DOI: HTTPS://DOI.ORG/10.15446/RCP.V34N1.105537

Development of Additive Reasoning in Young Children: The Case of Partitioning and the Successor Function[1]

ANA CRISTINA SANTANA ESPITIA
Escuela de Psicología, Universidad Pedagógica y Tecnológica de Colombia

YENNY OTÁLORA
Instituto de Investigaciones en Ciencias del Desarrollo, del Aprendizaje y Subjetividades CIDEAS, Facultad de Psicología, Universidad del Valle

HERNANDO TABORDA-OSORIO
Departamento de Psicología, Pontificia Universidad Javeriana

How to cite this article: Santana Espitia, A. C., Otálora, Y., and Taborda-Osorio, H. (2025). Development of additive reasoning in young children: The case of partitioning and the successor function. *Revista Colombiana de Psicología*, 34*(1)*, pp 93-110. https://revistas.unal.edu.co/index.php/psicologia/article/view/105537

Correspondence: Hernando Taborda-Osorio (https://orcid.org/0000-0001-6119-7642); Pontificia Universidad Javeriana; Bogotá, Colombia; Email: hernando.taborda@javeriana.edu.co

SCIENTIFIC RESEARCH ARTICLE

RECEIVED: OCTOBER 28^ST^, 2022–ACCEPTED: NOVEMBER 17^TH^, 2023

1 This research article is product of the research projects "Aprendizaje de la función de sucesión del sistema de números naturales en niños preescolares", funded by MINCIENCIAS (Código 80740-162-2020) awarded to the first author ACSE; "Influencias del desarrollo del razonamiento numérico y espacial en el aprendizaje de la matemática formal" funded by Universidad del Valle (CI: 5332; Convocatoria Interna 132-2021 para presentación de Proyectos de Investigación y Creación Artística en las ciencias, las artes, las humanidades, las tecnologías y la innovación Modalidad-2) awarded to the second author YO and "Innovación en intervención educativa para el desarrollo del pensamiento matemático en la primera infancia", funded por MINCIENCIAS, MEN and UNITED WAY (Código 1203-1003-72878, CT 517-2020) awarded to the third author HTO.

Development of Additive Reasoning in Young Children: The Case of Partitioning and the Successor Function

Abstract

Additive reasoning is a fundamental mathematical skill that children learn during elementary school. However, previous studies have suggested that children start their learning process in preschool. The current research aims to examine how two additive reasoning skills, successor function and partitioning, emerge in the preschool years. To this purpose, a group of 56 children of 4 and 5 years of age were tested on three additive tasks, a cardinality task, and a counting task. The results show a similar developmental trajectory for children's performance on the successor function task and the partitioning tasks, with significantly better performance in 5-year-olds. The results also show that children's cardinality knowledge and counting skills are good predictors in both additive reasoning tasks. These findings suggest that preschool is a critical period for learning the additive structure of the number system and that knowledge of verbal counting boosts this acquisition.

Keywords: preschool children, numerical knowledge, additive reasoning, successor function, partitioning, child development.

Desarrollo del razonamiento aditivo en niños pequeños: El caso de la partición y la función de sucesión

Resumen

El razonamiento aditivo es una habilidad matemática fundamental que los niños aprenden durante la escuela primaria. Sin embargo, estudios anteriores han sugerido que los niños inician su proceso de aprendizaje en preescolar. La presente investigación pretende examinar cómo emergen dos habilidades de razonamiento aditivo, la función de sucesión y la partición, en los años preescolares. Para ello, un grupo de 56 niños de 4 y 5 años fueron evaluados en tres tareas aditivas, una tarea de cardinalidad y una tarea de conteo. Los resultados muestran una trayectoria de desarrollo similar para el rendimiento de los niños en la tarea de función de sucesión y en las tareas de partición, con un rendimiento significativamente mejor en los niños de 5 años. Los resultados también muestran que el conocimiento de cardinalidad y las habilidades de conteo de los niños son buenos predictores en ambas tareas de razonamiento aditivo. Estos resultados sugieren que la etapa preescolar es un periodo crítico para el aprendizaje de la estructura aditiva del sistema numérico y que el conocimiento del conteo verbal potencia esta adquisición.

***Palabras clave*:** niños preescolares, conocimiento numérico, razonamiento aditivo, función de sucesión, partición, desarrollo infantil.

A central component of children's mathematical thinking is their understanding of additive reasoning, which is one of the main goals of early math education (Chamorro, 2005; Fuson, 2019; Gilmore, 2023). Additive reasoning is the representation of the number system in terms of part-whole relations and the use of different mathematical principles to organize this relationship, as commutativity and inversion (Canobi et al., 2002; Ching & Nunes, 2017). This logical organization is expressed in the base-10 system used to represent written quantities and to calculate the result of arithmetic operations. For instance, the number 32 can be decomposed in three units of 10s and two units of 1s, and thus the whole is represented as the result of the addition of several parts (Bower et al., 2022; Nunes & Bryant, 1996). The mental manipulation of numbers to solve basic arithmetic problems also requires various additive reasoning strategies. In this way, mathematical principles as commutativity and associativity allow solvers to flexibly change the positions of numbers and compose quantities to find out the results. The development of additive reasoning moves beyond these notions towards more complex situations (Robinson & Dubé, 2013); however, it is worth studying the first steps of this development during the preschool years to better understand how children advance in their construction of formal mathematical knowledge.

There are several additive concepts whose development can be traced back to the preschool years (Kullberg et al., 2020; Prather & Alibali, 2009; Resnick, 1989); for example, the principles of commutativity, associativity, and additive composition. Other two related concepts, namely the successor function and partitioning, are critical to understand the natural number system. In the current research, we explore the development of the successor function and partitioning. Particularly, partitioning is an ability closely related to additive composition (Saxton & Cakir, 2006). Below we present a summary of the study of the concepts of additive composition, partitioning and successor function.

Additive Composition and Partitioning

Additive composition is the principle that numbers are composed of other numbers (Bermejo, 2004; Rodríguez et al., 2008). Studies show that this knowledge develops gradually from the preschool years onward (Bower et al., 2022; Krebs et al., 2003; Kullberg et al., 2020; Medina, 2023). Two types of tasks are generally used to assess the acquisition and development of this principle: the "shop task" and simple addition tasks. In the shop task, children are asked to pay for an item with play coins with different values (e.g., 5-dollar coins and 1-dollar coins). Children reveal knowledge of additive composition when they are able to compose the target value using coins with different values instead of using only one-unit coins (Carraher et al., 1985; Ching & Kong, 2022). In the simple addition task, children are asked to add two numbers (e.g., 3 + 5) and the researcher observes the strategies children use to solve the task (Fuson, 1988). Younger children use a "counting-all" strategy, where they count the total units one by one, while older children use a "counting-on" strategy, where they start counting from an added. The more advanced "counting-on" strategy is associated with knowledge of additive composition, as children presumably realize that the total number is composed of both addends (Krebs et al., 2003; Nunes & Bryant, 1996; Obando & Vásquez, 2008).

Using the above tasks, studies have revealed that knowledge of additive composition emerges between 5 and 6 years old (Bower et al., 2022). In studies with Western populations, children typically pass the shop task at 6 years old, but not before (Krebs et al., 2003). However, in a recent study conducted with a Chinese population a basic ability to compose units of ones and fives was observed in 5-year-olds (Ching & Kong, 2022). The development of the "counting-on" strategy follows a similar trajectory, with 6-year-olds displaying a more robust knowledge of additive composition than younger children (Krebs et al., 2003).

Another strategy for studying the development of additive composition is by using a "conceptual judgment task" with concrete material (Canobi et al., 2002; Canobi et al., 2003; Sophian et al., 1995). In an example of this task, children observed a researcher handing out candy to two different puppets in a way consistent with the principle of additive composition (e.g., one puppet with two boxes of 3 and 2 candies each, and other puppet with a box of 3 and 2 candies). Then, children were asked whether both puppets had the same number of candies or not. Consistent with previous findings, the results of this study showed good scores for 5-and 6-year-old children, but relatively low scores for 4-year-olds (Canobi et al., 2002).

Finally, children also have been tested on partitioning tasks (Saxton & Cakir, 2006; Kullberg & Björkund, 2020). Partitioning is the ability to divide any natural number into different subsets. Crucial to this ability is the understanding that the cardinal of the whole is equivalent to the cardinal of the subsets (Saxton & Cakir, 2006). In the partitioning task children are asked to count and report the sum total of a set of objects (e.g., 6 cubes). Then, the set is divided into two separate groups (e.g., 3 and 3 cubes) and children are again asked to report the total number of objects (divided-whole task). The task can also be presented in the reverse order, from two separate groups to one set (united-parts task). If the children provide the answer quickly without hesitation or by counting the whole set again, they are credited with knowledge of number partitioning. In this research (Saxton & Cakir, 2006), most of the 6-year-old children completed the partitioning task successfully. Partitioning is a type of knowledge related to additive composition, since both involve reasoning about part-whole relationships in the numerical domain. This ability has also been proposed to be a predictor in the acquisition of base-10 system knowledge (Saxton & Cakir, 2006).

Successor Function

The successor function has been proposed as one of the main logical foundations of the natural number system (Badiou, 2008; Buijsman, 2020). Accordingly, some researchers have investigated the emergence of this additive notion in childhood (Carey, 2009; Schneider et al., 2020; Schneider et al., 2021a). One of the main tasks to assess the acquisition and development of the successor function is the Unit Task (Sarnecka & Carey, 2008; Schneider et al., 2021b), where children observe a number of objects that are placed inside an opaque box, and after children name the total (e.g., N = 5 objects) they observe one or two other objects being placed inside the same box. Once the children observe this sequence, they are asked the critical question about the total number of objects in the box (N+1 or N+2). Presumably, children who answer this question correctly know that adding one unit to a previous numerical value represents a proportional increase of one unit in the numeral list. The findings show that children solve this task at the age of 4 years, only when they have mastered the cardinal principle, that is, when they know that the last number label in a counting sequence represents the total amount of objects in the set (Piantadosi et al., 2014; Spaepen et al., 2018).

However, recent studies have shown that the acquisition of the successor function is a more gradual process that depends on the counting range of children or how high they can count (Cheung et al., 2017; Davidson et al., 2012; Schneider et al., 2020; Schneider et al., 2021a, 2021b). The higher the counting range, the better the children's performance on the Unit Task. Thus, for instance, preschoolers who are medium counters (e. g., count well to 20) perform well on the Unit Task when the numbers are small, but at the same time perform poorly when the numbers are larger. This pattern of results has led some researchers to propose that the full acquisition of the successor function is a protracted process that extends through the preschool years as children improve their counting range (Cheung et al., 2017; Cho et al., 2024). This same protracted acquisition has been observed in various cultures, but with different patterns of development, revealing that the particular counting system of each language may have an important influence on the acquisition of the successor function (Guerrero et al., 2020; Guerrero & Park, 2023; Schneider et al., 2020; Schneider et al., 2021b).

The Current Research

The literature review shows that the development of additive reasoning begins early in the preschool years. Additionally, studies on the development of the successor function show that the emergence of this notion depends both on the previous mastering of the cardinal principle and on the children's proficiency in counting (Cho et al., 2024; Sullivan et al., 2023; Wege et al., 2023). However, it is unknown how other additive notions emerge in the preschool years and whether the same knowledge of cardinality and counting skills have an effect on the development of these notions. The basic concept of partitioning seems to be already established by the age of 6 (Saxton & Cakir, 2006), but to our knowledge, no studies have shown its prior development. This is important as partitioning may be the earliest notion related to additive composition and its understanding can open the way to more complex numerical representations.

Accordingly, this study has three goals. First, to determine at what point in development children exhibit knowledge of both number partitioning and the successor function. Second, to investigate the possible role of children's knowledge of the cardinal principle and counting ability in the developmental trajectory of both number partitioning and the successor function. Three, to examine the relationship between the emergence of both additive notions, number partitioning and the successor function. Previous studies have suggested that the successor function is the conceptual core of children's understanding of the natural number system, and thus could be a precondition for learning other additive notions as number partitioning. Therefore, one possibility is that the development of the successor function predates the development of number partitioning.

To address these goals, we asked a group of 4- and 5-year-old children to solve three additive tasks: The Unit Task to measure children's knowledge of the successor function, the Divided-Whole Task, and the United-Parts Task to measure children's knowledge of number partitioning. All three tasks were preceded by a Cardinality Task and a Counting Task. The purpose of the Counting Task (Davidson et al., 2012) was both to identify the highest number correctly counted and to classify children into one of three counting ranges. The Cardinality Task was used to classify the children into two categories: knowers and no knowers of the cardinality principle. This assessment allowed us to determine the possible influence of the children's knowledge of cardinality on the development of both additive notions: partitioning and the successor function.

Importantly, this research was carried out during the Covid-19 pandemic and data collection took place amid the lockdown introduced across the country by the Colombian government in 2020 and 2021. Therefore, all children were tested online through specially designed tasks. The limitations associated with this contingency are discussed in the final section of this paper.

Method

Context and Participants

Study participants (n = 56; 24 boys and 32 girls) were 4 and 5 years old at the time of application, 28 children for each group (M= 60.36 months, SD= 6.76, Range 48-71 months). The sex distribution by age was as follows: Of the 28 4-year-old children, 14 were boys (50%) and 14 were girls (50%). Of the 28 5-year-old children, 10 were boys (37.7%) and 18 were girls (64.3%). Other 21 children were removed from the study due to parental interference during the testing session. All participants spoke Spanish as their first language.

Due to the health contingencies derived from the Covid-19 pandemic, a convenience sampling was carried out in which the invitation to participate in the research was made through direct contact with schools and kindergartens in Bogota, Colombia, and surrounding municipalities. All the institutions serve children from middle-income families. Parents interested in their children participating in the study completed a Google Forms registration form. Subsequently, a virtual session was held with the parents in which they were informed about the nature of the research and a verbal consent was read out to them, who in turn authorized their child's participation in the research. In addition, each child gave their informed assent. Both informed consent and informed assent were recorded on video. The ethical approval of the study was obtained from the Institutional Review Committee for Ethics in Research of the Pontificia Universidad Javeriana.

Measures and Data Collection

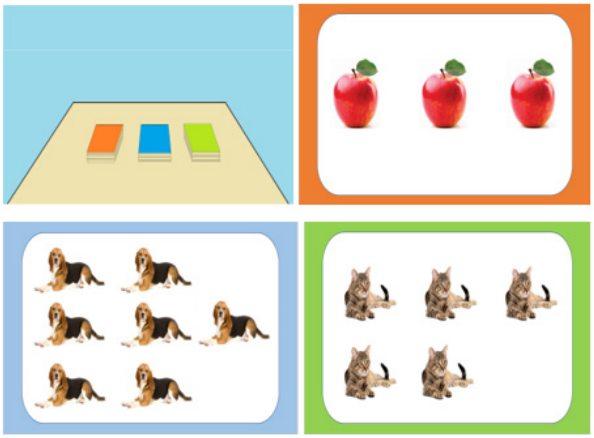
Highest Count Task

To identify the highest number correctly counted, children were asked: "Can you count as high as you can for me?" The experimenter registered counting errors, as omissions (e.g., "14, 15, 17") and cyclical repetitions (e.g., "29, 30, 21, 22"). The child was allowed to make a maximum of two errors in the application (not necessarily consecutive). On making the second counting error the child was asked to stop and the number counted before the error was recorded as the largest number counted. If the child reached 50 without making more than two errors, he/she was asked to stop counting. The counting ranges were defined as follows: 10-19 (low), 20-29 (medium), and 30-50 (high).

Cardinality Task

To assess children's understanding of cardinality, we adapted the "What's on this card?" (WOC) task from Gelman (1993) and Le Corre et al., (2006). Previous studies have shown the reliability of this task to assess children's knowledge of cardinality (Le Corre et al., 2006). The task was presented in PowerPoint slides and Adobe Animate. The children were presented with three decks of cards of various colors. The cards in each deck had different images: apples, dogs, and cats (see Figure 1). The researcher presented each card one by one and asked the child: "What is on this card?" The apple cards had 1 to 8 items and the dog and cat cards had 5 to 8 items. A correct response was scored when children said the correct number of items on the card. For all participants, the cards with the apples were presented first and in numerical ascending order, while the presentation order of the other cards were randomized in each deck.

Figure 1. *Material used in the Cardinality Task.*

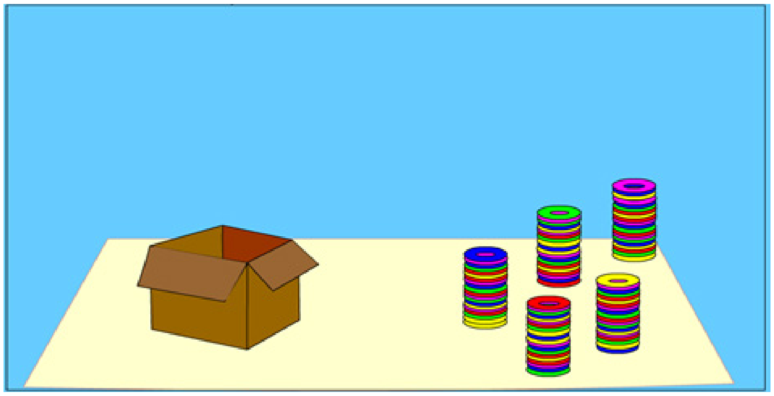


The application of the card game was stopped when the child made three consecutive errors (e.g., responses as "apples", "I don't know", or wrong numbers). The total score of the cardinality task was calculated based on the children's performance across the three decks in the numbers 5 to 8, for a total of 12 cards. If the children answered at least 8 of 12 questions correctly (66%), they were granted with cardinality knowledge.

Unit Task

The Unit Task was adapted from Davidson et al. (2012) and was set up in Adobe Animate. It was designed to assess children's understanding of the successor function. The task setup was presented on a blue background and a pale-yellow table in the foreground (see Figure 2). An opaque empty box and five stacks of colored rings were presented on the table.

Figure 2. *Unit Task Interface in Adobe Animate.*



At the beginning of each trial, the child was told that N rings will be placed in the box, then the video shows the corresponding N rings moving inside the box, all at once. After that, the experimenter asked a reminder question: "How many rings are in the box?" If the child answered incorrectly or failed to respond, the experimenter repeated the previous procedure by replaying the recording. Then, the child was told "Right! Now watch", as the experimenter added one or two more rings to the box. After that, the child was asked the critical question: "Now, how many rings are in the box in total, N or N+1 (or N+2)?" Once the child answered the question, the box was removed from the table in the video and a new box of a different color appeared from top to bottom, so that the child observed that the new box was empty.

All participants started with two familiarization trials (1+1 and 1+2). Afterwards, three conditions were used in the Unit Task: Small Numbers (all numbers in the arithmetic operation were in the 1-10 range), Medium Numbers (all numbers were in the 11-20 range), and Large Numbers (all numbers were in the 21-30 range). Four items were used for each condition: 4+1, 4+2, 5+1, and 5+2 for Small, 14+1, 14+2, 15+1, and 15+2 for Medium and 24+1, 24+2, 25+1, and 25+2 for Large. The number of trials administered varied according to the counting range of the participants. Consequently, children with a low counting range (Low Counters) were given only the 4 trials of the Small Numbers condition; children with a medium counting range (Medium Counters) were given the trials of the Small and Medium Numbers for a total of 8 trials; and children with a high counting range (High Counters) were given the trials of the Small, Medium, and Large Numbers for a total of 12 trials. The children who did not pass the Cardinality task were given only trials of the Small Numbers condition. The order in which the choice alternatives (N+1 or N+2) were presented was counterbalanced across trials. The total score of this task was calculated by counting the number of correct trials.

Partitioning Tasks

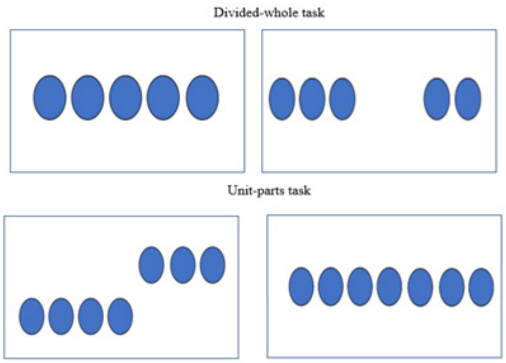
The partitioning tasks were adapted from Saxton and Cakir (2006) and they were set up in PowerPoint. In the Divided-whole task, children were shown a set of cubes on the screen and were asked to count and report the sum total. Once children reported the correct response, the cubes were divided into two sets through an animation video. Children were then again asked to report the sum total of cubes across the two groups ("please, tell me how many cubes are there in total all together now?"). The procedure for the United-part task was very similar to the previous task, except that children were first shown two separate sets of cubes on the screen and then both were united in a single set (see Figure 3 for examples of these tasks). Following Saxton and Cakir (2006), knowledge of partitioning was attributed when the child reported the correct answer without hesitation or by counting the cubes again. These tasks were applied in a counterbalanced manner across participants.

Two familiarization tasks were first presented to the children: one for the Divided-whole task, 3 (2 + 1), and another for the United-parts task, (2 + 2) 4. Afterwards, and as in the Unit Task, three conditions were used in the Partitioning tasks: Small Numbers (all numbers in the arithmetic operation were in the 1-10 range), Medium Numbers (all numbers were in the 11-20 range), and Large Numbers (all numbers were in the 21-30 range). Four items were used for each condition, two for the Divided-whole task and two for the United-parts task. The complete set of items is presented in Table 1. The number of trials administered varied according to the counting range of the participants in the same manner as in the Unit Task. The children who did not pass the Cardinality task were given only trials of the Small Numbers condition.

Table 1
Partitioning tasks according to count range

Condition	Item	Partitioning type	Stimuli
Familiarization	(3) 2+1	Divided-whole task	Orange cubes
	2+2 (4)	United-parts task	
Small	(5) 3+2	Divided-whole task	Blue circles
	(9) 6+3	Divided-whole task	
	4+3 (7)	United-parts task	
	7+2 (9)	United-parts task	
Medium	(14) 9+5	Divided-whole task	Red hearts
	(16) 12+4	Divided-whole task	
	11+6 (17)	United-parts task	
	13 + 2 (15)	United-parts task	
Large	(21) 15+6	Divided-whole task	Green triangles
	(25) 18+7	Divided-whole task	
	17+5 (22)	United-parts task	
	19 + 4 (23)	United-parts task	

Figure 3. *Selected sample of partitioning tasks.*



Procedure

The five tasks were delivered in two online sessions through the Zoom platform, each one lasting between 15-20 minutes. We set up test sessions based on previous online research studies (Johnston et al., 2019). Each child was accompanied by a parent during the sessions. At the beginning of each session, the parents were told to sit out of sight of the children and not to interfere during the tests. Then, the children were asked to sit in front of the computer screen at a comfortable distance to start the session. Before starting, we check the quality of sound and picture by asking the children several control questions. Once both the researcher and the participant were comfortable with the setup, the experiment was started.

Three tasks were delivered in the first session and two in the second session. Both sessions were recorded on video. In the first session, all children first completed the Highest Count Task and then the Cardinality Task. For half of the randomly selected participants, the third task of the first session was the Unit Task and for the other half were the Partitioning Tasks. In the second session, the children were given the missing task, either the Unit Task or the Partitioning Tasks. At the end of each session, the children rated on a 5-point Likert scale the degree of interest in the activities, and the quality of both the video and the sound. The children reported scores above 4 in the 3 aspects evaluated in both sessions. Therefore, they considered the tasks interesting and with good video and sound quality in both sessions.

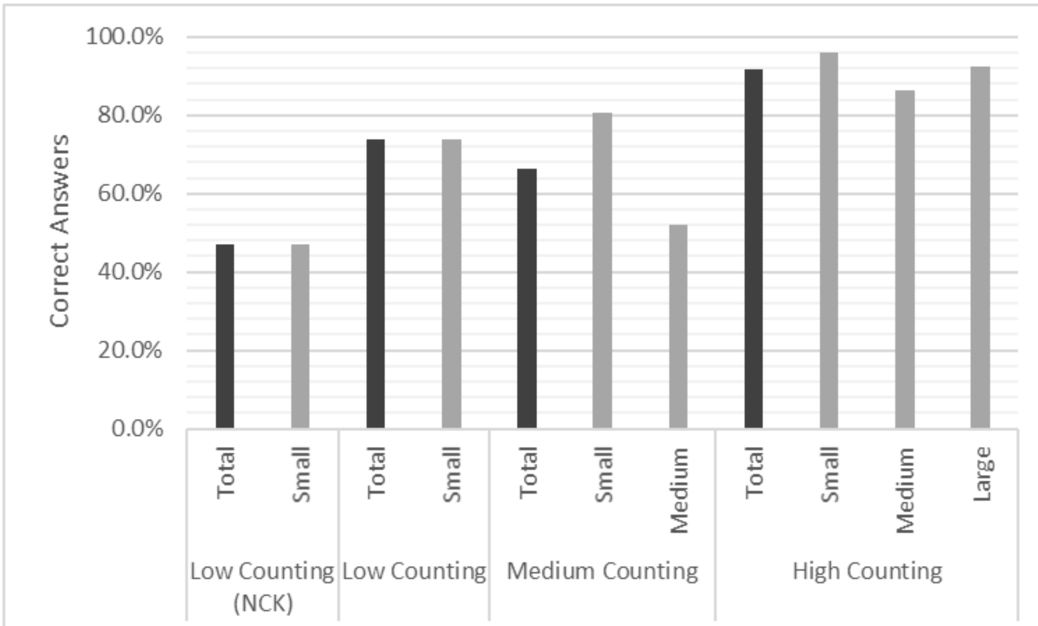
Results

The results show that out of 56 children, a total of 9 failed to pass the Cardinality Task (No Cardinality Knowledge group, NCK). Of the other 47 children, 21 were Low Counters (M = 14.5, SD = 3.8), 13 Medium Counters (M = 24.5, SD = 4.4), and 13 High Counters (M = 42.3, SD = 9.1). Following Davidson et al., (2012), the children's scores in both the Unit Task and The Partitioning Task were transformed into percentages for statistical testing. Preliminary analyses found no effect of Sex or counterbalance order; therefore, these variables were collapsed in further analyses. Below we present the results of the children's performance in each of the tasks.

Unit Task

The results shown in Figure 4 are discriminated by the three counting groups (Low Counters, Medium Counters, and High Counters) in the three numerical conditions (Small Numbers, Medium Numbers, and Large Numbers). To determine whether the children responded to this task by chance, we conducted a one-sample t test with 50 as the test value. The results with the percentages of success in the Small Numbers condition show that children's performance was significantly beyond chance (all *ps* $< .01$), except for those who did not pass the Cardinality Task, 47%, $t(8) = -0.26$, $p = 0.8$. Furthermore, the performance of children who failed the cardinality task (NCK) was significantly lower compared to children in the Low Counting group, $t(28) = -2.36$, $p = .02$. These results suggest that knowledge of the cardinal principle seems to be a cornerstone of the acquisition of the successor function.

Figure 4. *Percentage of correct answers in the Unit Task for High, Medium, and Low counters in the Small, Medium, and Large number conditions.*



A one-way analysis of variance (ANOVA) was performed on three counting groups to identify possible differences in knowledge of the successor function. First, we performed this analysis on Small Numbers across all three counting groups, and then on Medium Numbers between the Medium Counting and High Counting groups. The results for Small Numbers yielded a significant effect of counting range, $F(2, 44) = 4.3$, $p = .02$. Post hoc multiple comparisons with the Bonferroni test revealed significant differences only between the Low Counters and High Counters ($p = .01$). The results for the Medium Numbers also yielded a significant effect of counting range, $t(24) = -2.83$, $p < .01$. Thus, in general, the children who performed best on the counting task differed significantly from all other children on the Unit Task.

To determine whether knowledge of the successor function was homogeneous across the number conditions, we conducted comparisons between Small Numbers and Medium Numbers for the Medium Counting range and between Small, Medium, and Large Numbers conditions for the High Counting range. These analyses showed significant differences for the Medium Counting range, $t(12) = 2.4$, $p = .03$, but not for the High Counting range, $F(2, 24) = 1.66$, $p = .21$. These results show that the performance of children with the highest counting skills was homogeneous in all trials in the Unit Task. In contrast, knowledge of the successor function in the Medium Counting range appears to be less consolidated and specific to small numbers.

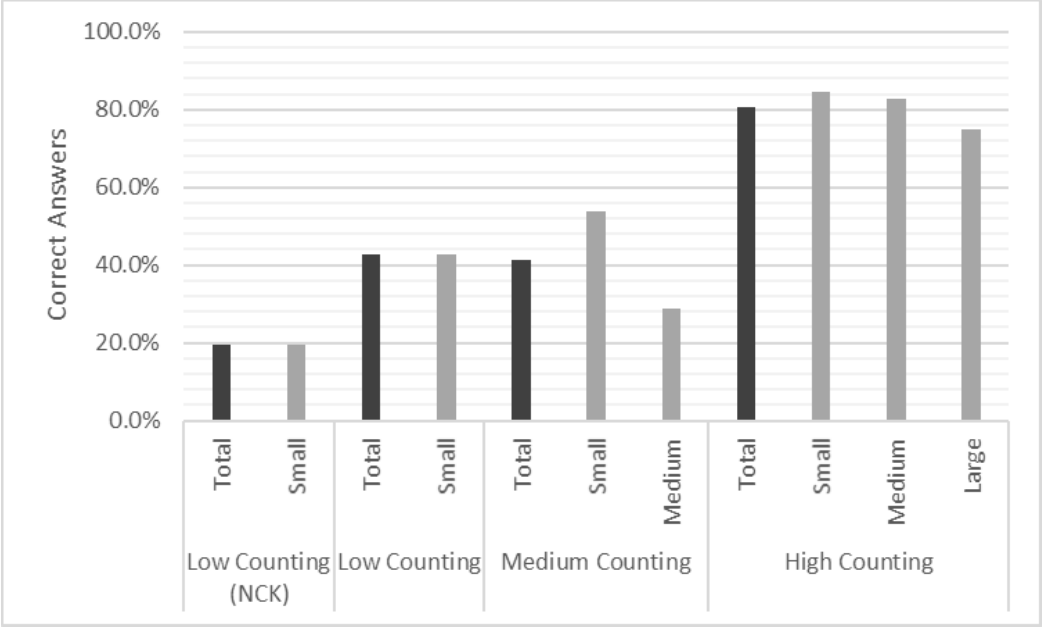
Finally, we examined the effect of age and counting on the children's performance in the Unit Task. The comparison between 4- and 5-year-old children revealed a statistically significant difference, $t(54) = -4.58$, $p < .01$, with 5-year-olds performing better than younger children (M = 85.5%, SD = 17.9 and M = 58.3%, SD = 25.7, respectively). A bivariate correlation between the performance on the Unit Task and the Counting Task shows a statistically significant association, $r = 0.4$, $p < .01$. This association partially holds out when controlling for age in months, $r = 0.26$, $p = .05$, revealing that counting ability is a good predictor of the children's performance in the Unit Task.

Partitioning Task

Figure 5 shows the results of the Partitioning Tasks in percentages, combining the results for both the Divided-Whole Task and the United-Parts Task. Previous analyses did not reveal significant differences between both tasks. As before, the results are discriminated by the three counting ranges: Low Counters, Medium Counters, and High Counters. A one-way analysis of variance (ANOVA) was performed to identify possible differences in knowledge of the successor function. First, we conducted this analysis on Small Numbers across all three counting groups, and then on Medium Numbers between the Medium Counting and High Counting groups. The results for Small Numbers yielded a significant effect of counting range, $F(2, 44) = 7.47, p < .01$. Post hoc multiple comparisons with the Bonferroni test revealed significant differences between the Low Counters and High Counters ($p < .01$), and between Medium Counters and High Counters ($p = .04$). The results for Medium Numbers also yielded a significant effect of counting range, $t(24) = -4.22$, p < .01. As in the Unit Task, the children who performed best on the counting task differed significantly from all other children on the Partitioning Task. Visual inspection of Figure 5 also shows a poor performance of the children who did not pass the cardinality task (NCK), their performance was significantly lower compared to children in the Low Counting condition, $t(25.2) = -2.21, p = .03$.

To determine whether knowledge of number partitioning was homogeneous across the number conditions, we performed comparisons between Small Numbers and Medium Numbers for the Medium Counting range and among Small, Medium, and Large Numbers conditions for the High Counting range. These analyses showed significant differences for the Medium Counting range, $t(12) = 3.12, p < .01$, but not for the High Counting range, $F(2, 24) = 0.95, p = .4$. Similar to children's performance on the Unit Task, these results show that performance for the children with the highest counting skills was homogeneous across trials in the Partitioning Task. Knowledge of number partitioning in the Medium Counting range seems to be less consolidated and specific to small numbers.

Figure 5. *Percentage of correct answers in the Partitioning Task for High, Medium, and Low counters in the Small, Medium, and Large number conditions.*



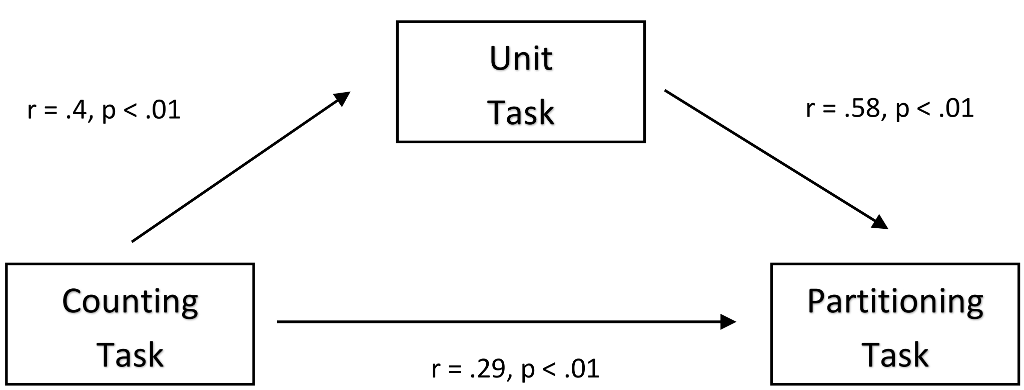
Next, we examined the effect of age and counting on the children's performance on the Partitioning Task. The comparison between 4- and 5-year-old children revealed a statistically significant difference, $t(54) = -3.64$, $p < .01$, with 5-year-olds performing better than younger children (M = 62.9%, SD = 33.9 and M = 32.1%, SD = 29.1, respectively). A bivariate correlation between the performance on the Partitioning Task and the Counting Task shows a statistically significant association, $r = 0.48$, $p < .01$. This association holds out when controlling for age in months, $r = 0.39$, $p < .01$, revealing that counting ability is also a good predictor of the children's performance in the Partitioning Task.

Relations Between the Unit Task and Partitioning Task

We compared children's performance between the Unit Task and the Partitioning Task across all children's Counting Ranges. The children's performance in the High Counting range did not reveal significant differences between both tasks (all *ps* > .05, controlling for multiple comparisons). In contrast, the children's performance in the Medium and Low Counting ranges did reveal significant differences in both Small, $t(20) = 4.65$, $p < .01$ for Low Counting; $t(12) = 2.8$, p = .03 for Medium Counting, and Medium Numbers conditions, $t(12) = 2.74$, $p = .03$. In all of these comparisons, children performed better on the Unit Task than on the Partitioning Task. These analyses reveal, in general, that only children with good counting skills performed similarly on the Unit and Partitioning Tasks, whereas children with lower counting skills show poor performance on the Partitioning task in all conditions.

The previous analyses show that most preschoolers in our sample still struggle with partitioning knowledge. However, there is a clear improvement over time. To find out the factors that can lead to a better performance in the Partitioning task, we conducted a mediation analysis to estimate whether the performance in the Unit Task mediates the association between the performance in the Counting Task and the Partitioning task (see Figure 6). The total effect of the Counting task on the Partitioning task had a $B = 0.48$, $p < .01$, [bootstrap confidence interval of 0.2 – 0.4]. When controlling for performance in the Unit Task the Beta was lower, but still significant, $B = 0.29$, $p = .01$ [bootstrap confidence interval of 0.1 – 0.3]. However, the indirect effect, although small, was statistically significant, $B = 0.19$, $p = .04$, [bootstrap confidence interval of 0.09 – 0.3]. These results suggest that the children's performance in the Unit Task partially mediates the association between the Counting Task and the Partitioning Task.

Figure 6. *Mediation analysis between Counting and Partitioning Task.*



Discussion

In the current study, 4- and 5-year-olds were tested on various numerical tasks to examine the developmental trajectory of early additive reasoning. The results show three main findings. First, both additive notions, successor function and partitioning, emerge throughout the preschool years with similar trajectories. Five-year-old children performed better on both tasks than four-year-olds. Moreover, the children's performance varies across the counting ranges of the additive tasks. The children with a high counting range performed significantly better on both tasks than all other children. However, the children with a medium and small counting range showed a similar performance in both additive tasks. To our knowledge, this is the first study to trace the developmental trajectory of number partitioning understanding in the preschool years and compares it to another additive notion.

The second finding is that both cardinality and counting knowledge have a strong effect on the emergence of additive reasoning. The children who failed to show cardinality knowledge performed poorly overall on both additive tasks. This finding suggests that cardinality is critical in the development of children's additive reasoning. This is in agreement with previous studies, where knowledge of cardinality has been shown to predict children's understanding of the successor function (Guerrero & Park, 2023; Schneider et al., 2021a, 2021b; Spaepen et al., 2018). Additionally, the strong correlation between counting proficiency and performance on both additive tasks suggests that the knowledge children learn from counting logic may contribute to the development of additive reasoning. This conclusion is reinforced by the fact that only children who are high performers in counting differ significantly from all other children.

Third, the strong correlation between both additive tasks suggests that there is a common underlying notion of addition that develops between 4 and 5 years old. However, the successor function seems to be an easier notion to acquire for children than the partitioning knowledge. Even in the group with the highest counting range there is an important number of trials in the partitioning tasks that children fail, while the number of trials in the Unit Task that children fail is low. Indeed, mediation analysis shows that knowledge of the successor function partially mediates the acquisition of number partitioning. Thus, although both tasks tap into a similar concept, they may differ regarding task complexity.

The results of the current study with the Unit Task replicate previous findings reported in other countries (Cheung et al., 2017; Davidson et al., 2012). Results in English language show that children with low and medium counting range have a mixed performance on the Unit Task. This pattern of results suggests that children who still struggle with the counting task have item-based knowledge of the successor function, rather than knowledge of an abstract general principle (Davidson et al., 2012). The current study goes further by showing that the developmental trajectory of the successor function could be generalized to other additive notions, such as number partitioning. Therefore, this research suggests that what is developing in the preschool years is a deeper understanding of the additive structure of natural numbers.

An open question is how this understanding develops and why counting proficiency is a good predictor of children's additive reasoning. One possibility suggested by others (Schneider et al., 2020), is that the learning of the productive linguistic rules of verbal counting may lead children to infer the N + 1 rule that governs the logic of the natural numbers. Thus, because the verbal counting system increases in units one by one (in Spanish: veintidós, veintitres, veinticuatro), it may allow children over time to infer that the N + 1 rule is a general principle of the natural number system. This hypothesis has been tested by studying children with different languages, some more opaque than others in the verbal counting (Schneider et al., 2020). Overall, the more opaque the verbal counting, the longer it takes children to pass the Unit Task. However, the fact that counting is also a good predictor of number partitioning suggests that what children also learn from the verbal counting is that numbers are composed of other numbers. Both additive tasks, number partitioning and the successor function, shared this common notion. A critical step in this learning could be the acquisition of knowledge of the base-10 system. Children with the highest counting range may understand this knowledge better (Guerrero et al., 2020; Schneider et al., 2021a, 2021b; Wege, 2023), and thus, they could be better prepared to understand the additive nature of numbers.

Future studies should address the development of additive reasoning in a more comprehensive way by adding other types of part-whole relationships, as commutativity and associativity (Eaves et al., 2021; Guerrero & Park, 2023; Wege et al., 2020). Likewise, it is important to study the developmental consequences of this early understanding of additive knowledge on children's formal mathematics learning throughout primary school. Several studies have shown the significant effect that early mathematics has 2 to 4 years later on children's mathematical performance (Hirsch et al., 2018; Jordan et al., 2009). However, the long-term effect of early additive knowledge is not well understood (see Ching & Kong, 2022).

Educational Implications

Important educational implications could be drawn from the findings of the current study in order to inform practice and policy at the preschool level. Typically, kindergarten mathematical activities tend to focus mainly on rote counting (Solovieva et al., 2022). Even when tasks on counting small sets of objects are used, the prompts emphasize one-to-one correspondence among objects and the verbal counting sequence. However, the current study shows that between 4 and 5 years of age is a critical period for learning additive notions of the natural number system. Therefore, preschool curricula should include a wide variety of additive composition tasks to foster this learning further (Zúñiga, 2014). Additive reasoning is critical for success in school mathematics and is required for good performance in STEM disciplines. For instance, Geary et al. (2013) found that early numeracy skills, including additive reasoning, predicted math performance in 13-years-olds. Thus, instructors should pay more attention to the development of additive reasoning skills in preschool children (Björklund et al., 2021).

The findings of the current research also show an association between counting skills and children's additive reasoning skills. However, it leaves in the dark what drives this association. A reasonable hypothesis presented above is related to how children can begin to learn the additive structure of natural numbers from the logic of the verbal counting system; therefore, at the educational level, it is important for instructors to develop activities that can promote the understanding of this additive logic (Alsina, 2022; Flores et al., 2024; Santana et al., 2018). For example, instructors could implement additive composition activities as finding all the *partners* that form a number or forming a number from two partners, through varied representations of number and quantities (numerical words, visual resources, math symbols) (Fuson, 2019). Diverse studies have shown that young children from different economic and cultural backgrounds can participate in and profit from early instructional activities centered on mathematical abilities (Clements et al., 2007; Dyson et al., 2015; Fuson et al., 1997; Siegler & Ramani, 2008). Therefore, young children at the kindergarten level could benefit from activities that allow them to comprehend the N+1 rule that governs the number system along with activities where they have to compose numbers from other numbers and decompose them back.

Limitations

As a result of the restrictions related to the Covid-19 pandemic, the test sessions were carried out through online platforms. Under these circumstances it is difficult to control for environmental factors and parental interference. In this study, we were very careful to avoid parental interference, but this increased the dropout rate of participants. Likewise, in relation to the test modality, all the numerical tasks were delivered through on-screen animations with non-manipulable objects. This could make it difficult for children to understand the mathematical situation presented in each task. However, it is worth noting the similarity between the current pattern of results on the Unit Task and previous studies. This could suggest that the children in our study had an appropriate understanding of the tasks. Finally, we believe that this research contributes to current efforts to develop online research strategies, as some potential advantages have been identified (Sheskin, et al., 2020). For example, online testing contributes to more diverse samples and can encourage family participation for longer periods and more sessions. Therefore, more effort should be devoted to studying the implications of online testing and developing tools to improve the efficiency of data collection.

References

Alsina, A. (2022). *Itinerarios didácticos para la enseñanza de las matemáticas (3-6 años).* GRAÓ.

Badiou, A. (2008). *Number and Numbers*. Polity Press.

Bermejo, V. (2004). *Cómo enseñar matemáticas para aprender mejor.* CCS.

Björklund, C., Ekdahl, A. L., & Kempe, U. R. (2021). Implementing a structural approach in preschool number activities. Principles of an intervention program reflected in learning. *Mathematical Thinking and Learning*, *23*(1), 72-94, https://doi.org/10.1080/10986065.2020.1756027

Bower, C. A., Mix, K. S., Yuan, L., & Smith, L. B. (2022). A network analysis of children's emerging place-value concepts. *Psychological Science, 33*(7), 1112-1127. https://doi.org/10.1177/09567976211070242

Buijsman, S. (2020). Two roads to the successor axiom. *Synthese, 197,* 1241–1261. https://doi.org/10.1007/s11229-018-1752-5

Canobi, K. H., Reeve, R. A., & Pattison, P. E. (2002). Young children's understanding of addition concepts. *Educational Psychology, 22*(5), 513-532. https://doi.org/10.1080/0144341022000023608

Canobi, K. H., Reeve, R. A., & Pattison, P. E. (2003). Patterns of knowledge in children's addition. *Developmental Psychology, 39*(3), 521–534. https://doi.org/10.1037/0012-1649.39.3.521

Carey, S. (2009). *The Origin of Concepts.* Oxford University Press.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology, 3*, 21-29. https://doi.org/10.1111/j.2044-835X.1985.tb00951.x

Chamorro, M. (2005). *Didáctica de las Matemáticas para Educación Infantil.* Pearson Education.

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology, 92*, 22-36. https://doi.org/ 10.1016/j.cogpsych.2016.11.002

Ching, B. H. H., & Kong, K. H. C. (2022). Understanding additive composition is important for symbolic numerical magnitude processing. *Cognitive Development*, *62*, 101170. https://doi.org/10.1016/j.cogdev.2022.101170

Ching, B. H. H., & Nunes, T. (2017). Children's understanding of the commutativity and complement principles: A latent profile analysis. *Learning and Instruction*, *47*, 65-79. https://doi.org/10.1016/j.learninstruc.2016.10.008

Cho, H.Y., Vasilyeva, M., & Laski, E. V. (2024) Statistical learning and mathematics knowledge: the case of arithmetic principles. *Frontiers in Developmental Psychology, 2:1370028*. https://doi.org/10.3389/fdpys.2024.1370028

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education*, *38*(2), 136-163. https://doi.org/10.2307/30034954

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*(1), 162-173. doi: 10.1016/j.cognition.2011.12.013

Dyson, N., Jordan, N. C., Beliakoff, A., & Hasinger-Das, B. (2015). A kindergarten number-sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education, 46*(3), 331-370. doi: 10.5951/jresematheduc.46.3.0331

Eaves, J., Gilmore, C., & Attridge, N. (2021). Conceptual knowledge of the associativity principle: A review of the literature and an agenda for future research. *Trends in Neuroscience and Education, 23:100152.* doi: 10.1016/j.tine.2021.100152

Flores, P., Coelho, E., Mourão-Carvalhal, M.I. & Forte, P. (2024) Motor activities to improve maths performance in pre-school children with typical development. *Frontiers in Psychology, 15*:1332741. doi: 10.3389/fpsyg.2024.1332741

Fuson, K. C. (1988). *Children's counting and concepts of number.* Springer-Verlag Publishing.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 28(2), 130-162. https://doi.org/10.2307/749759

Fuson K. (2019). Relating math words, visual images, and math symbols for understanding and competence. *International Journal of Disability, Development and Education*, *66*(2) 119-132, DOI: 10.1080/1034912X.2018.1535109

Geary, D., Hoard, M., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLOS ONE 8*(1): e54651. https://doi.org/10.1371/journal.pone.0054651

Gelman, R. (1993). A rational-constructivist account of early learning about numbers and objects. In D. L. Medin (Ed.). *The psychology of learning and motivation* (pp. 61-96). Academic Press.

Gilmore, C. (2023). Understanding the complexities of mathematical cognition: A multi-level framework. *Quarterly Journal of Experimental Psychology. 76*(9), 1953-1972. doi: 10.1177/17470218231175325.

Guerrero, D., Hwang, J., Boutin, B., Roeper, T., & Park, J. (2020). Is thirty-two three tens and two ones? The embedded structure of cardinal numbers. *Cognition, 203*(3), 104331. https://doi.org/10.1016/j.cognition.2020.104331

Guerrero, D., & Park, J. (2023). Arithmetic thinking as the basis of children's generative number concepts. *Developmental Review, 67*: 101062. https://doi.org/10.1016/j.dr.2022.101062

Hirsch, S., Lambert, K., Coppens, K., & Moeller, K. (2018). Basic numerical competences in large-scale assessment data: Structure and long-term relevance. *Journal of Experimental Child Psychology, 167,* 32-48. https://doi.org/10.1016/j.jecp.2017.09.015

Johnston, A. M., Sheskin, M., & Keil, F. (2019). Learning the relevance of relevance and the trouble with truth: Evaluating explanatory relevance across childhood. *Journal of Cognition and Development, 20*(4), 552-572. https://doi.org/10.1080/15248372.2019.1631167

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental psychology, 45*(3), 850-867. https://doi.org/10.1037/a0014939

Krebs, G., Squire, S., & Bryant, P. (2003). Children's understanding of the additive composition of number and of the decimal structure: what is the relationship? *International Journal of Educational Research, 39*(7), 677-694. https://doi.org/10.1016/j.ijer.2004.10.003

Kullberg, A., & Björklund, C. (2020). Preschoolers' different ways of structuring part-part-whole relations with finger patterns when solving an arithmetic task. *ZDM Mathematics Education, 52*, 767–778. https://doi.org/10.1007/s11858-019-01119-8

Kullberg, A., Björklund, C., Brkovic, I., & Kempe, U. R. (2020). Effects of learning addition and subtraction in preschool by making the first ten numbers and their relations visible with finger patterns. *Educational Studies in Mathematics, 103*, 157-172. https://doi.org/10.1007/s10649-019-09927-1

Le Corre, M., Brannon, E. M., Van de Walle, G., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*(2), 130-169. https://doi.org/10.1016/j.cogpsych.2005.07.002

Medina, D. A. (2023). Comprensión del valor de posición: equivalencia numérica y composición aditiva como facilitadores en la escritura de numerales. *Avances en Psicología Latinoamericana, 41*(2), 1-23. https://doi.org/10.12804/revistas.urosario.edu.co/apl/a.9882

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Blackwell.

Obando, G., & Vásquez, N. (Octubre, 2008). *Pensamiento numérico del preescolar a la educación básica* [curso]. 2018. 9 Encuentro Colombiano de Matemática Educativa. Valledupar, Colombia. http://funes.uniandes.edu.co/933/1/1Cursos.pdf

Piantadosi, S. T., Jara-Ettinger, J., & Gibson, E. (2014). Children's learning of number words in an indigenous farming-foraging group. *Developmental Science*, *17*, 553-563. https://doi.org/10.1111/desc.12078

Prather, R. W., & Alibali, M. W. (2009). The development of arithmetic principle knowledge: How do we know what learners know? *Developmental Review, 29*(4), 221-248. https://doi.org/10.1016/j.dr.2009.09.001

Resnick, L. B. (1989). Developing mathematical knowledge. *American Psychologist, 44*(2), 162-169. https://doi.org/10.1037/0003-066X.44.2.162

Robinson, K. M., & Dubé, A. K. (2013). Children's additive concepts: Promoting understanding and the role of inhibition. *Learning and Individual Differences, 23*, 101-107. https://doi.org/10.1016/j.lindif.2012.07.016

Rodríguez, P., Lago, M., Caballero, S., Dopico, C., Jiménez, L., & Solbes, I. (2008). El desarrollo de las estrategias infantiles. Un estudio sobre el razonamiento aditivo y multiplicativo. *Anales de Psicología, 24*(2), 240-252. https://www.redalyc.org/pdf/167/16711589007.pdf

Santana-Espitia, A. C., Fajardo-Santamaría, J. A., & Herrera-Rojas, A. N. (2018). El aprendizaje situado de la adición y la sustracción. *Revista Latinoamericana de Etnomatemáticas, 11*(2), 98-119. https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/515/440

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108*(3), 662-674. https://doi.org/10.1016/j.cognition.2008.05.007

Saxton, M., & Cakir, K. (2006). Counting-on, trading and partitioning: Effects of training and prior knowledge on performance on base-10 tasks. *Child Development, 77*(3), 767-785. https://doi.org/10.1111/j.1467-8624.2006.00902.x

Schneider, R. M., Pankonin, A., Schachner, A., & Barner, D. (2021a). Starting small: Exploring the origins of successor function knowledge. *Developmental Science 24*(4):*e13091*. https://doi.org/10.1111/desc.13091

Schneider, R. M., Sullivan, J., Guo, K., & Barner, D. (2021b). What counts? Sources of knowledge in children's acquisition of the successor function. *Child Development, 92*(4):e476-e492. https://doi.org/10.1111/cdev.13524

Schneider, R. M., Sullivan, J., Marušič, F., Žaucer, R., Biswas, P., Mišmaš, P., Plesničar, V., & Barner, D. (2020). Do children use language structure to discover the recursive rules of counting? *Cognitive Psychology, 117*, 101263. https://doi.org/10.1016/j.cogpsych.2019.101263

Sheskin, M., Scott, K., Mills, C. M., Bergelson, E., Bonawitz, E., Spelke, E. S., Fei-Fei, L., Keil, F. C., Gweon, H., Tenenbaum, J. B., Jara-Ettinger, J., Adolph, K. E., Rhodes, M., Frank, M. C., Mehr, S. A., & Schulz, L. (2020). Online developmental science to foster innovation, access, and impact. *Trends in Cognitive Sciences, 24*(9), 675-678. https://doi.org/10.1016/j.tics.2020.06.004

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*, 655-661. https://doi.org/10.1111/j.1467-7687.2008.00714.x

Solovieva, Y., Baltazar-Ramos, A. M., Quintanar-Rojas, L., Escotto-Córdova, E. A., & Sidneva, A. (2022). Analysis of mathematics teaching programmes at preschool age based on activity theory. *Culture and Education*, *34*(1), 72-101. https://doi.org/10.1080/11356405.2021.2006910

Sophian, C., Harley, H., & Manos Martin, C. S. (1995). Relational and representational aspects of early number development. *Cognition and Instruction, 13*(2), 253–268. https://doi.org/10.1207/s1532690xci1302_4

Spaepen, E., Gunderson, E., Gibson, D., Goldin-Meadow, S., & Levine, S. (2018). Meaning before order: Cardinal principle knowledge predicts improvement in understanding the successor principle and exact ordering.

Cognition, 180, 59-81. https://doi.org/10.1016/j.cognition.2018.06.012

Sullivan, J., Cramer-Benjamin, S., Alvarez, J., & Barner, D. (2023). Everything is infinite: Children's beliefs about endless space, time, and number. Open Mind, *7*, 715–731. https://doi.org/10.1162/opmi_a_00104

Wege, T. E., Batchelor, S., Inglis, M., Mistry, H., & Schlimm, D. (2020). Iconicity in mathematical notation: Commutativity and symmetry. *Journal of Numerical Cognition, 6*(3), 378-392. https://doi.org/10.5964/jnc.v6i3.314

Wege, T. E., Gilmore, C., & Inglis, M. (2023, preprint). Children's early understanding of the successor function. https://doi.org/10.31234/osf.io/5aytf

Zúñiga, M. (2014). El aprendizaje de la descomposición aditiva en la educación infantil: Una propuesta para niños y niñas de 5 y 6 años. *EDMA 0-6: Educación Matemática en la Infancia, 3*(2), 84-113. https://revistas.uva.es/index.php/edmain/article/view/5842/4361