Revista Colombiana de Matemáticas
Vol. IX, (1975), págs. 161-171

NEW TENDENCIES IN INDUSTRIAL STATISTICS: IS SQC IN A CRISIS?

by

*Viorel Gh. VODĂ**

Motto : "... is Statistics an academic tool?
...: a tool is not an end in itself"
J. M. Hammersley

1. *Introduction.* Sometimes, Industrial Statistics have been considered by some people as a collection of simple statistical techniques applied to certain peculiar problems raised from industrial activity.

Commonly, Industrial Statistics is associated with the application of some tests -- as distance tests for testing normality- t or F-tests, fitting straightlines or some curves or writing some models in Analysis of Variance.

The relative simplicity of control charts in Statistical Quality Control (SQC) has accredited also the belief that in fact Industrial Statistics are a little bit "low mathematics".

The situation is not at all just so. In fact, Industrial Statistics like all sta-

*) Center of Mathematical Statistics, Bucharest - Romania. Now exchange visitor under Grant of NAS at University of California, Berkeley, 94720.

tistical methodology is a body for obtaining knowledge on actual phenomena – in this case, on industrial processes.

Through mathematical branches, Statistics has a special place due to its strong interference with practice.

A valid statistical research always will try to solve actual problem by cons – tructing, improving or just simply using a mathematical model, but not to construct first a model and then look for the appropriate problem - which often may not exist. (See references Section A).

The purpose of these lines is to emphasize some new tendencies and possible future mathematical tools in Industrial Statistics. Since the matter is so large, we shall stress especially on SQC.

2. Recent Tendencies. If someone follows carefully the literature published in the matter last ten years, he will observe that :

a. Life testing and reliability techniques have received a tremendous development. Through the main topics we will enlist :

– accelerated life tests
– estimation problems from multicensored samples
– reliability of complex systems in connection with dependent subsystem failures
– new distribution functions as time– to– failure distributions
– availability theory
– fault – tree – analysis, a.s.o.

(See references, Section B)

b. In SQC the majority of works are on acceptance sampling techniques, es–

pecially on multiple sampling and Bayesian methods (see references, Section C).

c. There are not so many papers on Shewhart control charts : the literature devoted to the SQC during production regards mainly the extension of control charts to dependent variables, multivariate control charts, use of some auxiliary distributions - like Burr distribution- for constructing charts for largest and smallest sample values in a normal population, a.s.o. (see references, Section D).

This situation has implemented the idea - especially in Europe - that SQC is in a "crisis", the computerization and automation being two factors which will eliminate gradually classical control charts, the main attention remaining concentrated on Reliability, Sampling Plans and Multivariate Statistical Control. This idea has a simple but not a very obvious explanation :

In spite of the efforts paid to find more sophisticated tools in control chart theory, the classical Shewhart control charts are still the most efficient instrument for obtaining a close image of a process evolution.

Computerization and automation have given a valuable help to SQC and the visualization of control charts on TV sets does not mean their dismissing.

It is important to note that an automatic (or electronic) control device installed on certain, say milling machine, tool may give an image on the specific product but not on the process evolution (see references, Section C).

The evolution may be pursued only if one uses a statistical methodology even if this one is an automatized one.

As a practical tool, Shewhart control charts seemed to be until now almost perfect, and after some improvements (see references, Section D) nothing has been added without a loss of simplicity.

This may be an explanation of the point (b). One thing we have to note : an important step in control charts practice is the interpretation of those charts. This interpretation is a combination of science (engineering thinking) and art* (skills to make analogies with previous cases met, a.s.o.) which is in a way more important than charts themselves since the purpose is to know what happens in the process.

And the *process control by the aid of control charts is the key to all kinds of equipment reliability* (see References, Section D, [20]).

It is useless to apply complicated reliability and lifetesting techniques on products delivered by a process which is not well dominated.

A natural general flow of how things must be done seems to be the following:

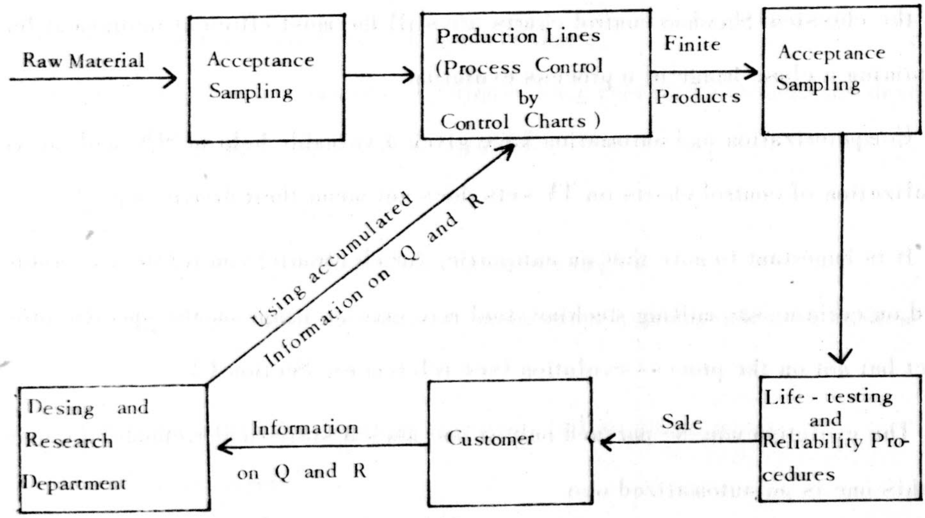

Figure 1 : Interferation between Q and R methods

*) In this field, the Handbook of SQC edited by Western Electric Company is still a masterpiece, in our opinion.

The fact that acceptance sampling techniques have to be applied to raw material of various kinds (pieces, bulk, a.s.o.) as well as to finite products which also may be delivered in different ways, has concentrated the attention of statisticians and the Congress in Tokyo (1969) on Quality Control has put into evidence an increasing interest – especially on bulk material sampling techniques.

3. *Possible future Tools.* It is a common fact that various scientific branches may interact, providing each other with ideas or specific analytical methods.

Attending this fall a course in Advanced Reliability Theory held by Professor R. E. Barlow at the University of California, Berkeley, seems to the present author that the tool of so -called "isotonic regression" could be a useful instrument not only reliability problems but also in Quality Control.

Briefly, the isotonic repression problem consists in finding :

$$\text{Minimum of} \quad \sum_{i=1}^{k} (g_i - x_i)^2 W_i \qquad (1)$$

in the condition $X_i \leq x_j$ when $i \leq j$ where $\leq$ "is a partial ordering on $\Omega = \{1, 2, \ldots, k\}$ and $w_i > o$" and g_i $(i = 1, \ldots, k)$ are given. (See Barlow and Brunk, Section E, [44]).

In a recent book (Barlow, Bartholomew, Bremner and Brunk Section E, [44])it has been shown that the isotonic repression solves also some restricted maximum likelihood estimation problems.

This fact may suggest immediate applications in Industrial Statistics .

Example 1 : Consider for instance the montly average production of a certain plant in a year – say, μ_i , $i = 1, \ldots, 12$ and let us know that this production has been constantly increased from month to month, that is :

$$\mu_1 \leq \mu_2 \leq \cdots \leq \mu_{12} \qquad (2)$$

In this way, the *a priori* knowledge on ordering, required by the isotonic regression methodology is satisfied.

The purpose is now to estimate the monthly average production .

In fact, we have to construct – in the assumption of normality – the likelihood function :

$$L = \prod_{i=1}^{12} \frac{\sqrt{\eta_i}}{\sigma\sqrt{2\pi}} \exp\left(- \frac{(\bar{X}_i - \mu_i)^2 \eta_i}{2\sigma^2} \right) \qquad (3)$$

and then to maximize L subject to (2), where n_i is the sample size of $\bar{X}_i$ (sample mean of the i^{th} month) and σ^2 is the variance which for the sake of simplicity has been assumed to be the same and known.

An equivalent problem is that considering $\bar{X}_i$ an average quality of a certain quality characteristic. The formulation changes in minor points.

Now the open problem is that of finding control procedures for an average quality subject to an order restriction.

Example 2 : Suppose that a production line under statistical control delivers batches of finite products. At the final control station, from every batch are extracted samples of size N_i and suppose that n_i defectives are detected in the sample of size N_i .

Assigning a Poisson law, we have hence :

$$Prob. \{ X_i = n_i \} = (\lambda_i N_i)^{n_i} \frac{e^{-\lambda_i N_i}}{n_i!} \qquad i = 1, 2, \ldots, k. \qquad (4)$$

where X_i is the handled variable and λ_i is the Poisson parameter which has the well-known significance of an average.

It is natural to suppose that as long as SQC is applied during production, the average of defectives is decreasing in time, namely :

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k \tag{5}$$

and hence, the problem of estimating average number of defectives reduces to :

$$\text{Maximize} \prod_{i=1}^{k} (\lambda_i N_i)^{n_i} \frac{e^{-\lambda_i N_i}}{n_i!} \tag{6}$$

subject to (5).

The problem has a very elegant solution in terms of isotonic regression theory (see Barlow and Brunk, Section E [44], Theorem 3.1).

The open problem is similar to that expressed in the above example.

4. *Conclusions.* Going back to the question in the title : "is SQC in crisis?", our answer is positively NO and we believe the motivation of this answer does not need more argumentation.

Only a last point we like to stress : as long as Technology will exist (and it seems that no reasons are against !) it will need a tool for observing and keeping it under control. Computers are a major aid in doing this. But identification, definition and investigation of a practical or theoretical problem is still an art. And we hope the art is still only a human attribute.

N. B. The references are given in such a way to help the reader to discover easily touched areas in the text.

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Centro de Estadística Matemática
Academia de Ciencias de Rumania
Bucarest, Rumania

(Recibido en enero de 1975)