival fraction (Log($\frac{N}{N_0}$)) of Bacillus cereus LSPQ 2872 exposed to gamma irradiation doses (Co-60) ranging from 0.00 to 2.50 kGy, fitted using dose-proportional effect (a), linear-quadratic dose-effect relationship (b), and Sotolongo et al. (c) models based on Tsallis entropy.*

Figure 2a shows the fit of the dose-proportional effect model with Tsallis entropy (Eq. 9) for the inactivation of *Escherichia coli* O157:H7 with an alpha coefficient (α) of 3.215, as shown in Table 2. The model indicates high bacterial sensitivity to irradiation, with the inactivation rate increasing exponentially with dose. The R^2 value of 0.994 shows an excellent model fit. This model effectively predicts bacterial inactivation, although more complex models may be needed to capture potential nonlinear effects.

Figure 2b shows the fit of the linear-quadratic dose-effect relationship model with Tsallis entropy (Eq. 11) for the inactivation of *Escherichia coli* O157:H7 with an alpha coefficient (α) of 3.103 and beta (β) of 0.137, as seen in Table 2. The alpha value indicates the high sensitivity of the bacteria, and the beta captures the non-linear effects, showing an accelerated increase in inactivation at higher doses. The R^2 value of 0.995 indicates that the model fits the data very well, similar to the dose-proportional effect model.

Models	**Sensitivity Coefficients**				**R^2**
	Alpha (α)	**Beta (β)**	**Gamma (γ)**	**D_0 (Gy)**	
Dose-proportional effect model	3.215	-	-	-	0.994
Linear-quadratic dose-effect relationship model	3.103	0.137	-	-	0.995
Sotolongo et al. model	-	-	172.165	53.981	0.995

TABLE 2. *Sensitivity coefficients of the models with Tsallis entropy in the survival fraction* $\log\left(\frac{N}{N_0}\right)$ *of Escherichia coli O157:H7 exposed to gamma radiation (Co-60) and the coefficient of determination* R^2.

Figure 2c shows the fit of the Sotolongo *et al.* model with Tsallis entropy (Eq. 24) for the inactivation of *Escherichia coli* O157:H7 with a gamma coefficient (γ) of 172.165 and a D_0 coefficient of 53.981 Gy, as shown in Table 2. The high value of γ indicates a nonlinear and highly sensitive relationship between irradiation dose and bacterial inactivation, characteristic of non-extensive entropy. In turn, the high value of D_0 suggests that a considerable dose is required to reduce the *Escherichia coli* population by 90%. The excellent fit of the model ($R^2 = 0.995$) reinforces its usefulness as a tool for optimizing irradiation doses.

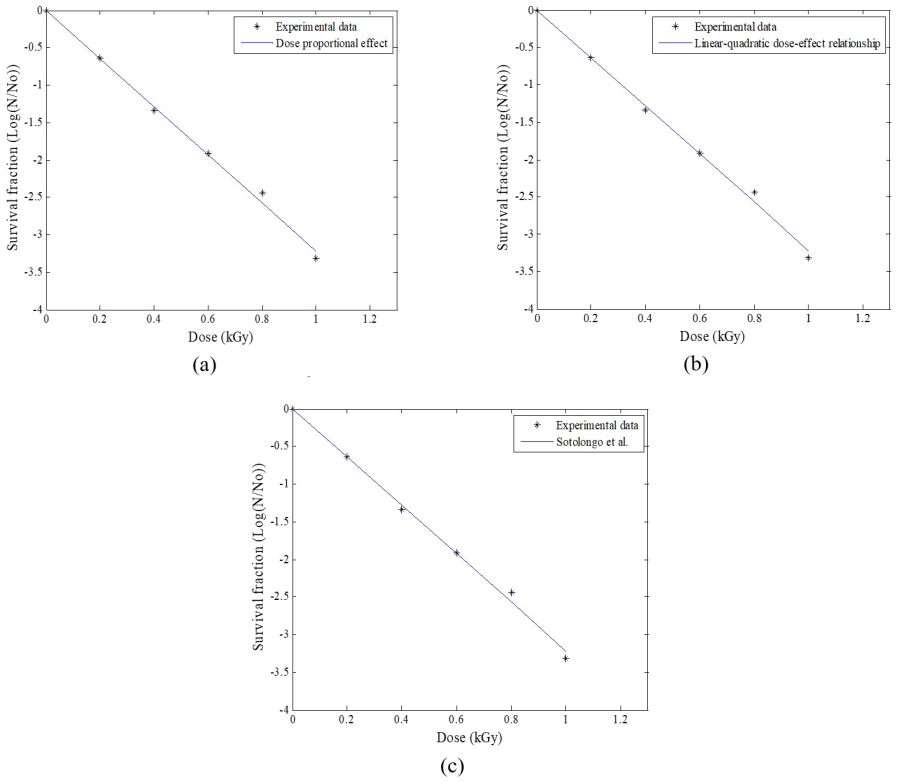

FIGURE 2. *Survival fraction (Log($\frac{N}{N_0}$)) of Escherichia coli O157:H7 exposed to gamma irradiation doses (Co-60) ranging from 0.00 to 1.00 kGy, fitted using dose-proportional effect (a), linear-quadratic dose-effect relationship (b), and Sotolongo et al. (c) models based on Tsallis entropy.*

Table 3 shows values of the lethal dose (D_{10}) for each model and bacteria evaluated, obtained through Monte Carlo simulation, and analyzed with Tukey's HSD test ($\alpha = 0.05$). For *Bacillus cereus* LSPQ 2872, the dose-proportional effect model and the linear-quadratic dose-effect relationship model present similar lethal doses (0.935 and 0.844 kGy, respectively), with no significant differences between them, while the Sotolongo *et al.* model predicts a lower lethal dose (0.406 kGy), significantly different from the other two models, indicating greater radiation efficacy. For *Escherichia coli* O157:H7, the dose-proportional effect and the linear-quadratic dose-effect relationship model present similar

lethal doses (0.716 kGy and 0.745 kGy, respectively); however, Sotolongo *et al.* predict a low dose (0.319 kGy), with a statistically significant difference from the other two models.

Models	**Lethal dose (D_{10}) (kGy)**	
	***Bacillus cereus* LSPQ 2872**	***Escherichia coli* O157:H7**
Dose-proportional effect	0.935^a	0.716^a
Linear-quadratic dose-effect relationship model	0.844^a	0.745^a
modelSotolongo et al. model	0.406^b	0.319^b

TABLE 3. *Lethal dose (D_{10}) values obtained by Monte Carlo simulations for the survival fraction of Bacillus cereus LSPQ 2872 and Escherichia coli O157:H7 exposed to gamma radiation (Co-60).*

The dose-proportional effect model (Eq. 9) fit the survival curve of bacteria well. This model is widely used in bacterial survival fraction studies [14, 22, 27]. However, it cannot capture nonlinear effects at high doses, so more complex models, such as the linear-quadratic dose-effect relationship or Sotolongo *et al.* models, are preferred [28, 29]. The linear-quadratic dose-effect relationship model (Eq. 11) showed a better fit in the survival curve of *Bacillus cereus* LSPQ 2872 and *Escherichia coli* O157:H7 against gamma radiation (Co-60), since this model is widely accepted to predict responses to ionizing radiation [22, 25]. However, despite its acceptance, doubts persist about its general applicability, although several studies have shown that the linear-quadratic dose-effect relationship model is complete enough to predict dose-time relationships [30, 31]. Moreover, the model of Sotolongo *et al.* (Eq. 24) showed a good fit in the bacterial survival curve, presenting a minimum annihilation dose (D_0). This model is appropriate for describing bacterial survival under radiation in various bacterial [22].

The lethal dose (D_{10}) results are comparable with those of Jo *et al.* (2004), who reported that the lethal dose (D_{10}) for *Bacillus cereus* was 0.663 kGy, and for *Escherichia coli* was 0.538 kGy, after exposure to different doses of gamma irradiation [2]. Similarly, Fernandes & Prakash (2020) determined D_{10} values between 0.303 and 0.370 kGy for *Escherichia coli* O157:H7 under gamma irradiation [5]. For their part, Ayari *et al.* (2012) reported

that the lethal dose for *Bacillus cereus* LSPQ 2872 was 0.420 kGy after applying different doses of gamma irradiation [26]. They also observed biochemical and structural changes in these irradiated bacteria, such as the generation of free radicals, damage to cell membranes, and the lethal dose, which affect bacterial viability.
Among the models, the linear-quadratic model best captures the non-linear effects of gamma irradiation, followed by the Sotolongo and dose-proportional models. Both account for complex bacterial responses and predict lower D_{10} values, indicating greater effectiveness at reducing the bacterial population by 90% [22, 29]. Although all three models show high coefficients of determination ($R^2 > 0.99$), the linear-quadratic model provides the best overall fit while maintaining biological interpretability through its α and β parameters. The Sotolongo model also shows a good fit but predicts significantly lower D_{10} values, which may reflect an overestimation of sensitivity. Thus, while both models capture non-linear behavior, their use depends on the balance between simplicity, interpretability, and predictive robustness.

Conclusions

The linear-quadratic dose-effect relationship and the Sotolongo *et al.* model showed a good fit to experimental data on survival fractions of *Bacillus cereus* and *Escherichia coli* irradiated with gamma radiation. However, the dose-proportional effect model, although a good fit ($R^2 > 0.99$), was less suitable for nonlinear responses. For *Bacillus cereus*, the proportional and linear-quadratic models yielded similar D_{10} (0.935 and 0.844 kGy), whereas the Sotolongo *et al.* model estimated a much lower value (0.406 kGy); in *Escherichia coli*, a similar trend was observed (0.716, 0.745, and 0.319 kGy, respectively). The incorporation of Tsallis entropy into the Sotolongo model captures complex nonlinear behaviors, offering an advantage over classical models and contributing to better estimation of microbiological risk. These findings have direct applications in the food industry, facilitating the design of more effective and safer irradiation treatments to ensure product safety.

Conflict of Interest

The authors state that they have no conflicts of interest.

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