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ANY EQUIVALENCE RELATION OVER A CATEGORY IS A SIMPLICIAL HOMOTOPY

by

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§ 1. *Simplicial Systems.*

Definition.([1]) A simplicial system over a category $\mathcal{C}$ is a triple $J=(\mathcal{H},\Phi,\lambda)$ where $\mathcal{H}:\mathcal{C}^o\times\mathcal{C}\to\Delta^o S$ ($\Delta^o S$ = the category of simplicial sets) is a covariant functor, Φ is an associative "composition law" with $\Phi_{XYZ}:\mathcal{H}(X,Y)\times\mathcal{H}(Y,Z)\to\mathcal{H}(X,Z)$ natural in X,Y,Z, and γ is a natural isomorphism $\gamma_{X,Y}:\mathcal{C}(X,Y)\to\mathcal{H}(X,Y)_o$ (We will denote $\alpha\bullet\beta=\Phi(\beta,\alpha)$ for $\alpha\in\mathcal{H}(X,Y)_n$ and $\beta\in\mathcal{H}(Y,Z)_n$). Moreover, J is subjected to the following conditions :

(i) for each morfism $u:X\to Y$ of $\mathcal{C}$, and each $f\in\mathcal{H}(Y,Z)_n$, then $f\bullet s^{(n)}(u)=\mathcal{H}(u,Z)(f)$;

(ii) for each $g\in\mathcal{H}(W,X)_n$ and each $u\in\mathcal{C}(X,Y)$, $s^{(n)}(u)\bullet g=\mathcal{H}(W,u)(g)$, where $s^{(n)}(u)$ stands for the image of u by the following composition $s_o\cdots s_o$ (n-times), where s_o denotes the *0-th* degeneracy in each dimension. Also we have used for a fixed Z in $\mathcal{C}$ the restriction $\mathcal{H}(-,Z):\mathcal{C}^o\to\Delta^o S$ of the functor $\mathcal{H}$ which for each $u:X\to Y$ in $\mathcal{C}$, induces a simplicial map $\mathcal{H}(u,Z):\mathcal{H}(Y,Z)\to\mathcal{H}(X,Z)$. Similarly, if one fixes the first variable.

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A simplicial category is a pair $(\mathcal{C}, J)$ where J is a simplicial system over a given category $\mathcal{C}$.

The homotopy relation over morphism associated with the system J is given as follows : $f, g : X \to Y$ (in $\mathcal{C}$) are J-homotopic, or more precisely, f is J-homotopic to g (in that order), if there exists $v \in \mathcal{H}(X,Y)_1$ such that $d_0(v) = f$ and $d_1(v) = g$. It is well known that if $\mathcal{H}(X,Y)$ is a Kan simplicial set – in lower dimensions– then this homotopy relation es an equivalence relation. Furthermore, it is compatible with composition. In fact, the categorical simplicial structure allows a composition of homotopies : if $H \in \mathcal{H}(X,Y)_1$ and $K \in \mathcal{H}(Y, Z)_1$ are such that $H : f \sim g$ and $K : u \sim v$ then $K \bullet H = \Phi(H, K)$ is a homotopy $uf \sim vg$.

§ 2. *Some examples of simplicial categories.*

a) In the category of topological spaces taking $\mathcal{H}(X,Y)_n = Top(\Delta(n) \times X, Y)$ with faces induced by the co-faces of the standard co-simplicial topological space Δ, we obtain a simplicial system .

b) The same construction in $\Delta^o S$ using $\Delta[n]$ instead of $\Delta(n)$.

c) Generalizing a) and b), above, if a category $\mathcal{C}$ is closed for finite products, then for each model $Y : \Delta \to \mathcal{C}$ (that is, a covariant functor) such that $Y[0]$ = final object of $\mathcal{C}$, whenever it exists, one defines $\mathcal{H}_Y(A, B)_n = \mathcal{C}(Y[n] \times A, B)$ and completes it by the same categorical procedures as in a) and b). Given the importance of this example and its generality we will devote the next paragraph to a detailed discussion of it.

d) In the paper "Homotopic Systems in categories with a Final Object" ([5]) it is shown that, if $Y : \Delta \to \mathcal{C}$ is a model in which $Y[0]$ is not necessarily the final object of $\mathcal{C}$, then one can consider the category of objects over $Y[0]$, denoted $\mathcal{C}/Y[0]$ · The model Y induces a model $Y/Y[0] = Y' : \Delta \to \mathcal{C}/Y[0]$, in which, of course, $Y'[0]$ is then the final object of $\mathcal{C}/Y[0]$. If $\mathcal{C}$ is closed for fibered products over $Y[0]$, then we can apply the procedure of part c) to induce over $\mathcal{C}/Y[0]$ a simplicial structure. For example, if $\mathcal{C} = Ab =$ the category of abelian groups and $Y : \Delta \to Ab$ is the *free abelian group functor*, restricted to Δ, then there exists over $Ab/\mathbb{Z}$ a simplicial structure associated with Y. Similarly, if one *tensorizes* Y by an abelian group M to get $(Y \otimes M)_n = Y[n] \otimes M$, then $Y \otimes M$ induces a simplicial structure over Ab/M (since $(Y \otimes M)[0] = M$), which is natural in M, in the sense that this assignent $M \to Ab/M$ can be completed to a functor from Ab into the category $C.Sim$ (cf. § 4).

e) A group G can be considered as a category with only one object, say e, and one morphism $\bar{g} : e \to e$ for each element g of G, the composition then given by $\bar{g} \bullet \bar{h} = \overline{gh}$. We will denote by G both the category and the group. $N(G)$ will represent the nerve of the category G (in [2], p. 32, this is denoted by $D(G)$). We will prove that there exists a non trivial (natural) simplicial structure over G when G is abelian. In fact we take $\mathcal{H}(e,e)$ to be the simplicial set $RC(N(G))$ where RC stands for the right - cut- functor $RC : \Delta^o S \to \Delta^o S$ (cf. [4]) defined for a simplicial set X by the formulae : (i) $RC(X)_n = X_{n+1}$, $(n \geq 0)$ (ii) $\partial_i^n : RC(X)_n \to RC(X)_{n-1}$ is the morphism $d_i^{n+1} : X_{n+1} \to X_n$ $(i = 0, \ldots, n)$; (iii) $\sigma_i^n : RC(X)_n \to RC(X)_{n+1}$ is the morphism $s_i^{n+1} : X_{n+1} \to X_{n+2}$ $(i = 0, \ldots, n)$. In order to complete the definition of $\mathcal{H} : G^o \times G \to \Delta^o S$ we associate to $x, y : e \to e$ the map $(x,y)_\# : \mathcal{H}(e,e) \to \mathcal{H}(e,e)$ defined by the following equality $(x,y)_\# (g_o, \ldots, g_n) = (g_o, \ldots, g_{n-1}, y\, g_x\, x)$. In order to this maps be simplicial it is necessary and sufficient that G be an abelian group. As for the simplicial composition $\Phi_{ee} = \Phi : \mathcal{H}(e,e) \times \mathcal{H}(e,e) \to \mathcal{H}(e,e)$, it is given by $\Phi((g_o, \ldots, g_n) ; (h_o, \ldots, h_n)) = (h_o g_o, \ldots, h_n g_n)$. Again, Φ so defined is a simplicial map if and only if G is an abelian group. Furthermore, $\mathcal{H}(e,e)_o = CR(N(G))_o = N(G)_1 = G = Hom_G(e,e)$. Now for $u \in Hom_G(e,e)$ it holds that $\Phi(f, s^{(n)}(u)) = \mathcal{H}(u,e)(f)$, since the right hand member of the equality is $(u, 1_e)_\# (f) = (f_o, \ldots, f_{n-1}, f_n u)$ for $f = (f_o, \ldots, f_n)$, and $s^{(n)}(u) = s_o \ldots s_o(u) = (1, \ldots, 1, u)$. Similarly $\Phi(s^{(n)}u, f) = \mathcal{H}(e,u)(f) = (1_e, u)_\# (f)$.

Remark: In the previous construction $\mathcal{H}(e,e)$ becomes the total space $W(\tilde{G})$ of $\overline{W}(\tilde{G})$, the classifying space of $\tilde{G}$, where $\tilde{G}_n = G$ for each n and the faces being the identity morphism. That is to say $W(\tilde{G}) = RC(N(G))$.

This construction can certainly be generalized to abelian monoids, in which case the homotopy obtained is non trivial (aggainst the case of abelian groups in which it is trivial): $f \sim g$ if there exists $a \in G$ such that $f\ a = g$. The problem of existence of homotopy is thus equivalent to the problem of solution of first degree equations in G.

f) There is a way to induce, trivially, a simplicial system on a category $\mathcal{C}$ by taking $\mathcal{H}(X,Y)_n = \mathcal{C}(X,Y)$, for each n, and faces to be the identity function. The homotopy relation obtained is the relation of equality.

§3. *The simplicial system associated to a model* $Y: \Delta \to \mathcal{C}$.

Let $\mathcal{C}$ be a category with a final object and with finite products. Let $Y: \Delta \to \mathcal{C}$ be a covariant functor such that $Y[0]$ = the final object of $\mathcal{C}$. We define, for each pair of objects A,B in $\mathcal{C}$, the simplicial set $\mathcal{H}(A,B)$ by the formulae : (i) $\mathcal{H}(A,B)_n = \mathcal{C}(Y[n] \times A, B)$; (ii) if $w: [n] \to [m]$ is a morphism in Δ, then $w^*: \mathcal{H}(A,B)_m \to \mathcal{H}(A,B)_n$ is the map $u \to u \circ (Y(w) \times A)$, where A stands for the identity morphism of A. The simplicial composition $\Phi : \mathcal{H}(A,B) \times \mathcal{H}(B,C) \to \mathcal{H}(A,C)$ is given for $f: Y[n] \times A \to B$ and $g: Y[n] \times B \to C$ by

$$Y[n] \times A \xrightarrow{\partial \times A} Y[n] \times Y[n] A \xrightarrow{Y[n] \times f} Y[n] \times B \xrightarrow{g} C,$$

where ∂ is the diagonal morphism. To prove that Φ is simplicial it suffices to prove that, for morphisms $w: K \to L$, $f: L \times A \to B$, and $g: L \times B \to C$ the following diagram commutes.

In order to do this it suffices to apply, for each X of $\mathcal{C}$, the functor $\mathcal{C}(X,-)$ to the diagram above. Then it becomes the same statement (or diagram) but in the category of sets. (recall that in order to prove that a diagram in a category commutes, it is necessary and sufficient that for each object X, the image of the diagram

$$
\begin{array}{ccccccc}
K\times A & \xrightarrow{\partial\times A} & K\times K\times A & \xrightarrow{K\times w\times A} & K\times L\times A & \xrightarrow{K\times f} & K\times B \\
\downarrow{\scriptstyle w\times A} & & & & & & \downarrow{\scriptstyle w\times B} \\
L\times A & & & & & & L\times B \\
\downarrow{\scriptstyle \partial\times A} & & & & & & \downarrow{\scriptstyle g} \\
L\times L\times A & & \xrightarrow{L\times f} & & L\times B & \xrightarrow{g} & C
\end{array}
$$

by $\mathcal{C}(X,-)$, resp. $\mathcal{C}(-,X)$, commutes in the category of sets). This is due to the fact that $\mathcal{C}(X,-)$ commutes with products and that $\mathcal{C}(X,\partial_Y) = \partial_{\mathcal{C}(X,Y)}$.

As far as associativity is concerned (of the simplicial composition Φ) it reduces to proving that the following diagram commutes in $\mathcal{C}$ for any morphism $f: K\times A \to B$

$$
\begin{array}{ccccc}
K\times A & \xrightarrow{\partial\times A} & K\times K\times A & \xrightarrow{K\times f} & K\times B \\
\downarrow{\scriptstyle \partial\times A} & & & & \downarrow{\scriptstyle \partial\times B} \\
K\times K\times A & \xrightarrow[K\times\partial\times A]{} & K\times K\times K\times A & \xrightarrow[K\times K\times f]{} & K\times K\times B
\end{array}
$$

§ 4. *The categories C. Sim and C. Rel.*

A simplicial functor $(\mathcal{C}, J) \to (\mathcal{C}', J')$ between simplicial categories is a pair (F, δ), where $F: \mathcal{C} \to \mathcal{C}'$ is a functor and $\delta: \mathcal{H}_{\mathcal{C}}(-,-) \to \mathcal{H}_{\mathcal{C}'}(F(-),F(-))$ is a natural transformation such that for any objects X, Y, Z of $\mathcal{C}$:

SF.1) the following diagram commutes

$$
\begin{array}{ccc}
\mathcal{H}_{\mathcal{C}}(X,Y)_o & \xrightarrow{\delta_{XY}} & \mathcal{H}_{\mathcal{C}'}(F(X),F(Y))_o \\
\downarrow{\scriptstyle \gamma_{XY}} & & \downarrow{\scriptstyle \gamma'_{F(X)F(Y)}} \\
\mathcal{C}(X,Y) & \xrightarrow[F]{} & \mathcal{C}'(F(X),F(Y))
\end{array}
$$

S.F.2) the following diagram commutes

$$
\begin{array}{ccc}
\mathcal{H}_{\mathcal{C}}(X,Y)\times \mathcal{H}_{\mathcal{C}}(Y,Z) & \xrightarrow{\quad\Phi\quad} & \mathcal{H}_{\mathcal{C}}(X,Z) \\
\Big\downarrow{\scriptstyle \delta\times\delta} & & \Big\downarrow{\scriptstyle \delta} \\
\mathcal{H}_{\mathcal{C}'}(F(X),F(Y))\times \mathcal{H}_{\mathcal{C}'}(F(Y),F(Z)) & \xrightarrow[\quad\Phi'\quad]{} & \mathcal{H}_{\mathcal{C}'}(F(X),F(Z))
\end{array}
$$

In this case we will say that (F, δ) is compatible with the simplicial composition.

We will denote by *C.Sim* the category of simplicial categories and simplicial functors, and by *C. Rel* the category of categories with *compatible relations* in the following since : (a) a category $(\mathcal{C}, R)$ with a compatible relation R, consists of a category $\mathcal{C}$ and, for each pair of objects X, Y, of a reflexive and transitive relation over the set $\mathcal{C}(X,Y)$ which is compatible with the composition in $\mathcal{C}$; (b) a morphism $F: (\mathcal{C}, R) \to (\mathcal{C}', R')$ between categories with compatible relations is a relation-preserving functor $F: \mathcal{C} \to \mathcal{C}'$ in the sense that if $(f,g) \in R_{XY}$ then $(F(f), F(g)) \in R'_{F(X)\,F(Y)}$.

The procedure that to a simplicial category $(\mathcal{C}, J)$ associates the reflexive and transitive relation generated by homotopy, denoted by $(\mathcal{C}, R(J))$, gives rise to a functor $\mathcal{R} : C.\,Sim \to C.\,Rel.$

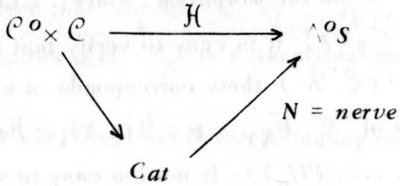
We now give the main theorem of this paper :

THEOREM. The functor $\mathcal{R}$ *admits a right adjoint* $\mathcal{S} : C.\,Rel \to C.\,Sim$.

We devote the rest of this paper to the proof of this theorem. To begin with we define the functor $\mathcal{S}$.

Let $(\mathcal{C}, R)$ be a category with a reflexive, transitive, and compatible relation. Since R_{XY} is reflexive and transitive it can be considered in itself as a category with objects the elements of $\mathcal{C}(X,Y)$ and a morphism $f \to g$ (and ohly one) if $(f,g) \in R_{XY}$. We define $\mathcal{H} : \mathcal{C}^o \times \mathcal{C} \rightarrow \Delta^o S$ by taking as $\mathcal{H}(X,Y)$ the nerve (see [2] p.32 for the definition of nerve, which is denoted by D) of the category R_{XY}. We will use $\mathcal{H}_{(\mathcal{C},R)}(X,Y)$ instead of $\mathcal{H}(X,Y)$ when emphasis on the category and the relation is necessary.

We remark that the functor $\mathcal{H}$ is the composite



where $\mathcal{C}^o \times \mathcal{C} \rightarrow Cat$ maps (X,Y) into the category associated with the relation R_{XY}. One also notices that if $\alpha : X' \rightarrow X$ and $\beta : Y \rightarrow Y'$ are morphisms of $\mathcal{C}$, then the functor $(\alpha, \beta)_{\#} : (\mathcal{C}(X,Y) \, ; \, R_{XY}) \rightarrow (\mathcal{C}(X',Y') \, ; \, R_{X'Y'})$ is the map $f \rightarrow \beta f \alpha$. $N((\alpha,\beta)_{\#})$ is given in dimension n by $N(\alpha,\beta)_{\#}(f_o,\ldots,f_n) = (\beta f_o \alpha, \ldots, \beta f_n \alpha)$, for each $(f_o,\ldots,f_n) \in N(\mathcal{C}(X,Y) \, ; \, R_{XY})_n$.

We now define the simplicial composition $\Phi_{XYZ} : \mathcal{H}(X,Y) \times \mathcal{H}(Y,Z) \rightarrow \mathcal{H}(X,Z)$. We recall that the nerve $N : Cat \rightarrow \Delta^o S$ commutes with products and since $\mathcal{H}(X,Y) = N(R_{XY})$ then we take $\Phi_{XYZ} = N(\varphi_{XYZ})$, where φ_{XYZ} is the functor (natural in X,Y,Z) defined on the objects by the composition $\mathcal{C}(X,Y) \times \mathcal{C}(Y,Z) \rightarrow \mathcal{C}(X,Z)$, and on the morphisms $R_{XY} \times R_{YZ} \rightarrow R_{XZ}$ by the compatibility of the relation R. More explicity the composition in dimension n is given by $(g_o,\ldots,g_n) \bullet (f_o,\ldots,f_n) = (g_o f_o, \ldots, g_n f_n)$, easily proved to be well defined.

As for the natural transformation γ it is, in our case, the identity of $\mathcal{C}(X,Y)$ since by the definition of nerve, $\mathcal{H}(X,Y)_o = N(\mathcal{C}(X,Y) \, ; \, R_{XY})_o = \mathcal{C}(X,Y)$.

In order to complete the proof that $(\mathcal{H}, \Phi, \gamma)$ is a simplicial system let $u : X \rightarrow Y \in \mathcal{C}$ and $f \in \mathcal{H}(Y,Z)_n$; then $f \circ s^{(n)}(u) = \mathcal{H}(u,Z)(f)$ because if $f = (f_o, \ldots, f_n)$ then $\mathcal{H}(u,Z)(f) = (u, 1_Z)_{\#}(f) = (1_Z \circ f_o \circ u, \ldots, 1_Z \circ f_n \circ u) =$

$= (f_o, \ldots, f_n) \bullet (u, \ldots, u) = f \circ s^{(n)}(u)$. Similarly, one can prove that $s^{(n)}(u) \circ g = \mathcal{H}(W,u)(g)$, for each $g \in \mathcal{H}(W,X)_n$ and each $u \in \mathcal{C}(X,Y)$.

We denote $J(\mathcal{C}, R) = (\mathcal{H}, \Phi, \gamma)$ given above and $\mathcal{S}(\mathcal{C}, R)$ the simplicial category $(\mathcal{C}; J(\mathcal{C}, R))$.

We proceed now to give $\mathcal{S}$ on the morphism : since $\mathcal{H}(X,Y)_n = \{(f_o, \ldots, f_n) \mid (f_i, f_{i+1}) \in \mathcal{R}_{XY}, 0 \leq i \leq n-1\}$ it is easy to verify that to a relation preserving functor $F : (\mathcal{C}, \mathcal{R}) \to (\mathcal{C}' \mathcal{R}')$ there corresponds a simplicial function for each pair X,Y of objects of $\mathcal{C}$, $\tilde{F}_{XY} = \tilde{F} : \mathcal{H}(X,Y) \to \mathcal{H}(F(X), F(Y))$, given by $(f_o, \ldots, f_n) \to (F(f_o), \ldots, F(f_n))$. It is also easy to verify that, if $f \in \mathcal{H}(X,Y)_n$ and $g \in \mathcal{H}(Y,Z)_n$, then $\tilde{F}(g \bullet f) = \tilde{F}(g) \bullet \tilde{F}(f)$ which proves the functorial condition $SF.2)$. Thus, to a functor $F: (\mathcal{C}, R) \to (\mathcal{C}', R')$ we have associated the pair $(F, \tilde{F}) : (\mathcal{C}, J(\mathcal{C}, R)) \to (\mathcal{C}', J(\mathcal{C}', R'))$ which also verifies $SF.1)$, and which we will denote by $\mathcal{S}(F)$, thus completing the definition of the functor $\mathcal{S} : C.\,Rel \to C.\,Sim$.

It remains to prove that the pair $(\mathcal{R}, \mathcal{S})$ is adjoint ($\mathcal{R}$ is left adjoint of $\mathcal{S}$).

We give first the natural transformations for adjointness : if $X = (\mathcal{C}, J)$ is a simplicial category with $J = (\mathcal{H}, \Phi, \gamma)$ and $V = (\mathcal{D}, R)$ is a category with a compatible, reflexive and transitive relation R, we will give $\theta_X : X \to \mathcal{S}\mathcal{R}(X)$ and $\mu_V : \mathcal{R}\mathcal{S}(V) \to V$ for which we notice that $\mathcal{R}\mathcal{S}(V) = V$ and therefore the functor $\mathcal{R}$ is a retract of the functor $\mathcal{S}$. Thus the transformation μ is simply the identity. θ_X is a pair (F, δ) where F is a functor with source and target the category $\mathcal{C}$ and $\delta : \mathcal{H}_X(\ .\ ) \to \mathcal{H}_{\mathcal{S}\mathcal{R}(X)}(F(\ ), F(\ ))$ is a natural transformation: F will be the identity of $\mathcal{C}$ hence it remains to give $\delta_{A,B}$ for objects A,B in $\mathcal{C}$. Notice that, in general, we can define $\delta_W : W \to W'$ where W is any simplicial set and W' is the following simplicial set : on W_o let R be the transitive relation associated to the homotopy relation of W. We take $W'' = N(R) =$ the nerve of R. We recall that W' so obtained is level-wise given by $W'_o = W_o$, $W'_1 = \{(u,v) \mid u,v \in W_o, (u,v) R\}$, and in general $W'_n = \{(u_o, \ldots, u_n) \mid (u_i, u_{i+1}) \in \mathcal{R}$, $i =$

$0, \ldots, n-1\}$, with faces $d_j(u_o, \ldots, u_n) = (u_o, \ldots, \hat{u}_j, \ldots, u_n)$, $s_j(u_o, \ldots, u_n) = (u_o, \ldots, u_j, u_j, u_{j+1}, \ldots, u_n)$. Now we can give δ_W. What is desired with this map is to associate with a simplex $x \in W_n$, the ordered set of its vertexes. More precisely, with each $w : [0] \to [m]$ we associate $w^* : W_n \to W_o$ and with this we construct the faces $w^*(x)$. If we denote by $w_k : [0] \to [n]$ the map $w_k(0) = k$, then we take $\delta_W(x) = (w_o^*(x), w_1^*(x), \ldots, w_n^*(x))$, which can be seen to belong to W'_n. Notice that $w_k^*(x) = d_o d_1 \ldots d_k \ldots d_n(x)$ $(0 \leq k \leq n)$. The following lemma implies that δ_W is a simplicial map.

LEMMA. In a simplicial set X the following relations hold :

$$d_o \ldots \hat{d}_i \ldots d_{n-1}(d_j(x)) = \begin{cases} d_o \ldots \hat{d}_i \ldots d_n(x) & \text{if } i < j \text{ and } x \in X_n \\ d_o \ldots \hat{d}_{i+1} \ldots d_n(x) & \text{if } i \geq j \text{ and } x \in X_n \end{cases}$$

$$d_o \ldots \hat{d}_i \ldots d_n(x) = \begin{cases} d_o \ldots \hat{d}_i \ldots d_{n+1} s_j(x) & \text{if } i \leq j \text{ and } x \in X_n \\ d_o \ldots \hat{d}_{i+1} \ldots d_{n+1} s_j(x) & \text{if } i > j \text{ and } x \in X_n \end{cases}$$

The desired natural transformation is precisely $\delta_{\mathcal{H}(A,B)} : \mathcal{H}(A,B) \to \mathcal{H}(A,B') = NR\mathcal{H}(A,B)$, A, B, in $\mathcal{C}$.

In order to prove that θ and μ are actualls the natural transformation of adjointness, one uses the fact that $\mathcal{R} : \Delta^o S \to Rel$ and $\mathcal{S} : Rel \to \Delta^o S$ are adjoint functors. Here $\mathcal{R}(X) = (X, \sim)$ is the transitive relation associated to the homotopy of X, Rel is the category of the reflexive transitive relations (on sets), and $\mathcal{S}(Y, R)$ is the nerve of R.

COROLLARY. On each category with a compatible reflexive transitive rela -

tion there exists a simplicial systems whose simplicial homotopy relation is the given relation. Moreover, if the original relation is a symmetric one then the simplicial systems lies within the category of Kan-simplicial sets [3] .

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