ENTANGLEMENT BETWEEN TWO SINGLE-PHOTON SOURCES DUE TO A BEAM SPLITTER

ENTRELAZAMIENTO ENTRE DOS FUENTES DE UN SOLO FOTÓN DEBIDO A UN DIVISOR DE HAZ

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Abstract

We consider a quantum dot of frequency ω_σ embedded in a semiconductor microcavity interacting with an electromagnetic mode of frequency ω_a. The cavity is subject to a pumping laser of power Ω and frequency ω_L, and to dissipative mechanisms such as photonic and excitonic relaxation, at rates γ_a and γ_σ, respectively. The power and frequency of the laser is set so that the cavity-dot system reaches a stationary state, in which it emits single photons. Taking two of this single-photon emitters as the input for a semi transparent beam splitter, we consider the entanglement of the two output states in the low excitation regime using the Peres criteria. Thus, we study the dependence of the entanglement with the photonic and excitonic relaxation rates.

Keywords: Entanglement, beam splitter, single-photon sources.

Resumen

Consideramos un punto cuantico de frecuencia ω_σ embebido en una microcavidad semiconductora e interaccionando con un modo del campo electromagnético de frecuencia ω_a.

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La cavidad es sometida a un bombeo coherente por un laser de potencia Ω y frecuencia ω_L, y a los mecanismos disipativos como el escape de fotones de la cavidad y decaimiento excitónico, los cuales ocurren a las tasas γ_a y γ_σ, respectivamente. La potencia y la frecuencia del laser se ajustan de manera que el sistema cavidad-punto cuántico alcanza un estado estacionario, en el cual emite un solo fotón. Tomando dos de estas fuentes de fotones individuales como las entradas de un divisor de haz semitransparente, consideramos el entrelazamiento de los dos estados de salida en el régimen de baja excitación usando el criterio de Peres. De esta manera, estudiamos la dependencia del entrelazamiento con las tasas de disipación γ_a y γ_σ.

Palabras clave: Entrelazamiento, divisor de haz, fuentes de fotones individuales.

Introduction

Entanglement is one of the most striking properties of the quantum systems, and it is at the heart of the quantum computation. Entangled systems allow for faster information processing [1–3], and have a great relevance in the security of encryption protocols in order to secure communications [4–7]. Because of this, the generation and characterization of entangled quantum states has been widely studied.

In particular, photon entangled states can be generated either with type-I or type-II spontaneous parametric downconversion [8–12], or with a beam splitter [13–17]. Specifically, Kim and co-workers [16] established that the nonclassicality of the input state of the beam splitter is a required condition for entanglement, e.g. coherent states do not get entangled with a beam splitter, and considered the entanglement due to a beam splitter for several pure and Gaussian mixed states.

In this paper, we study the entanglement between two single-photon sources, which has been proposed by Laussy and co-workers [18], due to a 50:50 beam splitter. The sources are characterized by the dissipative rates of the microcavity in which they are prepared, and therefore we compute the negativity of the beam splitter's output state as a function of such dissipative rates.

The rest of the paper is organized as follows: In the first place, we describe the theoretical model for the single-photon sources and for the transformation due to the beam splitter. Secondly, we present the results of our study and discuss them; and afterwards we give a summary and the conclusions of our study. Finally, we present the references used throughout the paper.

Model

Our model consists of two single-photon sources interacting with a 50:50 beam splitter. Each of the sources consist of a N-photon laser, which has been proposed by Laussy *et al.* in [18]. Such a source is obtained by a quantum dot, of excitation frequency ω_σ and decay rate γ_σ, interacting with an off-resonant electromagnetic mode inside a semiconductor microcavity. This interaction is modelled by the usual Jaynes-Cummings Hamiltonian ($\hbar$ is takes as 1 along the paper),

$$H_0 = \omega_a a^\dagger a + \omega_\sigma \sigma^\dagger \sigma + g\left(a^\dagger \sigma + \sigma^\dagger a\right), \tag{1}$$

where g describes the strength of the dipole coupling between the quantum dot and the cavity mode, $a^\dagger$ and a are the usual annihilation and creation operators of photons in the cavity, and $\sigma^\dagger$ and σ are the usual operators for the excitonic two-level systems consisting of a ground state $|G\rangle$ and a single exciton $|X\rangle$.

The microcavity losses photons at the rate γ_a and is coherently pumped at rate Ω with an external laser of frequency ω_L. The latter is included into the description by adding,

$$H_1 = \Omega\left(\sigma e^{i\omega_L t} + \sigma^\dagger e^{-i\omega_L t}\right), \tag{2}$$

to H_0.

Morover, the excitonic decay rate and photon leakage rate are taken into account in the master equation of the Lindblad form,

$$\dot{\rho} = i\left[\rho, H_0 + H_1\right] + \mathcal{L}\left(\rho\right), \tag{3}$$

where,

$$\mathcal{L}\left(\rho\right) = \frac{\gamma_a}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) + \frac{\gamma_\sigma}{2}\left(2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma\right). \tag{4}$$

Furthermore, in [18] is shown that the N-photon regimen is obtained when the pumping laser's frequency satisfies the following condition,

$$\omega_L = \omega_a + \frac{\sqrt{g^2\left(N+1\right) + \Delta^2/4}}{N+1}. \tag{5}$$

Considering this condition, we solve the eq. (3) for the steady state, i.e. for $\dot{\rho} = 0$, and set such state as the input state of the beam splitter. Thus, the input state in each of the beam splitter's arms is in general a mixed state so that the transformation must be carried away directly to the input's state operator.

Since each of the state operator's matrix elements may be described in the Fock state basis as $\rho_{ab,mn} = \langle a, b|\rho|m, n\rangle$, we have to obtain the transformation of every $|m, n\rangle$.

According to [16], the beam splitter transformation is given by:

$$\begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = B\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}B^\dagger, \tag{6}$$

where a_1 and a_2 (a_3 and a_4) are the input (output) annihilation operators, and B is the transformation operator given by,

$$B = \exp\left[\frac{\theta}{2}\left(a_1^\dagger a_2 e^{i\phi} - a_2^\dagger a_1 e^{-i\phi}\right)\right], \tag{7}$$

so that the transmission (t) and reflection (r) coefficients of the beam splitter are given by,

$$t = \cos\left(\frac{\theta}{2}\right), \tag{8}$$

$$r = \sin\left(\frac{\theta}{2}\right), \tag{9}$$

and ϕ is the phase difference between the reflected and transmitted fields. For a 50:50 beam splitter, $\phi = -\pi/2$ and $\theta = \pi/2$; and the transformation operator associated to the beam splitter is thus [19],

$$B = \exp\left[-i\frac{\pi}{4}\left(a_1^\dagger a_2 + a_2^\dagger a_1\right)\right], \tag{10}$$

so that the transmission and reflection coefficients are: $t = r = 1/\sqrt{2}$. In this sense, for a 50:50 beam splitter the transformation reduces to,

$$\begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \tag{11}$$

and the transformation due to the beam splitter of an output state given by $|m, n\rangle$ is the following,

$$|m\rangle_3|n\rangle_4 \to \sqrt{\frac{m!n!}{2^{m+n}}} i^{(m-n)} \sum_{k=0}^{m}\sum_{l=0}^{n} i^{(k-l)}(-1)^{m-k}$$

$$\times \frac{\sqrt{(n+k-l)!(m+l-k)!}}{k!l!(m-k)!(n-l)!}|n+k-l\rangle_1|m+l-k\rangle_2.$$

This equations yields to a transformation in the form,

$$\sigma_{ab,mn} = \sum c_{ij,kl}\rho_{ij,kl}, \tag{12}$$

where the $\sigma_{ij,kl}$ are the output state matrix elements, the $c_{ij,kl}$ are constants. In this way, we obtain the beam splitter's output state as a function of the input state.

Results

Following the directions in [18], we fixed the source parameters as $\omega_a - \omega_\sigma = 60g$, $\Omega = 10g$, and ω_L as in eq (5); and all the results presented in this paper were obtained using such parameters for both sources. Furthermore, as shown in Fig. 1, the most efficient single-photon source is obtained for γ_σ around 0,2g and γ_a around 0,001g; therefore, unless explicitly stated otherwise, those are the parameters use for the source 1 (reference source).

In Fig. 1, we also notice the effect of the dissipative rates, γ_a and γ_σ, on the mean photon number of the input states. The larger the γ_a is, the more vacuum-like is the obtained state. This makes sense, since the γ_a is associated to the cavity quality factor and the larger its value the worse the cavity is. In this way, large γ_a induce high photon leakage out of the cavity so the stationary state is almost a pure vacuum state. Moreover, setting γ_a constant, and varying the values of γ_σ, we see that the photon mean number is close to one for large γ_σ. This effect is explained by the fact that large values of γ_σ increases the number of photons inside the cavity, because the quantum dot decays at a high frequency.

On the other hand, once we have established the behaviour of the mean photon number of the sources as a function of the dissipative rates, we study the entanglement in the output state due to the beam splitter. In general, the density operator of the beam splitter's output state can't be written as a separable state, $\sum_i w_i \rho_i^3 \otimes \rho_i^4$, and therefore the state is said to be entangled. Nevertheless, at present there is no know method to compute the entanglement of states of dimension $N \times M$, and the usual measurements of entanglement, such as the negativity, can only detect the presence of entanglement.

To compute the output state's negativity, we perform a partial transposition of the state's density operator;

$$\sigma_{a\alpha;b\beta} \to \sigma^{T_2}_{a\beta;b\alpha}, \tag{13}$$

where σ is the density operator of the beam splitter's output state,

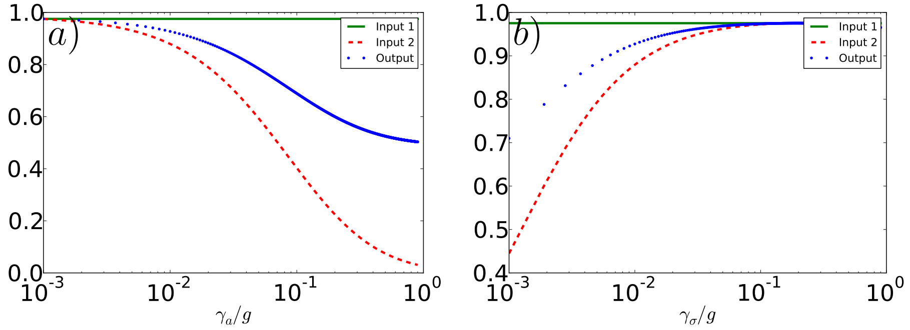

FIGURA 1. *(Color online). Mean photon number of the input 1 (green, straight line), input 2 (red, dashed line) and output (blue, dotted line) states as a function of: a) the cavity's photon leakage rate γ_a and b) the excitonic decay rate γ_σ of the source 2. The output photon mean number is calculated uniquely for a 50:50 beam splitter, in which the photon mean number for both the output 1 and output 2 are equal.*

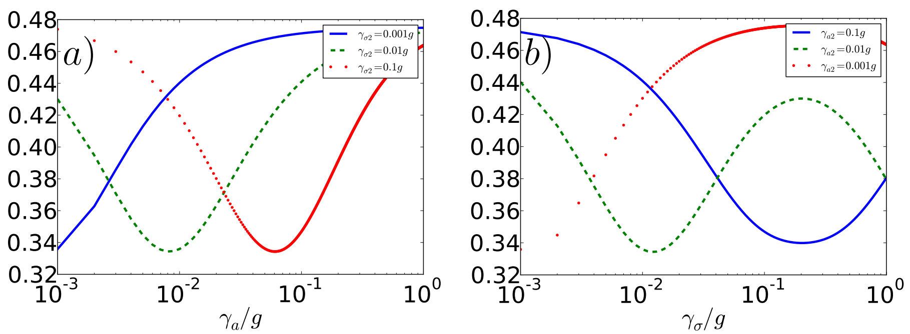

FIGURA 2. *(Color online). Negativity of the output state as a function of: a) γ_{a2} for three different $\gamma_{\sigma 2}$, and b) $\gamma_{\sigma 2}$ for three different γ_{a2}. In both cases, the negativity is maximum at the points for which $\gamma_{a2} = \gamma_{a1}$ and $\gamma_{\sigma 2} = \gamma_{\sigma 1}$.*

the latin indexes correspond to the system in the output arm associated to a_3, and the greek indexes correspond to the output arm associated to a_4. Then, we compute the negativity of the state σ as follows,

$$N(\sigma) = \frac{\|\sigma^{T_2}\|_1 - 1}{2}, \tag{14}$$

where $\|A\|_1 = Tr\left(\sqrt{A^\dagger A}\right)$ is the trace norm of the operator A.

In this way, we compute the negativity of the beam splitter's output state for different dissipative rates of the beam splitter's input states. In Fig. 2, we show the negativity behaviour as a function of: a) the photon leakage rate, and b) the excitonic decay rate of the source 2. In Figure 2.a we observe that the entanglement minimum shifts towards higher values of γ_a when the value of the excitonic decay rate increases. We also notice that the maximum negativity is reached when the photon leakage rate of the source two is equal to the one of the reference source ($\gamma_{a1} = 0{,}001g$ and $\gamma_{\sigma 2} = \gamma_{\sigma 1}$; red, dotted line), and when the value of $\gamma_{a2} \to g$, which yield to an entanglement between the reference source and a vacuum state, which for this study is not interesting.

Furthermore, in Figure 2.b we also observe that the negativity minimum shifts towards higher $\gamma_{\sigma 2}$ values, as the value of the photon leakage rate γ_{a2} reaches the value of the reference source. Nevertheless, we observe that the negativity maximum is only obtained when both the photon leakage rate and the excitonic decay rate of the two sources are equal (red, doted line).

Summary and conclusions

We studied the entanglement between two single photon sources due to the interaction via a semitransparent beam splitter. Each of the sources is characterized by the photon leakage rate (γ_a) and excitonic decay rate (γ_σ) of a semiconductor microcavity pumped coherently with a laser, and in which a quantum dot interacts with a mode of the electromagnetic mode. In this way, we established the range in which the source emit single photons: large values of γ_a yield to vacuum states, whereas large values of γ_σ yield to states containing approximately one photon. Then, we set the adequate combination of γ_a and γ_σ for which the source mean photon number is as close as possible to one, and took those values as a reference. Afterwards, we study the negativity associated to the beam splitter's output state as a function of both the photon leakage rate and excitonic decay rate of the second source, and we observed that the negativity maximum is reached when the two sources are equal.

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