

Bell's-type inequalities revisited: new constraints from objective reality

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Abstract

The empirical evidence tends to confirm the quantum – mechanically (QM) predicted violation of Bell's-type inequalities. The latter are widely accepted to be correct representation of the objective reality (OR) underlying EPR's gedankenexperiment. Contrariwise, we argue here that the starting premises leading to Bell's and CHSH's inequalities are not appropriate formulations of OR in the EPR context. Indeed, the essence of EPR argument is the *correlation* between pairs of particles travelling with some well defined orientations. To measure such correlation, one requires detectors with high resolution, and some appropriately defined relative orientations. On the contrary, experimental tests of Bell's inequalities typically involve counting the number of particles with some value of spin (or polarisation) using low-resolution detectors and completely arbitrary orientations. Depending upon the relative orientation of good-resolution detectors, we obtain two families of CHSH-type inequalities. One of these families is the most often used version of CHSH inequality, that was derived by Clauser and Horne with the supplementary no-enhancement assumption, here, it is obtained without special provisions. The other family is completely new and refers to coincidence counts from multiple events. Experimental tests to distinguish between the QM and OR descriptions of nature should take into account these new constraints.

Key words: Bell's inequalities, CHSH inequalities, EPR, (non) locality, objective reality, no-enhancement assumption, Kolmogorovian probability.

1 Introduction

To demonstrate the incompleteness of quantum mechanics (QM), Einstein, Podolsky, and Rosen (EPR) proposed a thought – experiment in 1935 [1], that led Bell [2], almost 30 years later, to formulate his theorem. a local reality model of the type propounded by EPR leads to an inequality that is inconsistent with QM predictions. Empirical evidence [3, 4] confirms QM stand. The derivation of the theorem is quite simple, thus leaving room for very few

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criticisms [5-8], satisfactorily answered [9, 10]. Suggestions for possible experimental loopholes range from scattering [11] and noise errors [12], passing through intrinsic detector inefficiencies [13], to far-fetched conspiracies between the detectors [14]. Other authors [15] argue that experimental tests refer to conditions stronger than Bell's original paper. The *apparent* inconsistency between EPR predictions and experiments induced a search for similar violations in other contexts [16-19] and have led to the conclusion that locality does not hold in QM. There are, however, some physicists of the opinion that locality/reality can be saved one way or another [20-25], as an example we mention the exchange about retro-causality in a leading journal [26].

The only aspect that, to our knowledge, has never been suspect is the empirical validity of Bell's and CHSH's starting premises. We argue here that both the detector resolution and the concept of independence leading to such inequalities are not appropriate to test objective reality (OR) models. As a consequence, both Bell's original inequality and CHSH's basic inequality are not acceptable representations of correlation in OR.

However, the standard version of CHSH's inequality that was derived by Clauser and Horne [27], CH hereafter, with the supplementary no-enhancement assumption can be derived from our OR model without any special provision, but with some constraints regarding the range of possible detector orientations. In addition, we derive a novel CHSH-type inequality for the balance of all other detector orientations. Both families of inequalities can be subject to empirical test by using experimental arrangements similar to the standard configurations [3, 4], but with high-resolution analysers/polarizers.

This paper is organized as follows. Section 2 describes an objective reality model, introduces the concept of detector resolution (new in the context of Bell's theorem) and applies it to the measurement of correlated particles. Sections 3 and 4 critically revisit the original derivations of Bell's inequality and CHSH's version of the theorem, respectively. Section 5 presents our derivation of CHSH-type inequalities, and a concluding section 6 closes the paper.

2 An objective reality model

2.1 The concept of detector resolution

Let us consider a source that emits spin-1/2 particles. The orientation of the spin is defined by $\lambda = (\theta, \phi)$ (see figure 1), whose probability density function (pdf) is $\rho(\lambda)$, so that the probability that the spin be oriented in the direction λ is

$$dP(\lambda) = \rho(\lambda) d\lambda = \rho(\theta, \phi) d\theta d\phi. \quad (1)$$

Following CH, a measuring apparatus is formed by an analyzer (A), a detector (D), and the associated electronics (see CH's figure 1, page 528). Consider now a **special** Stern-Gerlach analyzer with the magnetic field oriented along the z axis (i.e. orientation angle $\alpha = 0$) and the magnetic field gradient [28] adjusted to attain *partial separation* of a beam of particles into three classes according to the orientation of spin, as follows (see figure 2):

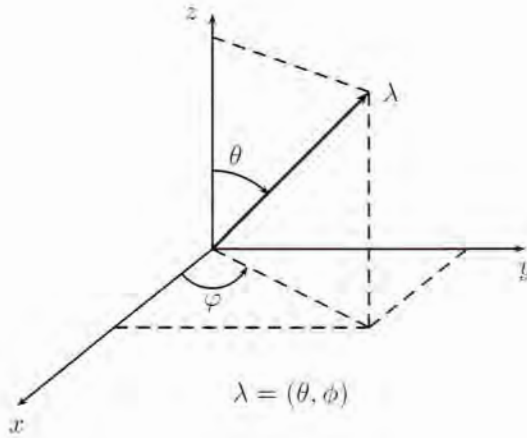


Figure 1. Spin orientation.

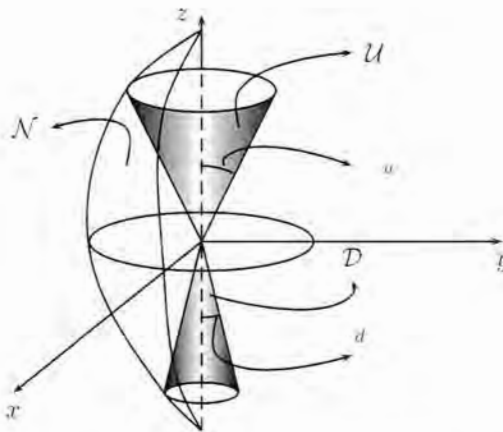


Figure 2. Partitioning of spin orientation space into three regions according to the orientational resolution of the up and down detectors ω_u and ω_d : $\mathcal{U}(\omega_u)$ $\mathcal{D}(\omega_d)$, $\mathcal{N}(\omega_u, \omega_d)$

Class \mathcal{U} (spin-**up** particles) formed by particles with λ belonging to the set $\mathcal{U}(\omega_u, \mathbf{a})$ defined by

$$\mathcal{U}(\omega_u, \mathbf{a}) \equiv \{ \lambda = (\theta, \phi) \quad 0 \leq \theta < \omega_u, 0 \leq \phi < 2\pi, \ni 0 < \omega_u \leq \pi/2 \} \quad (2a)$$

Class \mathcal{D} (spin-**down** particles) formed by particles with λ belonging to the set

$$\mathcal{D}(\omega_d; \mathbf{a}) = \{ \lambda = (\theta, \phi) \quad \pi - \omega_d < \theta \leq \pi, 0 \leq \phi < 2\pi, \ni 0 < \omega_d \leq \pi/2 \} \quad (2b)$$

Note that classes \mathcal{U} and \mathcal{D} may be pictured as cones of half-angles ω_u and ω_d around the orientation axis \mathbf{a} .

Class \mathcal{N} , a central class of **non**-projected spins formed by particles with λ belonging to the set

$$\mathcal{N}(\omega_u, \omega_d; \mathbf{a}) \equiv \{ \lambda = (\theta, \phi) \quad \omega_u \leq \theta \leq \pi - \omega_d, \quad 0 \leq \phi < 2\pi, \ni 0 < \omega_u \leq \pi/2, 0 < \omega_d \leq \pi/2 \} \quad (2c)$$

If each class of particles is directed towards a separate region of space where the particles are ionized and counted in an individual standard detector, the apparatus effectively becomes a three-channel analyzer. Extension of this definition to a multichannel analyzer is straightforward.

Let us focus on classes \mathcal{U} and \mathcal{D} , which are the subject of the various versions of Bell's theorem. The cone with half-angle ω_k ($k = u, d$) defines the resolution of the analyzer for channel k , that, for short, will be referred to as the resolution of detector D_k . Three levels of resolution may be identified:

1. Punctual detectors (p-detectors), defined by $\omega = 0$, are ideal and unattainable. Here, the resolution cone becomes a line along the axis \mathbf{a} .
2. Wide detectors (w-detectors), defined by $\omega = \pi/2$; the resolution cone becomes the open half-space perpendicular to the orientation axis \mathbf{a} . As discussed in section 3, Bell used w-detectors.
3. Real detectors are intermediate: $0 < \omega < \pi/2$. *Fine* detectors (f-detectors) are analyzers with ω small, while *coarse* detectors (c-detectors) have large ω .

Assuming ideal non-absorbing behavior, the efficiency for transmission of this particular Stern-Gerlach analyzer into the \mathcal{U} channel is then

$$\begin{aligned}\varepsilon_u(\lambda, \mathbf{a}) &= 1 && \text{for } 0 \leq \theta(\mathbf{a}) < \omega_u, \quad \forall \phi(\mathbf{a}) \\ \varepsilon_u(\lambda, \mathbf{a}) &= 0 && \text{for } \omega_u \leq \theta(\mathbf{a}) \leq \pi, \quad \forall \phi(\mathbf{a})\end{aligned}\quad (3a)$$

and into the \mathcal{D} channel is

$$\begin{aligned}\varepsilon_d(\lambda, \mathbf{a}) &= 0 && \text{for } 0 \leq \theta(\mathbf{a}) \leq \pi - \omega_u, \quad \forall \phi(\mathbf{a}) \\ \varepsilon_d(\lambda, \mathbf{a}) &= 1 && \text{for } \pi - \omega_u < \theta(\mathbf{a}) \leq \pi, \quad \forall \phi(\mathbf{a})\end{aligned}\quad (3b)$$

where $\theta(\mathbf{a}), \phi(\mathbf{a})$ refer to angles around the arbitrary orientation axis \mathbf{a} .

The dependence of λ defined in equation (3) is the main difference between our model and CH's model (see also the review paper by Clauser and Shimony [29], CS hereafter, and Shimony's textbook article [30]), but it is not new by any means. For instance, in their footnote 4 Freedman and Clauser [31] stated. "A hidden-variable theory need not require that [the individual counting rates with one polarizer removed] R_1 and R_2 be independent of the orientation of the inserted polarizer, and we do not assume this independence in our *data analysis*. However, the results are consistent with R_1 and R_2 being independent of angle, and for simplicity they are so denoted" (emphasis added). In our opinion, this λ -dependence should not be left for the data analysis, but should be built into the theory right from the beginning.

The **calculated** ensemble probability that a particle entering the analyzer will be deflected into channel k is thus

$$P_k(\mathbf{a}) = \int_{\Lambda} \varepsilon_k(\lambda, \mathbf{a}) dP(\lambda) = \int_{\Lambda} \varepsilon_k(\lambda, \mathbf{a}) \rho(\lambda) d\lambda \quad \text{for } k = u, d \quad (4)$$

where the integration is over all possible orientations Λ .

As an example, consider a uniform distribution of spin-orientation given by $\rho(\lambda) = 1/4\pi$. Substituting equation (3) into (4) yields

$$P_k = \frac{1 - \cos \omega_k}{2} \quad \text{for } k = u, d \quad (5)$$

Note that the isotropy of the distribution leads to independence of \mathbf{a} in equation (5). Also, as expected in an OR model, the probability in one channel is independent of the act of measurement in other channels.

2.2 The concept of OR correlation

The phenomena used to test Bell's theorem [3, 4] typically involve the emission of successive pairs of almost colinear particles, that travel in opposite directions with some definite properties, say spin or polarisation. To be specific, let us consider a source that emits pairs of *correlated* spin-1/2 particles, with particle 1 (p1) travelling along the +z axis and particle 2 (p2) along the -z axis. Let the orientation of the spin of p1 be defined by the pdf $\rho_1(\lambda_1)$, so that the probability that, for a given pair, the spin of p1 be oriented along λ_1 is given by equation (1).

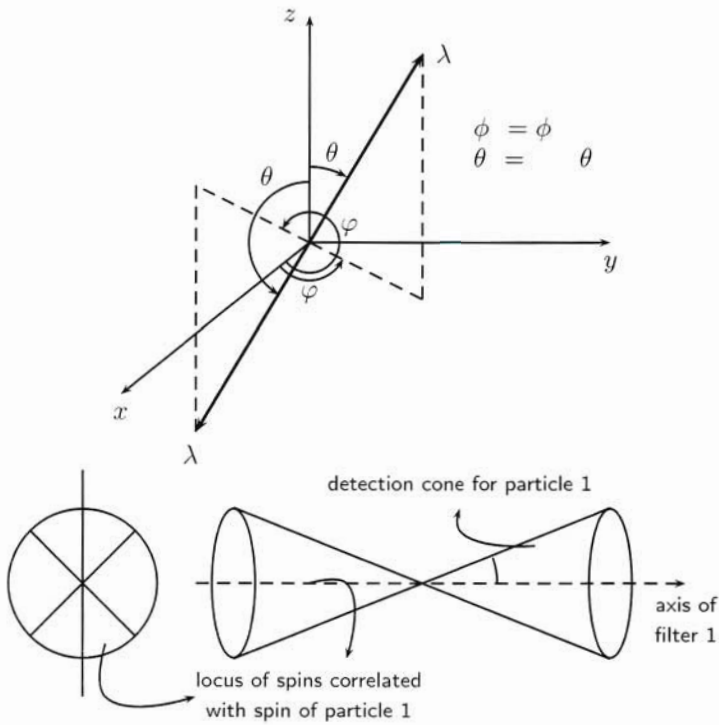


Figure 3. Correlation of spin in EPR experiment. If one particle has spin directed as $\lambda = (\theta, \phi)$ then the other has spin directed as $\lambda_c = (\theta_c, \phi_c) = (\pi - \theta, \pi + \phi)$

Let the total spin of the process be zero, so that if particle p1 has spin oriented along $\lambda_1 = (\theta_1, \phi_1)$, then the spin of p2 is necessarily oriented, right from the emission, as $\lambda_{2c} = (\pi - \theta_1, \phi_1 + \pi)$ (see figure 3). Clearly, this is an OR-rule, not a QM-rule. Hence, the (conditional) probability that the spin of p2 is

oriented as λ_2 , given that the spin of p1 is oriented as λ_1 is

$$\begin{aligned} P(\lambda_2 = \lambda_{2c} | \lambda_1) &= 1 \\ P(\lambda_2 \neq \lambda_{2c} | \lambda_1) &= 0 \end{aligned} \quad (6a)$$

or, in terms of Dirac s delta function

$$P(\lambda_2 | \lambda_1) = \delta(\lambda_2 - \lambda_{2c}) \quad (6a')$$

This means that associated with the cone $\mathcal{U}(\omega_u, \mathbf{a})$ of p1 particles that can be measured in analyzer A1, there exists a *correlated* cone

$$\begin{aligned} C(\omega_u, \mathbf{a}) &= \{ \lambda_2 = (\theta_2, \phi_2) \\ &\quad \theta_2 = \pi - \theta_1, \phi_2 = \phi_1 + \pi, \forall \lambda_1 \in \mathcal{U}(\omega_u, \mathbf{a}) \} \end{aligned}$$

formed by the correlated particles p2 travelling towards analyzer A2.

Similarly, if we initially focus on particle p2, the conditional probability that the spin of p1 is oriented as λ_1 , given that the spin of p2 is oriented as $\lambda_2 = (\theta_2, \phi_2)$, is

$$P(\lambda_1 | \lambda_2) = \delta(\lambda_1 - \lambda_{1c}) \quad (6b)$$

where $\lambda_{1c} = (\pi - \theta_2, \phi_2 + \pi)$

In this case, associated with the cone $\mathcal{D}(\omega_d; \mathbf{a})$ of p2 particles that can be measured in analyzer A2, there exists a *correlated* cone

$$\begin{aligned} C(\omega_d; \mathbf{a}) &= \{ \lambda_1 = (\theta_1, \phi_1) | \\ &\quad \theta_1 = \pi - \theta_2, \phi_1 = \phi_2 + \pi, \forall \lambda_2 \in \mathcal{D}(\omega_d; \mathbf{a}) \} \end{aligned}$$

formed by the correlated particles p1 travelling towards analyzer A1

Consequently, invoking Bayes rule [32], the (joint) probability that the spin of p1 be oriented along λ_1 **and** the spin of p2 be oriented along λ_2 is

$$dP(\lambda_1, \lambda_2) = P(\lambda_2 | \lambda_1) dP(\lambda_1)$$

Substituting eqs. (1) and (6),

$$dP(\lambda_1, \lambda_2) = \delta(\lambda_2 - \lambda_{2c}) \rho_1(\lambda_1) d(\lambda_1) \quad (7a)$$

Or, in terms of p2,

$$dP(\lambda_1, \lambda_2) = P(\lambda_1 | \lambda_2) dP(\lambda_2) = \delta(\lambda_1 - \lambda_{1c}) \rho_2(\lambda_2) d\lambda_2 \quad (7b)$$

If particles p1 and p2 are indistinguishable, then

$$\rho_1(\cdot) = \rho_2(\cdot) = \rho(\cdot)$$

It is stressed that equations (7) refers to calculated probabilities, **completely independent** of observation. A different matter is that to obtain empirical estimates for $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$, the experimenter has to resort to observations with detectors located on the $+z$ and $-z$ axes with the analyzers at *arbitrary* orientations. However, to measure the orientations of the spins for **two particles belonging to the same pair** the experimenter must choose *paired* orientations of the analyzers guided by equations (7). Furthermore, successive emissions are **independent** in a probabilistic sense, but the two components of a **correlated pair** are not.

2.3 Coincidences from single events

Bell's experimental setup [2] contained two analyzers A_i (one for p1, another for p2), each one followed by two detectors $D_{i,k}$ ($i = 1, 2, k = u, d$) whereas CH's experimental design contained only two detectors. D_{1u} to measure class \mathcal{U} following A_1 in p1's path, and D_{2d} to measure class \mathcal{D} following A_2 in p2's path.

From now on, let us focus on CH's arrangement. The probability that a particle arriving to detector D_i is registered as a count is given by the detector efficiency η_i ($i = 1, 2$), for the particular class of particles and energies involved in the experiment. In the ideal case, $\eta_k = 1$

Let N be the source intensity (pairs per unit time), and f_i be a geometrical factor, function of the pdf of directions of emission and the geometry source-detector. Consider a coincidence apparatus with the coincidence window open during non-overlapping time durations τ . Let $P(n = 1)$ be the probability that the source emits exactly one pair during a particular τ . The probability of observing in an interval τ one count in detector i due to a single emission is then

$$\begin{aligned} P_{ik}^{(1)}(\omega_k, \mathbf{a}_i) &= P(n = 1) f_i \eta_i P_k(\mathbf{a}_i) \\ &= f_i \eta_i P(n = 1) \int_{\Lambda} \varepsilon_k(\lambda, \mathbf{a}_i) dP(\lambda), \quad ik = 1u, 2d \quad (8) \end{aligned}$$

where we have substituted equation (4) with \mathbf{a}_i the orientation of the axis of analyzer i (A_i) relative to the z -axis. Likewise, the calculated probability of

single-event coincidences during τ , with analyzers A_1 and A_2 inserted at arbitrary orientations \mathbf{a}_1 and \mathbf{a}_2 is given by

$$P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2)$$

$$f_1 \eta_1 f_2 \eta_2 P(n=1) \int_{\Lambda} \varepsilon_u(\lambda_1, \mathbf{a}_1) \varepsilon_d(\lambda_2, \mathbf{a}_2) dP(\lambda_1, \lambda_2) \quad (9)$$

where $dP(\lambda_1, \lambda_2)$ is independent of observation (recall equation 7). The effect of measurement is contained in the transmission efficiencies $\varepsilon_k(,)$ (equation 3).

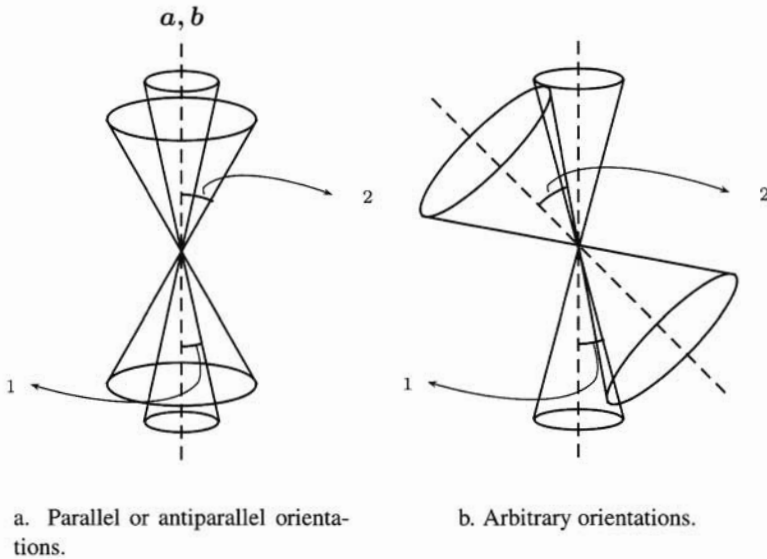


Figure 4. Overlapping configurations for detecting correlated spin pairs.

Let the resolution cones of two analyzers be $A_1(\omega_1, \mathbf{a}_1)$ and $A_2(\omega_2, \mathbf{a}_2)$, and the associated correlation cones be $C_1(\omega_1, \mathbf{a}_1)$ and $C_2(\omega_2, \mathbf{a}_2)$. The following taxonomy may be defined for the relative orientations of the two analyzers in arbitrary coincidence setups defined by the pair $(\mathbf{a}_1, \mathbf{a}_2)$

- 1 Class of overlapping configurations O defined by (see figure 4a.):

$$\begin{aligned} O &\equiv \{(\mathbf{a}_1, \mathbf{a}_2) \mid A_2(\mathbf{a}_2, \omega_2) \cap C_1(\mathbf{a}_1, \omega_1) \neq \emptyset\} \\ &= \{(\mathbf{a}_1, \mathbf{a}_2) \mid A_1(\mathbf{a}_1, \omega_1) \cap C_2(\mathbf{a}_2, \omega_2) \neq \emptyset\} \end{aligned}$$

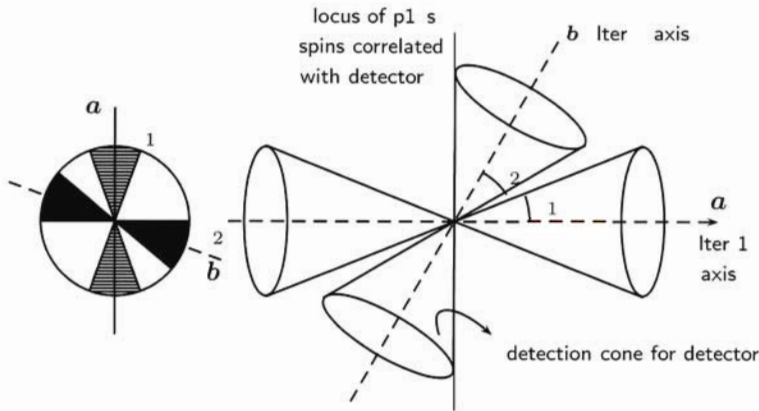


Figure 5. Non overlapping detector configurations. For wide filters ($\omega = \pi/2$), there is overlap for all orientations fulfilling $|a - b| < \pi$

Here, coincidences due to a single pair are always possible; evidently, the relative number of coincidence counts depends upon the extent of overlap of volumes A_2 and C_1 (or, equivalently, volumes A_1 and C_2). Parallel analyzers are the special case $\mathbf{a}_1 = \mathbf{a}_2$ (figure 4b.), where the extent of overlap is a maximum given by $\min[A_1, A_2]$, which depends of $\min[\omega_1, \omega_2]$

2. Class of non-overlapping configurations \tilde{O} defined by (see figure 5):

$$\begin{aligned} \tilde{O} &\equiv \{(\mathbf{a}_1, \mathbf{a}_2) | A_1(\mathbf{a}_1, \omega_1) \cap C_2(\mathbf{a}_2, \omega_2) = \emptyset\} \\ &= \{(\mathbf{a}_1, \mathbf{a}_2) | A_2(\mathbf{a}_2, \omega_2) \cap C_1(\mathbf{a}_1 + \omega_1) = \emptyset\} \end{aligned}$$

In this case, **coincidence counts due to single pairs are impossible.**

In order to appreciate the fundamental rôle of detector resolution, let us consider the idealized case of a planar source $\phi = 0$. In this two-dimensional example $\lambda = (\theta, \phi = 0)$ reduces to the angle θ on the xz plane. Assume first that equation (9) is evaluated by integrating over p_1 (equation 7a). As shown in figure 4a., particles arriving to detector D_1 after passing through analyzer A_1 have spins oriented in the open interval $A_1 = (\mathbf{a}_1 - \omega_u, \mathbf{a}_1 + \omega_u)$, which corresponds to the term $\varepsilon_u(\lambda_1, \mathbf{a}_1)$ in equation (9). The product $\delta(\lambda_2 - \lambda_{2c}) \varepsilon_u(\lambda_1, \mathbf{a}_1)$ generates the locus C_1 of particles p_2 oriented as λ_{2c} . The 2-D coincidence cone C_1 represents the group of p_2 particles potentially travelling towards detector D_2 , however, the analyzer A_2 only accepts spins oriented in the open interval $A_2 = (\mathbf{a}_2 - \omega_d, \mathbf{a}_2 + \omega_d)$. The latter is the term $\varepsilon_d(\lambda_2, \mathbf{a}_2)$ in

equation (9). Hence, there are coincidence counts if, and only if (iff), the intersection of A_2 and the locus C_1 is non-empty. A similar representation may be obtained when the integration is over p_2 (equation 7b) in such case there are coincidences iff the intersection of A_1 and C_2 is non-empty.

As a numerical example consider pairs of indistinguishable particles with spins oriented according to a uniform pdf $\rho(\theta) = 1/2\pi$, and let the two planar analyzers A_1 and A_2 have the same resolution $\omega_u = \omega_d = \omega$, and be oriented as $\mathbf{a}_1 = \mathbf{a}$ and $\mathbf{a}_2 = \mathbf{b}$ (in CH's notation). Substituting into equation (9) one obtains two cases: overlapping analyzers defined by $|\mathbf{a} - \mathbf{b}| < 2\omega$

$$P_c^{(1)}(\omega; \mathbf{a}, \mathbf{b}) = f_1 \eta_1 f_2 \eta_2 P(n=1) \frac{2\omega - |\mathbf{a} - \mathbf{b}|}{2\pi} \quad (10a)$$

Parallel analyzers are the particular case $\mathbf{a} = \mathbf{b}$

$$P_c^{(1)}(\omega; \mathbf{a}) = f_1 \eta_1 f_2 \eta_2 P(n=1) \frac{\omega}{\pi} \quad (10b)$$

Non-overlapping analyzers defined by $|\mathbf{a} - \mathbf{b}| \geq 2\omega$ yield

$$P_c^{(1)}(\omega; \mathbf{a}, \mathbf{b}) = 0. \quad (10c)$$

Hence, there is a large class of relative orientations $|\mathbf{a} - \mathbf{b}| \geq 2\omega$ for which coincidences from single-events are impossible. Even in the case of w-detectors ($\omega = \pi/2$), all relative orientations fulfilling $|\mathbf{a} - \mathbf{b}| = \pi$ yield

$$P_c^{(1)}(\pi/2; \mathbf{a}, \mathbf{b}) = 0.$$

2.4 Coincidences from multiple events

There is, however, a mechanism for observing coincidence counts under all possible orientations $(\mathbf{a}_1, \mathbf{a}_2)$ multiple events. Depending upon the source intensity N and the size of the coincidence window τ , the probability of emission of two or more pairs during τ may be significant. Let us consider double events only, which occur with probability $P(n=2)$ –triple and higher events add nothing new conceptually.

Given that two pairs have been emitted during τ , the probability of observing a coincidence count is

$$\begin{aligned} P(\text{coincidence}|n=2) &= P(\text{observing } p_1 \text{ from pair 1}) \times P(\text{observing } p_2 \text{ from pair 2}) \\ &\quad + P(\text{observing } p_1 \text{ from pair 2}) \times P(\text{observing } p_2 \text{ from pair 1}) \end{aligned}$$

And, after substituting equation (4),

$$P(\text{coincidence} | n=2) = 2f_1\eta_1 f_2\eta_2 P_{1u}(\mathbf{a}_1)P_{2d}(\mathbf{a}_2) \quad (11)$$

The probability of registering a double-event coincidence count during τ is thus

$$P_c^{(2)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2) = P(\text{coincidence} | n=2)P(n = 2) \\ = 2f_1\eta_1 f_2\eta_2 P_{1u}(\mathbf{a}_1)P_{2d}(\mathbf{a}_2)P(n = 2) \quad (12)$$

Continuing with the same planar example of previous section 2.3, the probability of a double-emission coincidence count is

$$P_c^{(2)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2) = 2f_1\eta_1 f_2\eta_2 P(n = 2) \frac{\omega_u}{\pi} \frac{\omega_d}{\pi} \quad (13a)$$

For detectors with the same resolution ω this expression becomes,

$$P_c^{(2)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2) = 2f_1\eta_1 f_2\eta_2 P(n = 2) \left(\frac{\omega}{\pi}\right)^2 \quad (13b)$$

Note that equation (13) is independent of analyzer orientation due to the assumed isotropy of spins.

The probabilities $P(n = 1)$ and $P(n = 2)$ depend upon the details of the coincidence system (in particular τ and dead-time), and upon the source intensity N (several probabilistic models are scattered throughout Feller's book [32]). As an illustration, assume that $P(n = k)$ is described by a Poisson distribution with $\mu = N\tau$, then

$$P(n = k) = \frac{\mu^k e^{-\mu}}{k!} \quad (14)$$

Under such circumstances, $P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2)$, given by equation (9), depends of N and $P_c^{(2)}(\omega_u, \omega_d; \mathbf{a}_1, \mathbf{a}_2)$ given by equation (12), depends of N^2

2.5 The importance of detector resolution

Consider a planar source $\phi = 0$ that emits particles with spins in the xz plane oriented along some privileged direction. To be specific, choose the z axis parallel to the Earth's magnetic field, with $+z$ pointing due north. Consider three completely different gedanken-sources j ($j = 1, 2, 3$) emitting particles with spin orientations $\rho_j(\theta)$, see figure 6, and let us further assume that the three sources have the same intensity (N particles per unit time):

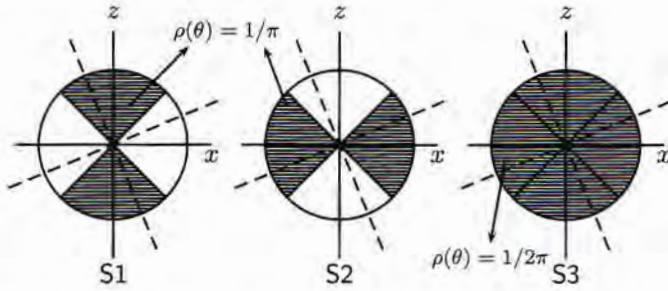


Figure 6. Wide detectors at arbitrary orientation \mathbf{a} can not distinguish between three emitters S1, S2, S3 –different according to OR.

Source 1 S1 emits particles with spin approximately parallel to the Earth's magnetic field.

$$\rho_1(\theta) = 1/\pi \quad \text{for} \quad \pi/4 < \theta < 3\pi/4 \quad \text{and} \quad 5\pi/4 < \theta < 7\pi/4 \quad (15a)$$

$$\rho_1(\theta) = 0 \quad \text{elsewhere} \quad (15b)$$

Source 2 S2 emits particles with spin approximately perpendicular to the Earth's magnetic field.

$$\rho_2(\theta) = 1/\pi \quad \text{for} \quad \pi/4 < \theta < 3\pi/4 \quad \text{and} \quad 5\pi/4 < \theta < 7\pi/4 \quad (16a)$$

$$\rho_2(\theta) = 0 \quad \text{elsewhere} \quad (16b)$$

Source 3 S3 emits particles with isotropic spin distribution given by

$$\rho_3(\theta) = 1/2\pi \quad \text{for} \quad 0 < \theta < 2\pi \quad (17)$$

Let us use a **w-detector** at different arbitrary orientations \mathbf{a} to obtain an empirical estimate for $\rho_j(\theta)$. It is easy to see from equation (4) that the calculated counting rates will be

$$N_1(\mathbf{a}) = N_2(\mathbf{a}) = N_3(\mathbf{a}) = N f_1 \eta_1 / 2, \quad (18)$$

which are independent of **both** the detector orientation and the type of source. Contrariwise, it is also easy to see that f-detectors would yield $N_j(\mathbf{a})$ with a quite definite orientation and source dependence. Hence, given that w-detectors are incapable of distinguishing between the very different sources of this section, it may be concluded that, in general, **w-detectors are not appropriate to measure distributions of orientational properties of OR particles.**

3 Bell's original inequality critically revisited

Bell's fundamental assumption is equation 2 in his original paper [2], reprinted as paper 2 in his collected papers [33] (references to this book are denoted by B followed by the page number, say B15). This equation is the same equation (5) in paper 4 (B36): “our notion of locality requires *that A does not depend on b, nor B on a*. We then ask if the mean value $P(\mathbf{a}, \mathbf{b})$ of the product AB , i.e.

$$P(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \quad (19)$$

can equal the quantum-mechanical prediction” (Bell's emphasis).

We do not take issue with Bell's interpretation of EPR's objective reality as independence of observations at two distant detectors $A(\mathbf{a}, \lambda)$ and $B(\mathbf{b}, \lambda)$, where vectors \mathbf{a} and \mathbf{b} contain information on the individual detectors, chiefly the orientation, and λ is the set of EPR's objective reality (OR) parameters; in Bell's words, “(for example) a unit vector λ with uniform probability distribution” (B16). Indeed, they are the same parameters determining the value of probability in our equation (9). Although at first sight the structure of equations (9) and (19) is the same, there are two important differences:

- (a) The integration in (9) is over the **joint** probability $dP(\lambda_1, \lambda_2)$ given by equation (7) whereas the integration in Bell's equation (19) is over the individual probability $dP(\lambda)$ given by equation (1). And,
- (b) Equation (9) is a probability whereas equation (19) is the expected value of the product AB of two arbitrarily defined two-valued random variables: $A(\mathbf{a}, \lambda) = \pm 1$, $B(\mathbf{b}, \lambda) = \pm 1$, given by the direction of projection of spin upon the axis of the analyzer (see, for instance, Bell's equation 9 in paper 2, B16):

$$A(\mathbf{a}, \lambda) = \text{sing}(\mathbf{a} \cdot \lambda) \quad (20)$$

In the 2-D example of section 2.3, Equation (20) becomes

$$A(\mathbf{a}, \lambda) = A(\mathbf{a}, \theta) = \text{sign}(\cos |\mathbf{a} \cdot \theta|) = \begin{cases} +1, & 0 \leq \mathbf{a} \cdot \theta < \pi/2 \\ 1, & \pi/2 < \mathbf{a} \cdot \theta \leq \pi \end{cases}$$

But this necessarily implies that $\omega_u = \omega_d = \omega = \pi/2$ (see equation (2) above). That is, Bell's analyzers are perfect w-detectors with $\varepsilon_u(\lambda, \mathbf{a}) = \varepsilon_d(\lambda, \mathbf{a}) = 1$ for $0 \leq \theta(\mathbf{a}) < \pi/2$, $\forall \phi(\mathbf{a})$, recall equation (3).

The two remarks above prompt several criticisms:

Criticism 1. By using $dP(\lambda)$ instead of $dP(\lambda_1, \lambda_2)$, Bell is neglecting the term $\delta(\lambda_2 - \lambda_{2c})$ that represents the *correlation* between the pair of particles, which is the essence of EPR s argument.

Criticism 2. As argued in section 2.5, w-detectors are not the best choice for studying orientational properties to OR particles; in particular, w-detectors are not suited to measure the pdf of spin orientations.

Criticism 3. Bell s partitioning of λ space into two variables A and B leaves out some possible points in λ space. In the 2-D example of section 2, the value $\theta = \pi/2$ is assigned to neither A , nor B . Bell was aware of this fact, but he put it aside stating as the probability of this is zero we will not make special prescriptions for it [2, p.196]. This is completely correct under the ideal *continuous* representation of nature, where the pdf $\rho(\lambda)$ has zero measure at individual values of λ . However, this leaves out the intrinsic graininess of nature, that in this case manifests as discrete probability mass functions (pmf). Take for instance, a gedankensource emitting correlated pairs of particles, with p1 s spin oriented along the z and the x axes only as given by the pmf $\rho_1(\lambda_1) = [\delta(\theta) + \delta(\theta - \pi/2)]/2$. The p2 s spin will be oriented along z and $-x$ axes respectively. Let both analyzers be oriented along the z axis. $a = 0, b = 0$, then $A(a, \theta = 0) = +1, A(a, \theta = \pi/2) = 0, B(b, \theta = \pi) = -1, B(b, \theta = 3\pi/2) = 0$. Substituting into equation (19), one (correctly) obtains

$$P(a, b) = 1/2, \quad (21)$$

which differs from $P(a, a) = 1$ (equation 8 in Bell s original paper [2, p.197]).

Criticism 4. In deriving his original inequality

$$1 + P(b, c) \geq P(a, b) - P(a, c) \quad (22)$$

Bell changed the term $A(a, \lambda)A(c, \lambda)$ into $A(b, \lambda)A(b, \lambda)A(a, \lambda)A(c, \lambda)$ (see [2, p.198], or paper 2, B18) which implies that

$$A(b, \lambda)A(b, \lambda) = 1 \quad (23)$$

Due to the possibility of normalization, equation (23) is valid in general as long as $A(\cdot, \lambda)$ is a **two-valued** random variable $A(\cdot, \lambda) = \pm k$, where k is a real constant. Assuming continuous pdf s, this is indeed the case

with **perfect** w-detectors. However, from the viewpoint of experimental testing this fact opens an additional loophole because real analyzers are either **imperfect** w-detectors, or, else, c-detectors.

Criticism 5. Furthermore, in subsequent papers Bell himself made $A(., \lambda)$ a **three-valued** random variable: “In practice, there will be some occasions on which one or both instruments simply fail to register either way. One might then count A as zero.” (paper 4, B37). From this passage one cannot determine whether Bell was thinking in coarse detectors ($\omega < \pi/2$), and hence in a third class \mathcal{N} (see 2.1 above), or in the detector efficiency $\eta < 1$ (see 2.3 above). At any rate, Bell’s **original** derivation of inequality (22) does not get through for three-valued random variables.

However, Bell offered in paper 4 [33] an alternative derivation of inequality (22) as a general case of an inequality similar to CHSH. This derivation does not impose restrictions on the number of possible values that A and B can take but depends on the assumption $P(a, a) = 1$ **always** (equation 10, B38). As demonstrated by our equation (21) this is not always the case.

4 The CHSH inequality critically revisited

Let us turn now to CHSH inequalities derived by Clauser, Horne, Shimony and Holt [3] in the process of devising an experimental test of Bell’s theorem. Here we will refer to CH’s proof [27] (see also CS [29] and [30]). The CHSH inequalities differ from Bell’s original inequality in several aspects thoroughly reviewed by CS. We stress two differences: (a) CHSH refer to **calculated** probabilities, not to calculated expectations over arbitrarily defined random-variables, and (b) The probabilities appear as ratios, thus making unnecessary the (difficult) estimation of N

In the following we repeat CH’s derivation of the CHSH inequalities. The ensemble probabilities are given by (CH, equation 2, page 527)

$$p_k(\mathbf{a}) = \int_{\Lambda} p_k(\lambda, \mathbf{a}) \rho(\lambda) d\lambda \quad \text{for } k = 1, 2 \quad (24a)$$

$$p_{12}(\mathbf{a}, \mathbf{b}) = \int_{\Lambda} p_{12}(\lambda, \mathbf{a}, \mathbf{b}) \rho(\lambda) d\lambda \quad (24b)$$

We have no query with equations (24), which are more general than our equations (4) and (7). Next, they invoke a completely general lemma (see CH s appendix A):

$$1 \leq p_1(\lambda, \mathbf{a}) p_2(\lambda, \mathbf{b}) - p_1(\lambda, \mathbf{a}) p_2(\lambda, \mathbf{b}') + p_1(\lambda, \mathbf{a}') p_2(\lambda, \mathbf{b}) + p_1(\lambda, \mathbf{a}') p_2(\lambda, \mathbf{b}') - p_1(\lambda, \mathbf{a}') p_2(\lambda, \mathbf{b}) \leq 0 \quad (25)$$

Multiplying by $\rho(\lambda)d\lambda$ and integrating one obtains

$$p_1(\mathbf{a}') + p_2(\mathbf{b}) - 1 \leq \int_{\Lambda} [p_1(\lambda, \mathbf{a}) p_2(\lambda, \mathbf{b}) - p_1(\lambda, \mathbf{a}) p_2(\lambda, \mathbf{b}') + p_1(\lambda, \mathbf{a}') p_2(\lambda, \mathbf{b}) + p_1(\lambda, \mathbf{a}') p_2(\lambda, \mathbf{b}')] \rho(\lambda) d\lambda \leq p_1(\mathbf{a}') + p_2(\mathbf{b}) \quad (26)$$

So far, so good. At this point, however, CH (page 528) focussed their attention on a special case of formulation (24b) in which

$$p_{12}(\lambda, \mathbf{a}, \mathbf{b}) = p_1(\lambda, \mathbf{a}) p_2(\lambda, \mathbf{b}) f \quad (27)$$

Substituting equation (27) into expression (26) and using equation (24b) one obtains CHSH inequalities

$$p_1(\mathbf{a}') + p_2(\mathbf{b}) - 1 \leq p_{12}(\mathbf{a}, \mathbf{b}) - p_{12}(\mathbf{a}, \mathbf{b}') + p_{12}(\mathbf{a}', \mathbf{b}) + p_{12}(\mathbf{a}', \mathbf{b}') \leq p_1(\mathbf{a}') + p_2(\mathbf{b}) \quad (28a)$$

CS paper focussed on the rhs of inequalities (28a) written as

$$U = \frac{p_{12}(\mathbf{a}, \mathbf{b}) - p_{12}(\mathbf{a}, \mathbf{b}') + p_{12}(\mathbf{a}', \mathbf{b}) + p_{12}(\mathbf{a}', \mathbf{b}')}{p_1(\mathbf{a}') + p_2(\mathbf{b})} \leq 1 \quad (28b)$$

Let the empirical estimate for U be \widehat{U} given by

$$\widehat{U} = \frac{R(\mathbf{a}, \mathbf{b}) - R(\mathbf{a}, \mathbf{b}') + R(\mathbf{a}', \mathbf{b}) + R(\mathbf{a}', \mathbf{b}')}{r(\mathbf{a}') + r(\mathbf{b})} \quad (29)$$

where $R(., .)$ are coincidence rates and $r(.)$ are individual counting rates for the relevant analyzer orientations. Hence, the empirical data analysis reduces to checking

$$\widehat{U} \leq 1 \quad (28c)$$

The advantage of U is that \widehat{U} is independent of N (which is difficult to measure). It is noted in passing that the literature on Bell's inequality does not distinguish between the theoretical (or calculated) U and its empirical estimate \widehat{U} , such distinction is standard practice in probability theory [32] and in mathematical statistics [34]. To our knowledge, the only example of such distinction is the recent paper by Ou and Mandel [35].

In our opinion, the only weak link in CHSH's derivation is equation (27). We were particularly stricken by CH's hesitant defense of equation (27): "What considerations motivate this factored form? Clearly, if each source emission consists of two well-localized subsystems, e.g., a pair of objective particles, and there is no action at a distance, then the factored form is a *reasonable* locality condition. *we conjecture* that equation (27) is implicit in the thinking of many physicists. Whether or not this is correct, it is apparent that quantum mechanics is not of the form 27" (emphasis added; CH, page 528).

From this passage it is clear that CH did not claim that equation (27) is the only possible locality condition; they simply conjectured that equation (27) might represent the thinking of other (QM??) physicists. It is our contention that a more appropriate locality condition is furnished by our equation (9) (section 2.3 above), where CH's term $p_{12}(\lambda, \mathbf{a}, \mathbf{b})$ is given by

$$p_{12}(\lambda, \mathbf{a}, \mathbf{b}) = \varepsilon_1(\lambda_1, \mathbf{a})\varepsilon_2(\lambda_2, \mathbf{b})\delta(\lambda_2 - \lambda_{2c}) \quad (30)$$

When particles 1 and 2 are indistinguishable: $\rho_1(\lambda_1) = \rho_2(\lambda_2) = \rho(\lambda)$ It is easy to see that substitution of equation (30) into inequality (26) no longer leads to the **original** CHSH's inequalities (28) (CH's equation 4, page 528). Such result can be expected in advance because equation (30) contains joint probabilities that can not possibly yield the individual probabilities $p_1(\mathbf{a})$ and $p_2(\mathbf{b})$ in equation (28).

Thus, the only difference between our equation (30) and CHSH's equation (27) is the term $\delta(\lambda_2 - \lambda_{2c})$ which contains the **correlation** between the particles, *central core of EPR's argument*. This exactly is the same criticism 1 to Bell's derivation (see section 3). Note that the general derivation of CHSH inequalities imposes no constraints on the detector resolution, as such CHSH are immune to all other criticisms in previous section.

However, experimental tests arranged so-far depend upon imperfect w-detectors, thus introducing a loophole through the back door. Indeed, the majority of experiments to test CHSH inequalities measure pairs of cascading photons with correlated polarizations [4, 29, 36]. The analyzers typically are piles-of-plates polarizers inclined at nearly the Brewster's angle. Let a beam

I_0 of (light linearly polarized in a direction λ) be incident upon a perfect polarizer oriented at angle \mathbf{a} , the transmitted light intensity I is given by the law of Malus [37]:

$$I = I_0 \cos^2 \lambda \cdot \mathbf{a} \quad (31)$$

This means that the polarizer has a λ dependence, that in the context of our OR model can be interpreted as $\varepsilon(\lambda, \mathbf{a}) = \cos^2 \lambda - \mathbf{a}$, i.e., the cone of resolution is $\omega = \pi/2$. Such analyzers are then w-detectors with λ -dependent transmission efficiency. The present author is not familiar with the details of the polarizers used in more recent experiments [35, 38]

5 Constrained CHSH inequalities

5.1 Single-event inequalities

Consider an experimental arrangement for the measurement of correlated spin particles with f-detectors (or, at least, c-detectors). In order to assure the existence of coincidences from single events, let the relative detector orientations (\mathbf{a}, \mathbf{b}) , $(\mathbf{a}', \mathbf{b})$, $(\mathbf{a}, \mathbf{b}')$, and $(\mathbf{a}', \mathbf{b}')$ belong to the class O of overlapping configurations (section 2.3). Let the electronics be adjusted for *true coincidences* (i.e. to measure coincidences due to single events only). Under such conditions the probability of coincidence counts in detectors 1 and 2 is given by equation (9) (section 2.3).

Let us apply CH's general inequality (25) to the analyzer efficiencies $\varepsilon(\cdot, \cdot)$

$$\begin{aligned} 1 \leq \varepsilon_u(\lambda_1, \mathbf{a})\varepsilon_d(\lambda_2, \mathbf{b}) - \varepsilon_u(\lambda_1, \mathbf{a})\varepsilon_d(\lambda_2, \mathbf{b}') + \varepsilon_u(\lambda_1, \mathbf{a}')\varepsilon_d(\lambda_2, \mathbf{b}) \\ + \varepsilon_u(\lambda_1, \mathbf{a}')\varepsilon_d(\lambda_2, \mathbf{b}') - \varepsilon_u(\lambda_1, \mathbf{a}') - \varepsilon_d(\lambda_2, \mathbf{b}) \leq 0 \quad (32) \end{aligned}$$

Upon introducing the constant

$$K_1 = f_1\eta_1 f_2\eta_2 P(n=1), \quad (33)$$

and multiplying (32) by $K_1 dP(\lambda_1, \lambda_2)$, integration yields

$$\begin{aligned} P_c^{(1)}(\pi, \pi; \infty, \infty) \leq P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}, \mathbf{b}) - P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}, \mathbf{b}') \\ + P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}', \mathbf{b}) + P_c^{(1)}(\omega_u, \omega_d; \mathbf{a}', \mathbf{b}') \\ - P_c^{(1)}(\omega_u, \pi; \mathbf{a}', \infty) - P_c^{(1)}(\pi, \omega_d, \infty, \mathbf{b}) \leq 0 \quad (34) \end{aligned}$$

where

$$P_c^{(1)}(\omega_u, \pi; \mathbf{a}', \infty) = K_1 \int_{\Lambda} \varepsilon_u(\lambda_1, \mathbf{a}') dP(\lambda_1, \lambda_2) \quad (35a)$$

and

$$P_c^{(1)}(\pi, \omega_u, \infty, \mathbf{b}) = K_1 \int_{\Lambda} \varepsilon_d(\lambda_2, \mathbf{b}) dP(\lambda_1, \lambda_2) \quad (35b)$$

are the probabilities of single-event coincidences with one analyzer removed. Similarly,

$$P_c^{(1)}(\pi, \pi; \infty, \infty) = K_1 \int_{\Lambda} dP(\lambda_1, \lambda_2) \quad (35c)$$

is the probability of single-event coincidences with both analyzers removed.

It can be immediately recognized that inequality (34) is the same equation 11 of CH's paper, that was obtained with the introduction of the supplementary “*no-enhancement assumption*” (page 530). This standard inequality is obtained here without additional assumptions but with two constraints:

- 1 Detectors with resolution better than w-detectors, i.e.
 $\omega_u, \omega_d < \pi/2$,
2. Analyzers oriented in overlapping configurations O (see 2.3).

5.2 Multiple-event inequalities

Consider an experimental arrangement for the measurement of correlated spin particles with f-detectors or, at least, c-detectors. Let the relative detector orientations (\mathbf{a}, \mathbf{b}) , $(\mathbf{a}', \mathbf{b})$, $(\mathbf{a}, \mathbf{b}')$, and $(\mathbf{a}, \mathbf{b}')$ belong to the class \tilde{O} of non-overlapping configurations (section 2.3). Under such conditions the probability of coincidence counts in detectors 1 and 2 is given by equation (12):

$$\begin{aligned} p^{(2)}(\mathbf{a}, \mathbf{b}) &\equiv P_c^{(2)}(\omega_1, \omega_2, \mathbf{a}, \mathbf{b}) = 2f_1\eta_1f_2\eta_2P(\omega_1, \mathbf{a})P(\omega_2, \mathbf{b})P(n=2) \\ &= K_2P(\omega_1, \mathbf{a})P(\omega_2, \mathbf{b}) \end{aligned} \quad (36)$$

where

$$K_2 = 2f_1\eta_1f_2\eta_2hP(n=2) \quad (37)$$

and the term $h \geq 1$ represents the contribution from triple and higher events during the coincidence period τ , its exact value depending, inter alia, upon the details of the experimental arrangement. The individual values of probability $P(\omega_1, \mathbf{a})$ and $P(\omega_2, \mathbf{b})$ are calculated from equation (4).

Our equation (36) is formally similar to CH's equation (27) above but has a different empirical content. Hence, one can parallel CH's derivation given in section 4. Starting from the lemma (25) applied to the ensemble probabilities $P(\omega_1, \mathbf{a})$ and $P(\omega_2, \mathbf{b})$ one gets

$$1 \leq P(\omega_1, \mathbf{a})P(\omega_2, \mathbf{b}) - P(\omega_1, \mathbf{a})P(\omega_2, \mathbf{b}') + P(\omega_1, \mathbf{a}')P(\omega_2, \mathbf{b}) + P(\omega_1, \mathbf{a}')P(\omega_2, \mathbf{b}') - P(\omega_1, \mathbf{a}') - P(\omega_2, \mathbf{b}) \leq 0 \quad (38)$$

Multiplying by K_2 and substituting equation (36),

$$K_2 \leq p^{(2)}(\mathbf{a}, \mathbf{b}) - p^{(2)}(\mathbf{a}, \mathbf{b}') + p^{(2)}(\mathbf{a}', \mathbf{b})p^{(2)}(\mathbf{a}', \mathbf{b}') - K_2P(\omega_1, \mathbf{a}') - K_2P(\omega_2, \mathbf{b}) \leq 0 \quad (39)$$

Focussing on the rhs inequality only

$$U_2 = \frac{p^{(2)}(\mathbf{a}, \mathbf{b}) - p^{(2)}(\mathbf{a}, \mathbf{b}') + p^{(2)}(\mathbf{a}', \mathbf{b}) + p^{(2)}(\mathbf{a}', \mathbf{b}')}{P(\omega_1, \mathbf{a}') + P(\omega_2, \mathbf{b})} \leq K_2 \quad (40a)$$

Similarly to the standard CHSH inequality, the empirical estimate of U_2 is \hat{U}_2 , and the empirically testable inequality becomes

$$\hat{U}_2 \equiv \frac{R^{(2)}(\mathbf{a}, \mathbf{b}) - R^{(2)}(\mathbf{a}, \mathbf{b}') + R^{(2)}(\mathbf{a}', \mathbf{b}) + R^{(2)}(\mathbf{a}', \mathbf{b}')}{r(\omega_1, \mathbf{a}') + r(\omega_2, \mathbf{b})} \leq K_2, \quad (40b)$$

where the $R^{(2)}(\cdot, \cdot)$ are coincidence rates from multiple-events, and the $r(\cdot)$ are individual counting rates, both cases for f-detectors (or, at least, c-detectors).

Inequalities (40) are formally similar to CHSH inequalities (28), except for the presence of the constant K_2 , which may be estimated by the coincidence rate from multiple-events with both analyzers removed. However, we stress that our inequality refers to coincidences from **multiple** events, whereas CHSH inequality presumably referred to coincidences from single events only. Hence, although the experimental arrangement to test our inequality (40) may be similar to the standard setup to test inequality (28), data collection and analysis are different.

It is not clear whether QM violates inequalities (34) and (40). In the case of (34) the difference arises from our condition $\omega_u, \omega_d < \pi/2$, which differs from the QM-condition $\omega_u = \omega_d = \pi/2$. This question can be answered by an analysis similar to section 4 in CS paper, and is deferred to another paper.

6 Concluding remarks

Summarizing, it is our contention that the basic premises for deriving both Bell's and CHSH's inequalities do not take particle correlation duly into account. Such correlation is the basic assumption in EPR's gedankenexperiment. As a consequence, the representation of objective reality offered by Bell (equation 19) is only appropriate for parallel analyzer orientations and the same detector resolution $\omega_u = \omega_d = \omega$. In general, for arbitrary detector orientations and resolutions, a different representation (equation 9) is required.

Likewise, the independence or locality assumption in CH's derivation (equation 27) does not contain EPR's correlation condition. As a substitute we offer a different locality condition that contains correlation (equation 30).

These two results lead us to conclude that Bell's inequalities and the basic CHSH inequalities are not appropriate representations of OR in the context of the EPR experiment. Consequently, the experimentally observed violation of such inequalities does not have any bearing on the validity of OR representations of nature.

For the case of single-event coincidences, we obtained from our OR model the standard version of CHSH inequalities (equation 34) without invoking the no-enhancement assumption, thus providing additional theoretical strength to its validity. This is particularly relevant because most experiments are designed to test inequalities (34) [4, 35, 39, 40], or the version for the special geometry of Freedman and Clauser [31, 36, 38]. Therefore, the matter at issue is now whether the experimental tests carried out thus far are consistent with our new constraints. good resolution of analyzers and overlapping orientations of detectors. This question is open.

For the case of multiple-event coincidences, we obtained a novel inequality (40) similar to CHSH's, but with different empirical content. Experimental tests *using good resolution detectors* can decide whether nature violates such OR hypothesis or not. The conditions for QM violations of inequalities (34) and (40) are left open.

At any rate, as long as our *constrained* inequality (34) is not violated, previous findings reopen a door to OR interpretations. In particular, invoking Occam's razor, there disappears, for the time being, the need for the following concepts: nonlocality, superluminal communications, breaches of causality in local interpretations.

We end with a conjecture regarding a particular instance of locality: Kolmogorovian probabilities [21, 41, 42]. It is known that Bell's inequality, arising from equation (19), "is a necessary but not sufficient condition for a local Kolmogorovian probability" [41, p. 13]. Taking into account that our deriva-

tion conforms to standard probability, and that our equation (9) contains equation (19) as a special case, it may be conjectured that our correlation conditions (9) and (12) are both sufficient and necessary for the existence of Kolmogorovian probability.

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