On the Fischer matrices of a group of shape $2^{1+2n}.G$

Sobre las matrices de Fischer de un grupo de la forma $2^{1+2n}.G$

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Abstract. In this paper, the Fischer matrices of the maximal subgroup $\overline{G} = 2^{1+8}: (U_4(2):2)$ of $U_6(2):2$ will be derived from the Fischer matrices of the quotient group $Q = \frac{2^{1+8}}{Z(2^{1+8})} \cong 2^8:(U_4(2):2)$, where $Z(2^{1+8})$ denotes the center of the extra-special 2-group $2^{1+8}$. Using this approach, the Fischer matrices and associated ordinary character table of $G$ are computed in an elegantly simple manner. This approach can be used to compute the ordinary character table of any split extension group of the form $2^{1+2n}:G$, $n \in \mathbb{N}$, provided the ordinary irreducible characters of $2^{1+2n}$ extend to ordinary irreducible characters of its inertia subgroups in $2^{1+2n}:G$ and also that the Fischer matrices $M(g_i)$ of the quotient group $\frac{2^{1+2n}:G}{Z(2^{1+2n})} \cong 2^{2n}:G$ are known for each class representative $g_i$ in $G$.

Key words and phrases. split extension, extra-special $p$-group, irreducible projective characters, Schur multiplier, inertia factor groups, Fischer matrices.

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Resumen. En este artículo, las matrices de Fischer del subgrupo maximal $\overline{G} = 2^{1+8}: (U_4(2):2)$ de $U_6(2):2$ serán derivadas a partir de las matrices de Fischer del grupo cociente $Q = \frac{2^{1+8}}{Z(2^{1+8})} \cong 2^8:(U_4(2):2)$, donde $Z(2^{1+8})$ denota el centro del grupo 2-extra especial $2^{1+8}$. Usando este enfoque, las matrices de Fischer y la tabla de caracteres asociadas de $\overline{G}$ son calculados de una manera elegante y simple. Este enfoque se puede utilizar para calcular la tabla de caracteres de cualquier extensión escindida de la forma $2^{1+2n}:G$, $n \in \mathbb{N}$, siempre y cuando los caracteres irreducibles ordinarios de $2^{1+2n}$ se extiendan a caracteres irreducibles ordinarios de sus subgrupos de inercia en $2^{1+2n}:G$. 

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1. Introduction

The maximal subgroup \( \overline{G} = 2^{1+8}:(U_4(2):2) \) (see [4]) of the automorphism group \( U_6(2):2 \) of the unitary simple group \( U_6(2) \) is a split extension of the extraspecial 2-group \( N = 2^{1+8} \) by \( G = U_4(2):2 \). The center \( Z(N) \equiv 2 \) is isomorphic to the cyclic group of order 2 and \( N_1 = \frac{N}{Z(N)} \equiv 2^8 \) can be considered as an eight-dimensional \( U_4(2) \)-module over the finite field \( GF(2) \). In fact, up to isomorphism \( 2^8 \) afforded the unique representation of \( U_4(2):2 \) of degree eight over \( GF(2) \) (see [9]).

Computing the table of marks within GAP it is noticed that there are 38 conjugacy classes of non-trivial subgroups of \( G \) having index less than 256. Hence \( G \) has 38 non-trivial subgroups \( G_i \), where the degree of each of the permutation characters \( \chi(G|G_i) \) of \( G \) acting on the classes of a subgroup \( G_i \) will be less than 256. Let \( \chi(G|N_1) \) be the permutation character of \( G \) acting on the non-trivial classes of \( N_1 = 2^8 \). Then \( \chi(G|N_1) \) will be the sum of some of these 38 permutation characters \( \chi(G|G_i) \) such that for any non-trivial \( g \in G \) it is required that \( \chi(G|N_1)(g) = 2^k - 1 \) for some \( k \in \{0, 1, 2, 3, 4, 5, 6, 7\} \), i.e. the number \( 2^k - 1 \) of elements of \( N_1 \) which is fixed by \( g \). Using Exercise 4.2.4 in [11], it can be shown that for any element \( g \) in the conjugacy class 5A of \( G \), we have that \( \chi(G|N_1)(g) = 0 \). Therefore, the only possibility for \( \chi(G|N_1) \) will be the sum of two permutation characters \( \chi(G|G_1) \) and \( \chi(G|G_2) \) with degrees of 120 and 135, respectively. Hence \( G \) has three orbits on \( N_1 \) of lengths 1, 120 and 135.

It is well known that \( N_1 \cong V_8(2) \) (considered as a vector space of dimension 8 over \( GF(2) \)) and its dual space \( N^*_1 := \text{Hom}(N_1, C^*) \) are isomorphic to each other. Since \( G \) has only one faithful irreducible eight-dimensional presentation over \( GF(2) \) it follows that \( N_1 \) and \( N^*_1 \) are also isomorphic as eight-dimensional modules for \( G \) over \( GF(2) \). Moreover, \( N^*_1 \) can be identified with set set \( \text{Irr}(N_1) \) and hence the action of \( G \) on the irreducible characters \( \text{Irr}(N_1) \) of \( N_1 \) will be the same as the action of \( G \) on \( N_1 \). Thus \( G \) has also three orbits of lengths 1, 120 and 135 on the 256 linear characters of \( N_1 \). Since the 256 linear characters of \( N \) come from \( N_1 \), \( G \) will also have three orbits on them with corresponding stabilizers \( H_1, H_2 \) and \( H_3 \) which have indices 1, 120 and 135 in \( H_1 = G \). The last outstanding character of \( N \) is the unique faithful irreducible character \( \theta_257 \) of degree sixteen, which form on its own an orbit. Hence \( G \) has four orbits on the set \( \text{Irr}(N) \) and by checking the indices of the maximal subgroups of \( G \) in the ATLAS, the inertia factor groups corresponding to these orbits are identified as \( H_1 = U_4(2):2, H_2 = 3_+^{1+2}:2D_8, H_3 = 2^4:S_4 \) and \( H_4 = U_4(2):2 \).
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Since $G$ has 4 orbits on $\text{Irr}(N)$ it follows that $G$ will also have 4 orbits on the 257 conjugacy classes of elements of $N$. Hence under the action of $G$, $N$ splits up into 4 conjugacy classes of $\overline{G}$. The first class contains the identity element, the second class the central element of order two, the third class $2^8 - 2^4 = 240$ elements of order 4 and the fourth class 270 elements of order two.

Having identified the inertia factor groups $H_i$, $i = 1, 2, 3, 4$, for the action of $G$ on $\text{Irr}(N)$ we proceed to use the technique of Fischer matrices (see [5] or [11]) to compute the ordinary irreducible characters of $\overline{G} = 2^{1+8}:(U_4(2):2)$.

A summary of the method of Fischer matrices will be given in Section 2. In Section 3, the Fischer matrix $M(1A)$ of $\overline{G}$ corresponding to the identity class $1A$ of $G$ will be computed and together with the decomposition of some ordinary irreducible characters of $U_6(2):2$ into the set $\text{Irr}(N)$ it will be shown that the irreducible characters $\text{Irr}(N)$ of $N$ extend to ordinary irreducible characters of their inertia subgroups in $\overline{G}$. The quotient group $Q = \frac{G}{Z(2^{1+8}):(U_4(2):2)}$ is isomorphic to a subgroup $\overline{G}_1$ of $O_{10}^+(2)$ with shape $2^8:(U_4(2):2)$ (see [6]). The current author and others determined the Fischer matrices and ordinary character table of $\overline{G}_1$ in [6]. It will be discussed in Section 4 how each Fischer matrix $M(g)$ of $\overline{G}$ can be derived from the corresponding Fischer matrix $\overline{M}(g)$ of $\overline{G}_1$ by just adding a row and a column to $\overline{M}(g)$.

Note that $\overline{G}$ is the pre-image of the maximal subgroup $U_4(2):2$ of index 28 in $Sp_6(2)$ under the natural epimorphism modulo $N = 2^{1+8}$. Hence an isomorphic copy of $\overline{G}$ sits maximally inside the maximal subgroup $\overline{G}_2 = 2^{1+8}:Sp_6(2)$ of $Co_2$ (see [4]). In Section 4, the fusion map of the conjugacy classes of $U_4(2):2$ into $Sp_6(2)$ together with the permutation character of $\overline{G}_2$ on $\overline{G}$ and, if necessary, some of the ordinary irreducible characters of small degrees of $\overline{G}_2$ are restricted to the set $\text{Irr}(\overline{G})$, to compute the orders of the elements of the conjugacy classes of $\overline{G}$ associated with each Fischer matrix $M(g)$ of $\overline{G}$. Note that the sizes of the centralizers of the classes of $\overline{G}$ coming from a coset $Ng$ are easily determined by using the column orthogonality relation (see Section 2) of a Fischer matrix $M(g)$. Having obtained the conjugacy classes and Fischer matrices of $\overline{G}$ from each conjugacy class $[g]$ of $G$ and together with the ordinary character tables of the inertia factor groups $H_i$, the ordinary character table of $\overline{G}$ (see Section 5) is constructed following the outline of the method discussed in Section 2. Using the algebra computer system GAP [8], the power maps of the elements of $\overline{G}$ are computed from the ordinary character table of $\overline{G}$ which was constructed in Section 5. Finally, the power maps of $\overline{G}$ and $U_6(2):2$ together with some restricted ordinary irreducible characters of $U_6(2):2$ to $\overline{G}$ are used to compute the fusion map of $\overline{G}$ into $U_6(2):2$.

Computations are carried out with the aid of the computer algebra systems MAGMA [3] and GAP and the notation of ATLAS is mostly followed. For an update on recent developments around Fischer matrices, interested readers are referred to the papers [17], [1], [2], [14], [15], [16] and [18].
The method used in this paper, to construct the Fischer matrices and ordinary character table of $G$, works for any finite split extension of the form $\mathbb{S} = 2^{1+2n}:G_1$, $n \geq 1$, provided the ordinary irreducible characters of the extra-special 2-group $2^{1+2n}$ (of type "," or type ",") extend to ordinary irreducible characters of their inertia subgroups $H_i$ in $\mathbb{S}$. Furthermore, the Fischer matrices of the quotient group $\mathbb{S}/2^{1+2n} \cong 2^{2n}:G$ are also known. In fact, this method can be extended to any extension group of the shape $E = p^{1+2n}:G_1$, $p$ a prime, if such a group $E$ exists.

2. Theory of Fischer Matrices

Since the ordinary character table of $G = 2^{1+8}:(U(4);2)$ will be constructed by the technique of Fischer matrices, a brief overview of this method is given as found in [20].

Let $G = N.G$ be an extension of $N$ by $G$, where $N$ is normal in $G$. Denote the set of all irreducible characters of a finite group $G_1$ by $\text{Irr}(G_1)$. Also, define $\mathbb{H} = \{ x \in G | \theta^x = \theta \} = I_G(\theta)$ as the inertia group of $\theta \in \text{Irr}(N)$ in $G$ then $N$ is normal in $\mathbb{H}$. Let $\mathbb{G} \in G$ be a lifting of $g \in G$ under the natural homomorphism $G \rightarrow G$ and $[g]$ be a conjugacy class of elements with representative $g$. Let $X(g) = \{ x_1, x_2, \ldots, x_{c(g)} \}$ be a set of representatives of the conjugacy classes of $G$ from the coset $N\mathbb{G}$ whose images under the natural homomorphism $G \rightarrow G$ are in $[g]$ and we take $x_1 = g$. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $G$ on $\text{Irr}(N)$ such that for $1 \leq i \leq t$, we have $\mathbb{H}_i$ with corresponding inertia factors $H_i$. By Gallagher [10] we obtain

$$\text{Irr}(\mathbb{G}) = \bigcup_{i=1}^t \{ (\psi_i, \overline{\beta})| \beta \in \text{IrrProj}(H_i), \text{ with factor set } \alpha_i^{-1} \}$$

where $\psi_i$ is a projective character of $\mathbb{H}_i$ with factor set $\overline{\pi}_i$ such that $\psi_i \downarrow_N = \theta_i$. Note that $\overline{\beta}$ is a lifting for $\beta$ into $\mathbb{H}_i$ and $\alpha_i$ is obtained from $\overline{\pi}_i$. We have that $\mathbb{H}_i \cong G$ and $H_1 = G$. Choose $y_1, y_2, \ldots, y_r$ to be representatives of the $\alpha_i^{-1}$-regular conjugacy classes of elements of $H_i$ that fuse to $[g]$ in $G$. We define

$$R(g) = \{ (i, y_k) | 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r \}$$

and we note that $y_k$ runs over representatives of the $\alpha_i^{-1}$-regular conjugacy classes of elements of $H_i$ which fuse into $[g]$ in $G$. We define $y_k \in \mathbb{H}_i$ such that $y_k$ ranges over all representatives of the conjugacy classes of elements of $\mathbb{H}_i$, which map to $y_k$ under the homomorphism $\mathbb{H}_i \rightarrow H_i$ whose kernel is $N$.

Lemma 2.1. With notation as above,

$$(\psi_i, \overline{\beta})G(x_j) = \sum_{y_k(i, y_k) \in R(g)} \beta(y_k) \sum_{t=1}^t \frac{|C_G(x_j)|}{|C_{\mathbb{G}}(y_k)|} \psi_i(y_k)$$
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Proof. See [20]

Then the Fischer matrix $M(g) = (a_{i,j}^l)$ is defined as

$$
(a_{i,j}^l) = \left( \sum_i \frac{|C_G(x_j)| |C_{\overline{G}}(y_k)|}{|\overline{C}_{\overline{G}}(y_k)|} \psi_i(x_j) \right),
$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_j$ is the summation over all $l$ for which $y_{i_k} \sim x_j$ in $\overline{G}$. So, we can write Lemma 2.1 as

$$
(\psi_i)_{\overline{G}}(x_j) = \sum_{y_k:(i,y_k) \in R(g)} a_{i,y_k}^l \beta(y_k).
$$

The Fischer matrix $M(g)$ (see Figure 1) is partitioned row-wise into blocks, where each block corresponds to an inertia group $H_i$. We write $|C_{\overline{G}}(x_j)|$, for each $x_j \in X(g)$, at the top of the columns of $M(g)$ and at the bottom we write $m_j \in \mathbb{N}$, where we define $m_j = |C_{\overline{G}}:C_{\overline{G}}(x_j)| = |N| |C_{\overline{G}}(g)|$ and $C_\overline{G} = \{x \in \overline{G}|x(N\overline{G}) = (N\overline{G})x\}$. On the left of each row we write $|C_{H_i}(y_k)|$, where the $\alpha_i^{-1}$-regular class $[y_k]$ fuses into the class $[g]$ of $G$. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks $M_i(g)$ corresponding to the inertia groups $H_i$ are separated by horizontal lines.

$$
M(g) = \begin{pmatrix}
|C_G(g)| & |C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
|C_{H_1}(y_1)| & a_{1,1} & a_{1,2} & \cdots & a_{1,c(g)} \\
|C_{H_2}(y_1)| & a_{2,1} & a_{2,2} & \cdots & a_{2,c(g)} \\
|C_{H_2}(y_2)| & a_{3,1} & a_{3,2} & \cdots & a_{3,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_1)| & \vdots & \vdots & \vdots & \vdots \\
|C_{H_i}(y_2)| & a_{i,1} & a_{i,2} & \cdots & a_{i,c(g)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
m_1 & m_2 & \cdots & m_{c(g)}
\end{pmatrix}
$$

Figure 1. The Fischer Matrix $M(g)$
In practice we will never compute the $y_k$ or the ordinary irreducible character tables of the inertia subgroups $H_i$. The reason for this is that the ordinary irreducible characters of the $H_i$ are in general much larger and more complicated to compute than the one for $\mathcal{G}$. Instead of using the above formal definition of a Fischer matrix $M(g)$, the arithmetical properties of $M(g)$ below are used to compute the entries of $M(g)$ (see [12]).

(a) $a^2_{(1,g)} = 1$ for all $j = \{1, 2, \ldots, c(g)\}$.
(b) $|X(g)| = |R(g)|$.
(c) $\sum_{j=1}^{a(g)} m_j a^j_{(i,y_k)} \overline{a^j_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_{\mathcal{G}}(g)|}{|C_{H_i}(y_k)|} |N|$.
(d) $\sum_{(i,y_k) \in R(g)} a^j_{(i,y_k)} a^{j'}_{(i,y_k)} |C_{H_i}(y_k)| = \delta_{jj'} |C_{\mathcal{G}}(x_j)|$.
(e) $M(g)$ is square and nonsingular.

If $N$ is elementary abelian, then we obtain the following additional properties of $M(g)$:

(f) $a^1_{(1,y_k)} = \frac{|C_{\mathcal{G}}(g)|}{|C_{H_i}(y_k)|}$.
(g) $|a^1_{(1,y_k)}| \geq |a^j_{(1,y_k)}|$.
(h) $a^j_{(1,y_k)} \equiv a^1_{(1,y_k)} \pmod{p}$, if $|N| = p^n$, for $p$ a prime and $n \in \mathbb{N}$.

The matrix $M(g)$ is square, where the number of rows is equal to the number of $\alpha_i^{-1}$-regular classes of the inertia factors $H_i$'s, $1 \leq i \leq t$, which fuse into $[g]$ in $G$ and the number of columns is equal to the number $c(g)$ of conjugacy classes of $\mathcal{G}$ which is obtained from the coset $Ng$. Then the partial character table of $\mathcal{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by

$$
\begin{bmatrix}
C_1(g) & M_1(g) \\
C_2(g) & M_2(g) \\
\vdots \\
C_t(g) & M_t(g)
\end{bmatrix}
$$

where the Fischer matrix $M(g)$ (see Figure 1) is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\mathcal{H}_i$ and $C_i(g)$ is the partial character table of $H_i$ with factor set $\alpha_i^{-1}$ consisting of the columns corresponding to the $\alpha_i^{-1}$-regular classes that fuse into $[g]$ in $G$. We obtain the characters of $\mathcal{G}$ by multiplying the relevant columns of the projective characters of $H_i$ with factor set $\alpha_i^{-1}$ by the rows of $M(g)$. We can also observe that

$$
|\text{Irr}(\mathcal{G})| = \sum_{i=1}^{t} |\text{IrrProj}(H_i, \alpha_i^{-1})|.
$$
### 3. On the type of characters of the inertia factors $H_i$

The group $\overline{G} = 2_1^{+8}.(U_4(2):2)$ can be regarded as the 2-fold cover $Z(N) \cdot (2^8:(U_4(2):2))$ for the group $\overline{G}_1 = 2^8:(U_4(2):2)$. Using Fischer-Clifford theory we noticed that $\overline{G}_1$ will have two orbits on $Z(N) = Z(\overline{G}) \cong 2$, with one orbit containing the identity element $1_{Z(N)}$ and the second orbit the central element $z$ of order two. Hence the orbits will have both $\overline{G}_1$ as their respective set stabilizer. By Lemma 5.2 in [7], $\overline{G}_1$ will also have two orbits on $\text{Irr}(Z(N))$ containing one element each and with corresponding inertia factors $H_1 = H_2 = \overline{G}_1$. To construct the ordinary character table of $2^8:(U_4(2):2)$ via the technique of Fischer matrices, we will then require the ordinary character table of $H_1$ and an irreducible projective character table of $H_2$ with factor set $\alpha$ of order two. The ordinary character table of $\overline{G}_1 = 2^8:(U_4(2):2)$ was constructed in [6] using Fischer matrices and it was shown that $U_4(2):2$ acts irreducibly on its unique eight-dimensional module $2^8$, where $U_4(2):2$ has three orbits of lengths 1, 120 and 135 on $\text{Irr}(2^8)$ with corresponding inertia factor groups $U_4(2):2, 3_1^{+2}:(2D_8)$ and $2^4:S_4$. Since $\overline{G}_1$ is a split extension and $2^8$ elementary abelian, only the ordinary character tables of the inertia factor groups of $2^8$ in $\overline{G}_1$ were used in construction the set $\text{Irr}(\overline{G}_1)$ as a consequence of Mackey’s Theorem in [10]. Hence we will also use the ordinary character tables of the inertia factors $H_1 = U_4(2):2, H_2 = 3_1^{+2}:(2D_8)$ and $H_3 = 2^4:S_4$ of $\text{Irr}(2_1^{+8})$ in $\overline{G}$ to construct the ordinary character table of $\overline{G} = 2_1^{+8}:(U_4(2):2)$ using Fischer matrices. To determine which type of irreducible characters (ordinary or projective) will be used for $H_4 = U_4(2):2$, we will use the Fischer matrix $M(1_G)$ together with decompositions of some ordinary characters of small degrees of $U_6(2):2$ into the ordinary irreducible characters of $N$. We can add here that for the first $c(g) - 1$ rows of each Fischer matrix $M(g)$ of size $c(g)$ the properties (f), (g) and (h) found in Section 2 are also applicable since $2^8$ is elementary abelian.

Having obtained the inertia factors $H_1 = U_4(2):2, H_2 = 3_1^{+2}:(2D_8), H_3 = 2^4:S_4$ and $H_4 = U_4(2):2$ for the action of $G$ on $\text{Irr}(N)$, we can form the Fischer matrix $M(1A)$ corresponding to the identity coset $N1_G = N$ as below. Properties (a) and (f) (see Section 2) were used to find the entries for the first row and the first three entries of the first column of $M(1A)$.

\[
M(1A) = \begin{pmatrix}
51840 & 26542080 & 26542080 & 110592 & 98304 \\
432 & 1 & 1 & 1 \\
384 & 135 & a & b & c \\
51840 & g & h & i & j \\
1 & 1 & 240 & 270
\end{pmatrix}
\]

The column weights above the matrix $M(1A)$ are the centralizer orders $|C_G(x_j)|$ of the four classes $[x_j]$ of $G$ coming from the identity coset $N$ and the weights below are the values $m_j$. Whereas, the row weights to the left of the matrix $M(1A)$ represent the centralizer orders $|C_H(1A)|$ of the inertia factors.
\( H_i \) on the identity element \( 1A \). Applying the remaining Fischer matrix \( M(g) \) properties in Section 2 to the above matrix, the entries of \( M(1A) \) are completed and shown below.

\[
M(1A) = \begin{pmatrix}
51840 & 1 & 1 & 1 & 1 \\
432 & 120 & 120 & 8 & -8 \\
384 & 135 & 135 & -9 & 7 \\
51840 & 16 & -16 & 0 & 0 \\
1 & 1 & 240 & 270
\end{pmatrix}
\]

| \( [x|_{\gamma} \) | \( 1A \) | \( [x_2] \) | \( [x_3] \) | \( [x_4] \) |
|---|---|---|---|
| \( \chi_1 \) | 1 | 1 | 1 | 1 |
| \( \chi_{26} \) | 120 | 120 | 8 | -8 |
| \( \chi_{40} \) | 135 | 135 | -9 | 7 |
| \( \chi_{60} \) | 16c | -16c | 0 | 0 |

Table 1. The partial character table of \( \overline{G} \) for coset \( N \)

Table 1 is the partial ordinary character table of \( \overline{G} \) on the classes \( 1A, [x_2], [x_3] \) and \( [x_4] \) of \( \overline{G} \) coming from \( N \), where each of the 4 lines of Table 1 corresponds to the first row of entries of the sub-matrices \( C_i(1A)M_i(1A), i = 1, 2, 3, 4 \). \( M_i(1A) \) and \( C_i(1A) \) correspond to the rows of the Fischer-Clifford matrix \( M(1A) \) and columns of the character tables (ordinary or projective) of the inertia factors \( H_i \), respectively, which are associated with the classes \( [1A|_{H_i} \) of the inertia factors \( H_i \) which fuse into the class \( [1A|_{G} \) of \( G \). Also, note that the character values in the 1st column of Table 1 are the degrees of the ordinary irreducible characters \( \chi_1, \chi_{26}, \chi_{40} \) and \( \chi_{60} \) of \( \overline{G} \). The characters \( \chi_1, \chi_{26}, \chi_{40} \) and \( \chi_{60} \) occupy the first position for each block of characters coming from an inertia subgroup \( H_i \) of \( G \). Also note that \( H_1, H_2 \) and \( H_3 \) will contribute 25, 14 and 20 ordinary irreducible characters, respectively, towards the set \( \text{Irr}(\overline{G}) \). The reason for this is that it was found earlier that we will use the ordinary irreducible characters of the inertia factors \( H_1, H_2 \) and \( H_3 \) in the construction of the set \( \text{Irr}(\overline{G}) \) and it only remains to determine whether we will use the ordinary character table of \( H_4 \) or the set \( \text{IrrProj}(H_4, \alpha) \) of irreducible projective characters with factor set \( \alpha \) of order 2. It can be readily being verified in GAP that the Schur multiplier \( M(H_4) \cong 2 \) of \( H_4 \) is a cyclic group of order 2 and hence will have two sets of irreducible projective character tables, i.e. \( \text{Irr}(H_4) \) and \( \text{IrrProj}(H_4, \alpha) \). Now \( \deg(\chi_1) = 1, \deg(\chi_{26}) = 120 \) and \( \deg(\chi_{40}) = 135 \) and \( \deg(\chi_{60}) = 16c \) (see Table 1) are the degrees of the ordinary irreducible characters of \( \overline{G} \) which occupy the first position in each block of the set \( \text{Irr}(\overline{G}) \) which corresponds to the inertia groups \( H_1, H_2, H_3 \) and \( H_4 \). The number \( c \) is the degree of one of the irreducible characters which is contained in either \( \text{Irr}(H_4) \) or \( \text{IrrProj}(H_4, \alpha) \).
A small part of the ordinary character table of $U_6(2):2$ (see ATLAS or GAP library) is found in Table 2, which contains the values of the irreducible characters $22a$ and $231a$ on the classes $1A$, $2A$, $2B$, $2D$ and $4A$ of $U_6(2):2$.

| $y|U_6(2):2$ | $1A$ | $2A$ | $2B$ | $2D$ | $4A$ |
|--------------|------|------|------|------|------|
| $|C_{U_6(2):2}(y)|$ | 18393661440 | 26542080 | 294912 | 2903040 | 110592 |

| 22a | 22 | -10 | 6 | 8 | 6 |
| 231a | 231 | 39 | 7 | 21 | 23 |

Table 2. The partial character table of $U_6(2):2$

Taking into account how the centralizer orders of the classes $1A$, $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ (see Table 1) can divide those of the classes of $U_6(2):2$, we obtain that the classes $1A$, $2A$, $2B$, $2D$ and $4D$ of $U_6(2):2$ (see Table 2) are the only candidates for the classes $1A$, $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ to fuse into. Notice that $[x_4]$ can either fuse into $2A$ or $2B$ of $U_6(2):2$. Now it is obvious that the other two non-trivial classes $[x_2]$ and $[x_3]$ of $\overline{G}$ will fuse into the classes $2A$ and $4A$ of $U_6(2):2$, respectively. Suppose that $[x_4]$ will fuse into $2A$ of $U_6(2):2$, then the inner product $(22a)_N, 1_N > N = -2,4375$ of the restriction of $22a \in \text{Irr}(U_6(2):2)$ to $N$ with the identity character $1_N$ of $N$ will give us a negative rational number which is impossible. Now, if we assume that $[x_4]$ will fuse into $2B$ of $U_6(2):2$ then $< (22a)_N, 1_N > N = 6$ and this shows that $[x_4]$ will fuse into $2B$ of $U_6(2):2$. See Table 3 for the fusion map of classes of $\overline{G}$ coming from the identity coset $N$ into the classes of $U_6(2):2$.

| $|C_{\overline{G}}(x_i)|$ | $[x_i]|_{\overline{G}}$ | $y|U_6(2):2$ | $|C_{U_6(2):2}(y)|$ |
|-----------------|-----------------|-----------------|-----------------|
| 26542080        | 1A              | 1A              | 18393661440     |
| 26542080        | $[x_2]$         | $2A$            | 26542080        |
| 110592          | $[x_3]$         | $4A$            | 110592          |
| 98304           | $[x_4]$         | $2B$            | 294912          |

Table 3. The fusion map of classes of $\overline{G}$ from $N$ into classes of $U_6(2):2$

By obtaining the orders of the elements in the classes $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ and also their fusion into $U_6(2):2$ we can now proceed to decompose the ordinary irreducible character $22a$ of $U_6(2):2$ with degree of 22 into the set $\text{Irr}(N)$ which is represented in Table 1. Now

$$(22a)_N = < (22a)_N, 1_N > (\chi_1)_N + < (22a)_N, (\chi_{20})_N > (\chi_{20})_N + < (22a)_N, (\chi_{40})_N > (\chi_{40})_N + < (22a)_N, (\chi_{60})_N > (\chi_{60})_N = 6 \times 1_N + 0 \times (\chi_{20})_N + 0 \times (\chi_{40})_N + c \times (\chi_{60})_N = 6 \times 1_N + c \times (\chi_{60})_N$$

Since the $\deg(22a)=22=6\deg(1_N)+c\deg(\chi_{60})=6(1)+c(16c)=6+16c^2$, it follows that $c = 1$ because $c$ is the degree of one of the irreducible characters belonging either to $\text{Irr}(H_4)$ or $\text{IrrProj}(H_4, \alpha)$. Therefore it shows that we will...
use the ordinary irreducible character table of $H_4$. Hence each of the irreducible characters of $\text{Irr}(N)$ extends to an ordinary irreducible character of its inertia group $\overline{H_i}$.

4. Fischer matrices and conjugacy classes of $\overline{G}$

In this section, the Fischer matrices and the conjugacy classes of $\overline{G}$ will be determined from those of a subgroup $\overline{G}_1$ of $G_{10}^+(2)$ with shape $2^8:(U_4(2):2)$ (see [6]) which is an isomorphic copy of the quotient group $Q = \frac{G}{Z(2^1+8)}$. Also, the fusion of $\overline{G}$ into the group $\overline{G}_2$ will help to determine the orders of the classes of $\overline{G}$.

In [11] and [13] the Fischer matrices of the maximal subgroups $\overline{G}_2 = 2_+^{1+8}:Sp_6(2)$ and $\overline{G}_3 = 2_+^{1+22}:Co_2$ of the sporadic simple groups $Co_2$ and the Baby Monster $B$, respectively, were computed. It was mentioned in these publications (see also Remark 7 in [2]) that the Fischer matrices of their quotients groups $Q_2 = \frac{G_2}{Z(2^1+8)} \cong 2^8:Sp_6(2)$ and $Q_3 = \frac{G_3}{Z(2^1+22)} \cong 2^{22}.Co_2$ (see proof of Lemma 7 in [13]) can be obtained by removing the first column and last row of each Fischer matrix of $\overline{G}_2$ and $\overline{G}_3$. In both cases, as in our case, the ordinary irreducible characters of $2_+^{1+8}$ and $2_+^{1+22}$ extend to ordinary irreducible characters of their inertia subgroups in $\overline{G}_2$ and $\overline{G}_3$. Therefore, only the ordinary character tables of the inertia factors are involved in the construction of the character tables of $\overline{G}_2$ and $\overline{G}_3$. Since the action of our group $\overline{G}_1$ on $\text{Irr}(2_+^{1+8})$ follows a similar pattern as the actions of $\overline{G}_2$ and $\overline{G}_3$ on $\text{Irr}(2_+^{1+8})$ and $\text{Irr}(2_+^{1+22})$, respectively, the results obtained in [11] and [13] will be applicable to $\overline{G}$. Also, an isomorphic copy of $\overline{G}_s$ sits maximally inside $\overline{G}_2$ and so, the Fischer matrices of $\overline{G}$ can be obtained by adding a first row and a last column to the Fischer matrices of $\overline{G}_1$. The nature of these rows and columns are described in the two lemmas below which were taken from [13] and adjusted for $\overline{G}$. The proofs of these lemmas for the case of $\overline{G}_1$ follow the exact pattern as that for $\overline{G}_3$ [13] with differences in notation. For the notation use in Lemma 4.1 and Lemma 4.2 the reader is referred to Section 2 of this paper.

Lemma 4.1. For every $c(g) \times c(g)$ Fischer matrix $M(g)$ of $\overline{G}$ the sum of the first $c(g) - 1$ rows equals the (componentwise) square of the last row.

Proof. See proof of Lemma 6 of [13].

Lemma 4.2. For each $M(g)$ of $\overline{G}$, the $x_j$‘s in the set $X(g)$ (in Section 2) can be ordered in such a way that the last row of each $M(g)$ is of the form $[q_j, -q_j, 0, ..., 0]$ with $q_j$ a power of 2 and we may choose $x_2 = z x_1$ with $z$ the central involution in $\overline{G}$. Also $a_{i}(y_k) = a_{i}(y_k) = \frac{[C_G(g)]}{[C_{H_i}(y_k)]}$ for $1 \leq i \leq 3, 1 \leq k \leq r$.

Proof. See proof of Lemma 7 of [13].
From Lemma 4.1 and Lemma 4.2 the first two columns and last row of each matrix $M(g)$ of $G$ are known and so are the values of all $\chi \in \text{Irr}(G)$ on the classes $[x_1]$ and $[x_2]$ of $G$ coming from a coset $Ng$. Note that the character $\chi_{60} \in \text{Irr}(G)$ in Table 1 is the extension of the unique faithful irreducible character $\theta_{257}$ of $N$ of degree sixteen. Also, $\chi_{60}$ is the equivalent of the character $\eta \in \text{Irr}(G_3)$ used in the proofs of Lemma 6 and Lemma 7 in [13]. Moreover, $(\chi_{60}^2)_N = \theta_{257}^2$ is the lifting of the regular character of $N/Z(N) \cong N_1$ and hence the sum of the 256 linear characters of $N$. Observe that

$$\chi_{60}^2 = \chi_1 + \chi_{26} + \chi_{40},$$

where $\chi_1, \chi_{26}$ and $\chi_{60}$ (see Table 1) are the extensions $\psi_i$, $i = 1, 2, 3$, of the representatives of the three orbits of $G$ on the linear characters of $N$ to their respective inertia groups $H_i$, which are induced to $G$. This shows that $\psi_i$, $i = 1, 2, 3$, are uniquely determined linear characters of the inertia subgroups $H_i$ in the construction of the ordinary character table of $G$ (as it was established in Section 3). In addition, the ordinary irreducible character $\chi_{60}$ of $G$ is made completely known by Lemma 4.2 and therefore also all the faithful irreducible characters of $G$.

\[ \chi_{60}^2 = \chi_1 + \chi_{26} + \chi_{40}, \]

Where $\chi_1, \chi_{26}$ and $\chi_{60}$ (see Table 1) are the extensions $\psi_i$, $i = 1, 2, 3$, of the representatives of the three orbits of $G$ on the linear characters of $N$ to their respective inertia groups $H_i$, which are induced to $G$. This shows that $\psi_i$, $i = 1, 2, 3$, are uniquely determined linear characters of the inertia subgroups $H_i$ in the construction of the ordinary character table of $G$ (as it was established in Section 3). In addition, the ordinary irreducible character $\chi_{60}$ of $G$ is made completely known by Lemma 4.2 and therefore also all the faithful irreducible characters of $G$.

Since $G_1$ is a split extension of an elementary abelian group $2^8$ by $U_4(2):2$ we have that if the first column and last row of a Fischer matrix $M(g)$ of $G$ is removed then we are left with the Fischer matrix $\tilde{M}(g)$ of $G_1$. Having computed the Fischer matrices of $G_1$ in Table 4 of [6], we can just add to the Fischer matrices of $G_1$ a first column and a last row (as described in Lemma 4.2) to obtain the Fischer matrices of $G$. The fusion maps for the inertia factors $H_2$ and $H_3$ into $H_1$ are available in [6]. For example, consider the Fischer matrix $\tilde{M}(2D)$ of $G_1$ corresponding to the coset $N_1g$ of $2^8$ in $G_1$, where $g$ is a representative of the class $2D$ of involutions in $U_4(2):2$ (see [6]). The coset $N_1g$ splits into four classes $\{2H, 4F, 4G, 4H\}$ of $G_1$ with their respective centralizer orders indicated in the row above the matrix $\tilde{M}(2D)$.

\[ \tilde{M}(2D) = \begin{pmatrix}
1536 & 1536 & 256 & 192 \\
96 & 1 & 1 & 1 & 1 \\
12 & 8 & -8 & 0 & 0 \\
96 & 1 & 1 & 1 & -1 \\
12 & 6 & 6 & -2 & 0
\end{pmatrix} \]

Using Lemma 4.2, a first column and a last row are inserted to the matrix $\tilde{M}(2D)$ to obtained the required Fischer matrix $M(2D)$ (see below) of $G$ corresponding to the class $2D$ of $U_4(2):2$. 

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Notice that the coset $Ng$ corresponding to the class $2D$ of $U_4(2):2$ splits into five classes $[x_1]$, $[x_2]$, $[x_3]$, $[x_4]$ and $[x_5]$ of $G$ with their centralizer orders found in the row on top of the matrix $M(2D)$, respectively. These centralizer orders $|C_G(x_j)|$ were computed using the column orthogonality relation (d) for Fischer matrices in Section 2. The next step is to find the orders of the elements contain in the five classes. For this purpose, we will make use of the permutation character $\chi(G_2|G) = 1 + 27a$ of $G_2$ on the conjugacy classes of $G$ together with Proposition 7.5.1 in [12]. Moreover, the elements of the above five classes will have orders $o(x_j) \in \{2, 4, 8\}$, since $2^{1+8}$ is an extra-special 2-group.

The class of involutions $2D$ of $U_4(2):2$ is the only conjugacy class of $U_4(2):2$ that is fusing into the class of involutions $2D$ of $Sp_6(2)$ (see Table 7.7 in [12]). Then it follows from Proposition 7.5.1 in [12] that the classes of $G_2$ coming from the coset $Ng$, $g \in 2D$ of $U_4(2):2$, will fuse into the classes of $G_2$ coming from the coset $Ng$, $g \in 2D$ of $Sp_6(2)$. In Example 3.8.17 of [11], the technique of Fischer matrices is applied to the group $G_2$ which is a maximal subgroup of the Conway sporadic simple group $Co_2$. The computation of the Fischer matrix of $G_2$ corresponding to the class $2D$ of $Sp_6(2)$ is left as Exercise 3.8.3 in [11]. Using a permutation representation of $G_2$ obtained from the online ATLAS [21] and a similar GAP routine as used in [16], the five classes $[y_j]$, $j = 1, 2, ..., 5$, of $G_2$ corresponding to the coset $Ng$, $g \in 2D$ of $Sp_6(2)$ are computed and the information about them are shown in Table 4 below. Taking in consideration, the sizes $|C_{G_2}(x_j)|$ of the centralizers of the elements in the classes $[x_j]$, $j = 1, 2, ..., 5$, of $G$ and those of the corresponding classes $[y_j]$ of $G_2$ together with the values of the permutation character $\chi(G_2|G)$ of $G_2$ on the classes of $G$, we deduce that the orders $o(x_j)$ of the elements in the classes $[x_1]$, $[x_2]$ and $[x_4]$ will be all 4 whereas the orders of elements in conjugacy classes $[x_3]$ and $[x_5]$ will be 2 and 8 respectively. All of the above-mentioned information is summarized in Table 4 below.

| $[y_j]|G_2$ | $[y_1]$ | $[y_2]$ | $[y_3]$ | $[y_4]$ | $[y_5]$ |
|----------|--------|--------|--------|--------|--------|
| $o(y_j)$ | 4 | 4 | 2 | 4 | 8 |

$|C_{G_2}(y_j)|$ | 12288 | 12288 | 6144 | 1024 | 768 |

| $[x_j]|G$ | $[x_1]$ | $[x_2]$ | $[x_3]$ | $[x_4]$ | $[x_5]$ |
|----------|--------|--------|--------|--------|--------|
| $o(x_j)$ | 4 | 4 | 2 | 4 | 8 |

$|C_{G}(x_j)|$ | 3072 | 3072 | 1536 | 256 | 192 |

$\chi(G_2|G)$ | 4 | 4 | 4 | 4 | 4 |

Table 4. The orders $o(x_j)$ of elements of $G$ from the coset $Ng$, $g \in 2D$.
In a similar manner, as described above, we obtained the conjugacy classes \([x_i]\) and the Fischer matrices \(M(g_i)\) of \(G\) corresponding to the remaining classes \([g_i]\) of \(G\) and this information is listed in Table 5 and Table 6, respectively.
\[
\begin{align*}
M(1A) &= \begin{bmatrix}
1 & 1 & 1 & 1 \\
128 & 120 & 8 & -8 \\
135 & 135 & -9 & 7 \\
16 & -16 & 0 & 0
\end{bmatrix} \\
M(2A) &= \begin{bmatrix}
1 & 1 & 1 \\
15 & 15 & -1 \\
4 & -4 & 0
\end{bmatrix} \\
M(2B) &= \begin{bmatrix}
1 & 1 & 1 & 1 \\
24 & 24 & 8 & -8 \\
3 & 3 & 3 & 3 \\
36 & 36 & -12 & 4
\end{bmatrix} \\
M(3A) &= \begin{bmatrix}
1 & 1 & 1 & 1 \\
8 & -8 & 0 & 0
\end{bmatrix} \\
M(3B) &= \begin{bmatrix}
1 & 1 & 1 & 1 \\
6 & 6 & 2 & -2 \\
6 & 6 & 6 & -2 \\
4 & -4 & 0 & 0
\end{bmatrix} \\
M(3C) &= \begin{bmatrix}
1 & 1 & 1 & 1 \\
3 & 3 & 3 & -1 \\
2 & 2 & -2 & 0
\end{bmatrix} \\
M(4A) &= \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & -1 \\
1 & 1 & -1
\end{bmatrix} \\
M(4B) &= \begin{bmatrix}
1 & 1 & 1 \\
4 & 4 & 4 & -4 & 0 \\
8 & -8 & 0 & 0 \\
3 & 3 & 3 & 3 & -1 \\
4 & -4 & 0 & 0 & 0
\end{bmatrix} \\
M(4C) &= \begin{bmatrix}
1 & 1 & 1 \\
8 & 8 & -8 & 0 & 0 \\
2 & 2 & -2 & 0 & 0
\end{bmatrix} \\
M(4D) &= \begin{bmatrix}
1 & 1 & 1 \\
3 & 3 & 3 & -1 \\
2 & 2 & -2 & 0 & 0
\end{bmatrix}
\end{align*}
\]

Table 6. The Fischer Matrices of \( \overline{G} \)
5. The character table of $\overline{G}$

Using the information of the classes of $G$ in Table 5, the ordinary irreducible characters of the inertia factors $H_i$ and the Fischer matrices in Table 6, the ordinary character table of $\overline{G}$ (see Table 7) is successfully constructed using the outline given in Section 2 of this paper. Consistency and accuracy checks of the character table of $\overline{G}$ have been carried out the aid of Programme E in [19] together with the computation of the class multiplication coefficients of the classes of $\overline{G}$. The set of ordinary irreducible characters of $\overline{G}$ will be partitioned into 4 blocks $\Delta_1 = \{\chi_j|1 \leq j \leq 25\}$, $\Delta_2 = \{\chi_j|26 \leq j \leq 39\}$, $\Delta_3 = \{\chi_j|40 \leq j \leq 59\}$ and $\Delta_4 = \{\chi_j|60 \leq j \leq 84\}$ corresponding to the inertia factor groups $H_1$, $H_2$, $H_3$ and $H_4$, respectively, where $\chi_j \in \text{Irr}(\overline{G})$. Since $\overline{G} \cong 2\overline{G_1}$ is a two-fold cover of $\overline{G_1}$, the ordinary characters of $\overline{G_1} = 2^8:((U_4(2):2)$ (see Table 5 in [6]) are found in blocks $\Delta_1$ to $\Delta_3$ and a set $\text{IrrProj}(G_1, \alpha)$ of irreducible projective characters with factor set $\alpha$ of order 2 for $\overline{G_1}$ can be obtained from block $\Delta_4$.

Using Programme E in GAP, the unique $p$-power maps of the elements of $\overline{G}$ are computed (see Table 5) from our Table 7. Also, using the power maps of $\overline{G}$ and $U_6(2):2$, the permutation character $\chi(U_6(2):2|\overline{G}) = 1a + 252a + 440a$ of $U_6(2):2$ on the classes of $\overline{G}$ and the restriction of some characters of small degrees of $U_6(2):2$ to the set $\text{Irr}(\overline{G})$ in Table 7, the fusion map of the classes of $\overline{G}$ into the classes of $U_6(2):2$ is computed (see last column of Table 5).
### Table 7. The character table of $\overline{G} = 2_+^{1+8}: (U_4(2):2)$

<table>
<thead>
<tr>
<th>(g)</th>
<th>1A</th>
<th>2A</th>
<th>2B</th>
<th>2C</th>
<th>2D</th>
<th>3A</th>
<th>3B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1_{id})</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(A)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(B)</td>
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<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>(C)</td>
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<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(D)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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| $|g|/|c|$ | 1A | 2A | 2B | 2C | 2D | 3A | 3B |
|---|---|---|---|---|---|---|---|
| $x_0$ | 135 | 135 | 135 | 135 | 135 | 0 | 0 |
| $x_1$ | 39 | 39 | 39 | 39 | 39 | 0 | 0 |
| $x_2$ | 15 | 15 | 15 | 15 | 15 | 0 | 0 |
| $x_3$ | 15 | 15 | 15 | 15 | 15 | 0 | 0 |
| $x_4$ | 6 | 6 | 6 | 6 | 6 | 0 | 0 |
| $x_5$ | 20 | 20 | 20 | 20 | 20 | 0 | 0 |
| $x_6$ | 405 | 405 | 405 | 405 | 405 | 0 | 0 |
| $x_7$ | 45 | 45 | 45 | 45 | 45 | 0 | 0 |
| $x_8$ | 45 | 45 | 45 | 45 | 45 | 0 | 0 |
| $x_9$ | 540 | 540 | 540 | 540 | 540 | 0 | 0 |
| $x_{10}$ | 60 | 60 | 60 | 60 | 60 | 0 | 0 |
| $x_{11}$ | 1080 | 1080 | 1080 | 1080 | 1080 | 0 | 0 |

The character table of $\overline{G} = 2_{+}^{1+8} : (U_4(2):2)$ (continued)
The character table of $\mathcal{C} = 2_1^{1+8}:A_4$ (continued)
<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$3C$</th>
<th>$4A$</th>
<th>$4B$</th>
<th>$3C$</th>
<th>$4D$</th>
<th>$5A$</th>
<th>$6A$</th>
<th>$6B$</th>
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<td>-1</td>
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<td>-1</td>
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<tr>
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<td>1</td>
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<tr>
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<td>1</td>
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<td>1</td>
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<td>1</td>
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<tr>
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The character table of $\mathcal{G} = 2_1^{1+8}:(U_4(2):2)$ (continued)
The character table of $G = 2_{+}^{4}:(U_{4}(2):2)$

where $A = 2\sqrt{2}i$, $B = -1 - 2\sqrt{3}i$

The character table of $G = 2_{+}^{4}:(U_{4}(2):2)$ (continued)
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References


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