Revista Colombiana de Matemáticas
Volumen X (1976), págs. 83 - 92

THE INVERSION FORMULAE FOR SOME BESSEL AND HYPERGEOMETRIC TRANSFORMS

by

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ABSTRACT

By an appeal to the Laplace transform and their inverses we have obtained five inversion formulae for some Bessel function transforms. The inversion formula for hypergeometric transforms have been given whose kernel are the confluent hypergeometric function ${}_1F_1(\gamma-\alpha;\gamma;\mu x)$ and Gauss hypergeometric function ${}_2F_1(\alpha,\beta,\gamma;-1/\mu\, x)$. The results given earlier by Gómez López [8] follow as special cases of our results given in section 3.

§ 1. *Introduction.* **The Laplace transform of $\phi(x)$ is defined by :**

$$(1) \qquad \mathfrak{L}\{\phi(x)\} = \int_0^\infty e^{-xt}\phi(x)\,dx = \psi(t) \quad .$$

If we denote with $\mathfrak{L}^{-1}$ the inverse Laplace transform, then we have

$$\mathfrak{L}^{-1}\{\psi(t)\} = \phi(x)$$

with $\phi(x)$ and $\psi(t)$ related as in (1) ; we denote with $M\{f(u)\}$ or by $F(s)$ the Mellin transform of $f(u)$; generaly u is real and positive, s is a complex va-

riable of the form $s = \sigma + i\tau$, where σ and τ are real. The Mellin transform of $f(u)$ [9, p. 46] is defined by :

$$(2) \qquad M\{f(u)\} = F(s) = \int_0^\infty f(u)\, u^{s-1}\, du$$

with $f(u)\, u^{s-1} \in L_2(0,\infty)$. The inverse Mellin transform of $F(s)$ with $f(u)$ and $F(s)$ related as in (2), is written as follows [9, p. 60] :

$$(3) \qquad M^{-1}\{F(s)\} = f(u) = \frac{1}{2\pi i}\int_C F(s)\, u^{-s}\, ds \quad ,$$

where $F(s) \in L_2(\sigma - i\tau, \sigma + i\tau)$; $-\infty < \tau < \infty$, and C is a suitable contour in the complex s-plane. If $H(s)$ and $F(s)$ are, respectively, the Mellin transform of $h(u)$ and $f(u)$, then we have :

$$(4) \qquad \int_0^\infty h(u)\, f(u)\, du = \frac{1}{2\pi i}\int_C H(s)\, F(1-s)\, ds \, ,$$

known as the Parseval theorem for Mellin transforms [9, p. 94] . In this paper we use $\mathfrak{L}$ and $\mathfrak{L}^{-1}$ operators to obtain five inversion formulae for some Bessel function transforms and hypergeometric transforms whose kernels are ${}_1F_1(\gamma-\alpha, \gamma : \mu x)$ and ${}_2F_1(\alpha, \beta, \gamma, -\frac{1}{\mu}x)$, respectively. In section 3 we give two results whose particular cases where given earlier by Gómez López. We use the following results [4] .

$$(5) \qquad M[e^{\alpha x} K_\nu(\alpha x) ; s] = \frac{\pi^{\frac{1}{2}}\Gamma(s+\nu)\Gamma(s-\nu)}{2^s \cdot \alpha^s\, \Gamma(s+\frac{1}{2})}, \qquad \Re e(s) > |\Re e(\nu)|, \quad \Re e(\alpha) > 0$$

$$(6) \quad M[sen(ax) J_\nu(ax) ; s] = \frac{2^{\nu-1}\Gamma(\frac{1}{2}-s)\Gamma(\frac{1}{2}+\frac{1}{2}\nu+\frac{1}{2}s)}{a^s\, \Gamma(1+\nu-s)\Gamma(1-\frac{1}{2}\nu-\frac{1}{2}s)}, \qquad a > 0, \quad -1 < \Re e(\nu) < \Re e(s) < \tfrac{1}{2}$$

$$(7) \quad M[cos(ax) \cdot J_\nu(ax) ; s] = \frac{2^{\nu-1}\Gamma(\frac{1}{2}-s)\Gamma(\frac{1}{2}\nu+\frac{1}{2}s)}{a^s\Gamma(\frac{1}{2}-\frac{1}{2}\nu-\frac{1}{2}s)\Gamma(1+\nu-s)}, \qquad a > 0, \quad -\Re e(\nu) < \Re e(s) < \tfrac{1}{2}$$

$$(8) \quad M[e^{-\alpha x} I_\nu(\alpha x) ; s] = \frac{\Gamma(\frac{1}{2}-s)\,\Gamma(s+\nu)}{2^s a^s \pi^{\frac{1}{2}}(1+\nu-s)}, \quad -\Re e(\nu) < \Re e(s) < \tfrac{1}{2} .$$

$$(9) \quad M\left[H_\nu^{(1)}(ax); s\right] = \frac{(1-i)\,2^{s-1}\,\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)}{a^s\,\Gamma(\frac{1}{2}\nu-\frac{1}{2}s+1)}, \quad \Re e(\nu) < \Re e(s) < \frac{3}{2}$$

We shall also use the following important theorem given by C. Fox [6,p. 300].

If: (i) $\alpha > 0$, $(\frac{1}{2}\alpha + \beta) > 0$, $t > 0$,

(ii) $s = \sigma + i\mu$, σ *and* μ *both real and*

$F(s) \in L(\frac{1}{2} - i\infty, \frac{1}{2} + i\infty)$

then

$$(10) \quad \mathfrak{L}^{-1}\left\{\frac{1}{2\pi i}\int_C \Gamma(\alpha s+\beta)\,F(s)\,t^{-\alpha s-\beta}\,ds\right\} = \frac{1}{2\pi i}\int_C F(s)\,x^{\alpha s+\beta-1}\,ds$$

where, for both integrals, the contour C *may be the line* $\sigma = \frac{1}{2}$, *a line parallel to the imaginary axis in the complex s-plane. And*

$$(11) \quad \mathfrak{L}\left\{\frac{1}{2\pi i}\int_C \frac{H(s)}{\Gamma(\alpha s+\beta)}\,x^{\alpha s+\beta-1}\,ds\right\} = \frac{1}{2\pi i}\int_C H(s)\,t^{-\alpha s-\beta}\,ds\,.$$

§ 2. *Inversion Formulae for Bessel function transforms.* **We shall obtain an inversion formula for the following transform :**

$$(12) \qquad h(\alpha) = \int_0^\infty e^{\alpha x} K_\nu(\alpha x)\, f(x)\, dx$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind [2].

THEOREM 1 : If

(i) $\nu + \frac{1}{2} < 0$, $\Re e(s) > |\Re e(\nu)|$, $R(\alpha) > 0$,

(ii) $f(x) \in L_2(0,\infty)$, *thus* $h(\alpha)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{-\nu-\frac{1}{2}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(v) $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)$,

where $f(y)$ *is of bounded variation near the point* $y = x$, *then the inversion formula for the transform (12) is :*

$$(13) \qquad f(x) = \pi^{-\frac{1}{2}} x^\nu\, \mathfrak{L}^{-1}\{t^{\nu+\frac{1}{2}}\,\mathfrak{L}\{x^{-\nu+\frac{1}{2}}\,\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}h(\alpha)\}\}\}$$

Proof : We apply (4) to the right hand of (12) and by (5) we have :

$$(14) \qquad h(\alpha) = \frac{1}{2\pi i} \int_C F(1-s)\pi^{\frac{1}{2}}(2\alpha)^{-s} \frac{\Gamma(s+\nu)\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} ds .$$

For large positive x and $\alpha > 0$, the kernel of our integral equation behaves as

$$e^{\alpha x} K_\nu(\alpha x) \approx \sqrt{\frac{\pi}{2}} (\alpha x)^{-\frac{1}{2}} .$$

For small positive x and $\alpha > 0$, we have

$$e^{\alpha x} K_\nu(\alpha x) \approx 2^{\nu-1}\Gamma(\nu) e^{\alpha x} (\alpha x)^{-\nu} ;$$

since we have the condition $\nu + \frac{1}{2} < 0$ it follows that the expression $e^{\alpha x} \cdot K_\nu(\alpha x)$ belongs to $L_2(0,\infty)$ and since $f(u) \in L_2(0,\infty)$, the application of (4) to the right hand of (12) is justified. The contour C in the s-plane is the straight line $s = \frac{1}{2} + i\tau$ and τ varies from $-\infty$ to ∞. We shall now try to eliminate the other gamma func – tion [10, p. 273] along the line $s = \frac{1}{2} + i\tau$, for large $|\tau|$ we have :

$$\frac{\Gamma(s+\nu)\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} F(1-s) = F(1-s)\,(1+O(s^{-1}))$$

$$\frac{\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} F(1-s) = s^{-\nu-\frac{1}{2}} F(1-s)\,(1+O(s^{-1}))$$

and the condition $\nu < -\frac{1}{2}$. Multiplying both sides of (13) by $(2\alpha)^{-\nu}$ and applying $\mathfrak{L}^{-1}$, we have

$$\mathfrak{L}^{-1}\{(2\alpha)^{-\nu} h(\alpha)\} = \pi^{\frac{1}{2}}\{\frac{1}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} F(1-s) \frac{\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} x^{s+\nu-1} ds\} .$$

Now multiplying this expression by $x^{-\nu+\frac{1}{2}}$ and using the $\mathfrak{L}$ operator, we get

$$(15) \; \mathfrak{L}\{x^{-\nu+\frac{1}{2}}\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}h(\alpha)\}\} = \int_0^\infty e^{-xt} x^{-\nu+\frac{1}{2}}\{\pi^{\frac{1}{2}} \frac{1}{2\pi i}\int_C F(1-s)\frac{\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} x^{s+\nu-1} ds\}dx$$

The integral of the relation (15) is absolutely convergent and

$$s^{-\nu-\frac{1}{2}} F(1-s) \in L_2(\tfrac{1}{2}-i\infty, \tfrac{1}{2}+i\infty) .$$

Hence, integrating with respect to x and eliminating $\Gamma(s+\frac{1}{2})$, we obtain

$$\mathfrak{L}\{x^{-\nu+\frac{1}{2}}\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}b(\alpha)\}\} = \pi^{\frac{1}{2}}\frac{1}{2\pi i}\int_C F(1-s)\Gamma(s-\nu)t^{-s-\frac{1}{2}}ds.$$

Multiplying both sides by $t^{\nu+\frac{1}{2}}$ and applying $\mathfrak{L}^{-1}$, we get :

$$\mathfrak{L}^{-1}\{t^{\nu+\frac{1}{2}}\mathfrak{L}\{x^{-\nu+\frac{1}{2}}\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}b(\alpha)\}\}\} = \pi^{\frac{1}{2}}\frac{1}{2\pi i}\int_C F(1-s)x^{s-\nu-1}ds.$$

Replacing s by $1-s$, we have :

$$\mathfrak{L}^{-1}\{t^{\nu+\frac{1}{2}}\mathfrak{L}\{x^{-\nu+\frac{1}{2}}\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}b(\alpha)\}\}\} = \pi^{\frac{1}{2}}\frac{1}{2\pi i}x^{-\nu}\int_C F(s)x^{-s}ds = \pi^{\frac{1}{2}}x^{-\nu}f(x)$$

which is justified by virtue of the condition : $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)$. Hence

$$f(x) = \pi^{-\frac{1}{2}}x^{\nu}\mathfrak{L}^{-1}\{t^{\nu+\frac{1}{2}}\mathfrak{L}\{x^{-\nu+\frac{1}{2}}\mathfrak{L}^{-1}\{(2\alpha)^{-\nu}b(\alpha)\}\}\}.$$

Proceeding in the same way, we can easily establish the following theorems.

THEOREM 2 : If

$$\varphi(a) = \int_0^\infty \operatorname{sen}(ax)J_\nu(ax)f(x)\,dx \tag{16}$$

and

(i) $\nu > 0$, $a > 0$,

(ii) $f(x) \in L_2(0,\infty)$, *thus* $\varphi(a)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{-\frac{1}{2}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+\infty)$,

$s^{\nu}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

$s^{\frac{1}{2}\nu+\frac{3}{4}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty$, $\frac{1}{2}+i\infty)$

(v) $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)$,

where $f(y)$ *is of bounded variation near the point* $y=x$ *, then the inversion formula* *for the transform (16) is :*

$$(17)\quad \mathfrak{L}^{-1}\{\tau^{-3/2}[\mathfrak{L}\{z^{-\frac{1}{2}\nu+\frac{1}{4}}\mathfrak{L}^{-1}\{t^{-\frac{1}{2}}[\mathfrak{L}\{a^{\nu}\varphi(a)\}]_{x=z^{\frac{1}{2}}}\}\}]_{t=\tau^{-1}}\} =$$

$$2^{\nu-1}\tau^{\nu+1}z^{\frac{1}{2}\nu}f(z^{\frac{1}{2}}).$$

THEOREM 3 : If :

$$(18)\qquad g(a) = \int_0^\infty \cos(ax)\, J_\nu(ax)\, f(x)\, dx$$

and

(i) $\nu > 0\,,\ a > 0\,,$

(ii) $f(x) \in L_2(0,\infty)$, *thus* $g(a)$ *is defined and also belongs to* $L_2(0,\infty)$.

(iii) $s^{-\frac{1}{2}}F(1-s) \in L_2(\frac{1}{2}-i\infty\,,\ \frac{1}{2}+i\infty)\,,$

$s^{\nu}F(1-s) \in L_2(\frac{1}{2}-i\infty\,,\ \frac{1}{2}+i\infty)\,,$

$s^{\frac{1}{2}\nu+\frac{1}{4}}F(1-s) \in L_2(\frac{1}{2}-i\infty\,,\ \frac{1}{2}+i\infty)$

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty\,,\ \frac{1}{2}+i\infty)$

(v) $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)\,,$

where $f(y)$ *is of bounded variation near the point* $y=x$, *then the inversion formula for the transform (18) is :*

$$(19)\quad \mathfrak{L}^{-1}\{\tau^{-\frac{1}{2}}[\mathfrak{L}\{z^{-\frac{1}{4}-\frac{1}{2}\nu}[\mathfrak{L}^{-1}\{t^{\frac{1}{2}+\nu}\mathfrak{L}\{a^{\nu}g(a)\}\}]_{x=z^{\frac{1}{2}}}\}]_{t=\tau^{-1}}\}$$

$$= 2^{\nu-1}z^{\frac{1}{2}\nu-\frac{1}{2}}f(z^{\frac{1}{2}})$$

THEOREM 4 : If :

$$(20)\qquad h(a) = \int_0^\infty H_\nu^{(1)}(ax)\, f(x)\, dx$$

and

(i) $a > 0\,,\ \nu > 0\,,$

(ii) $f(x) \in L_2(0,\infty)$, *thus* $h(a)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{-\frac{1}{2}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

$s^{\frac{1}{2}\nu+\frac{1}{4}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(v) $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)$,

where $f(y)$ *is of bounded variation near the point* $y=x$, *then the inversion formula for the transform (20) is:*

$$(21)\quad \mathfrak{L}^{-1}\{\tau^{-\nu-1}[\mathfrak{L}\{a^{\frac{1}{2}\nu}[h(2a)]_{a=a^{\frac{1}{2}}}\}]_{t=\tau^{-1}}\} = \frac{1-i}{2}\cdot x^{-\frac{1}{2}+\frac{1}{2}\nu}f(x^{\frac{1}{2}})$$

$H_\nu^{(1)}(\cdot)$ *stands for the Hankel function of the second kind* [2].

THEOREM 5: *If*

$$(22)\qquad g(\alpha) = \int_0^\infty e^{-\alpha x}\cdot I_\nu(\alpha x)\, f(x)\, dx$$

and

(i) $\alpha > 0$, $\nu > 0$,

(ii) $f(x) \in L_2(0,\infty)$, *thus* $g(\alpha)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{\nu+\frac{1}{2}}F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(v) $y^{-\frac{1}{2}}f(y) \in L_2(0,\infty)$,

where $f(y)$ *is of bounded variation near the point* $y=x$, *then the inversion formula for the transform (22) is:*

$$(23)\quad f(x) = \pi^{\frac{1}{2}}x^{-\nu}\mathfrak{L}^{-1}\{\tau^{-\nu+\frac{1}{2}}[\mathfrak{L}^{-1}\{t^{\frac{1}{2}+\nu}\mathfrak{L}\{a^\nu g(\tfrac{a}{2})\}\}]_{x=\tau}\}.$$

§ 3. *Inversion formulae for hypergeometric transform.* **In this section we establish two theorems, which provide us the inversion formulae for two hypergeometric transforms. Using the following results [4].**

$$(24)\qquad M\{x^{\beta} e^{-ux}\, {}_1F_1(\gamma-\alpha\,;\gamma\,;\mu x)\} = u^{-(s+\beta)}\, \frac{\Gamma(s+\alpha)\cdot\Gamma(\gamma)\Gamma(-s)}{\Gamma(\alpha)\cdot\Gamma(\gamma-s-\alpha)}$$

$$\Re(\mu)>0,\ \Re(\alpha)>\Re(s)>0,\ \gamma\neq 0,-1,-2$$

$$(25)\qquad M\left\{x^{\rho} H^{1,2}_{2,2}\left[\frac{1}{\mu x}\,\middle|\,\begin{matrix}(1-\alpha,1)(1-\beta,1)\\(0,1)\quad(1-\gamma,1)\end{matrix}\right]\right\} = u^{s+\rho}\,\frac{\Gamma(s+\rho)\Gamma(\alpha-s-\rho)\Gamma(\beta-s-\rho)}{\Gamma(\gamma-s-\rho)}$$

$$\Re\{s+1+\min(0,1-\gamma)\} > 0$$

$$\Re\{\max(-\alpha,-\beta)+s+1\} < 0$$

and proceeding in the same way as in the theorem 1, the following theorems can be proved easily.

THEOREM 6 : If

$$(26)\qquad \varphi(u) = \int_0^{\infty} x^{\beta} e^{-ux}\, {}_1F_1(\gamma-\alpha,\gamma;ux)\ f(x)\,dx$$

and

(i) $\beta > 0,\ \alpha > \frac{\gamma}{2}$; $\beta > \frac{\gamma}{2} - \frac{1}{2}$, $\alpha > 0$,

(ii) $f(x) \in L_2(0,\infty)$, *thus* $\varphi(u)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{\alpha-\gamma+\beta+\frac{1}{2}} F(1-s) \in L_2(\frac{1}{2}-i\infty,\ \frac{1}{2}+i\infty)$,

$s^{\alpha}\cdot F(1-s) \in L_2(\frac{1}{2}-i\infty,\ \frac{1}{2}+i\infty)$,

$s^{\alpha-\beta-\frac{1}{2}} F(1-s) \in L_2(\frac{1}{2}-i\infty,\ \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty,\ \frac{1}{2}+i\infty)$,

(v) $y^{-\frac{1}{2}} f(y) \in L_2(0,\infty)$,

where $f(y)$ *is of bounded variation near the point* $y=x$, *then the inversion formula for the transform (26) is :*

$$(27)\qquad \mathfrak{L}^{-1}\{t^{-\alpha+1}[\mathfrak{L}^{-1}\{\tau^{-\gamma}[\mathfrak{L}\{u^{\gamma-1}\varphi(u)\}]_{t=\tau^{-1}}\}]_{x=t}\} = \frac{\Gamma(\gamma)}{\Gamma(\alpha)}\, x^{\alpha-\beta-2} f(x^{-1}).$$

Particular case : **If we set** $\beta=0$ **, this theorem reduces to the one considered** recently by **Gómez López** [8, (26)] .

THEOREM 7 : If

$$(28) \qquad g(u) = \int_0^\infty x^{\rho}\, {}_2F_1\left(\alpha, \beta, \gamma ; -\frac{1}{u} x\right) f(x)\, dx$$

and

(i) $\rho > 0,\ \rho-\alpha < 0,\ \rho-\beta < -\frac{1}{2},\ \alpha+\beta-\gamma > 0,\ \alpha+\beta-\rho > \frac{1}{2},\ \alpha+\beta-2\rho > 1$

(ii) $f(x) \in L_2(0,\infty)$ *thus* $g(u)$ *is defined and also belongs to* $L_2(0,\infty)$,

(iii) $s^{\alpha+\beta-\gamma} F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

$s^{\alpha-\rho+\beta-\frac{1}{2}} F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

$s^{\alpha+\beta-2\rho-1} F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$

$s^{\beta-\rho-\frac{1}{2}} F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(iv) $F(1-s) \in L_2(\frac{1}{2}-i\infty, \frac{1}{2}+i\infty)$,

(v) $y^{-\frac{1}{2}} f(y) \in L_2(0,\infty)$

where $f(y)$ *is of bounded variation near the point* $y=x$ *then the inversion formula for the transform (28) is*

$$(29)\ \mathfrak{L}^{-1}\left\{ t^{-\beta+\alpha-1} \left[\mathfrak{L}^{-1}\left\{ t^{-\alpha+1} \left[\mathfrak{L}^{-1}\left\{ \tau^{-\gamma} \left[\mathfrak{L}\left\{ u^{\gamma-1} g(u^{-1}) \right\}\right]_{t=\tau^{-1}} \right\}\right]_{x=t} \right\}\right]_{x=t^{-1}} \right\}$$

$$= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\,\Gamma(\beta)}\, x^{\beta-\rho-2} f(x^{-1}) .$$

Particular case : **If we put** $\rho=0$ **this theorem reduces to the one considered recently by Gómez López** [8, (22)] .

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(Recibido en noviembre de 1975).