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ON PRESERVATION OF NORMALITY

by

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ABSTRACT. The theory of normal spaces is treated in this paper. In section 1 we define a new type of mappings called para normal, and we prove that a continuous mapping onto a normal space is paranormal if and only if its domain is normal. The section ends with the study of relativized notions of normality and paracompactness. In section 2 we study the normality of the products. Finally, section 3 contains the discussion of counterexamples relevant to the definition and theorems in the previous sections.

Introduction. It is known that the counter image of a normal space under a perfect mapping need not be normal. For instance, Tamano [7] proved that for a completely regular space X, the product X×βX is normal if and only if X is paracompact. Consequently, if X is a normal non-paracompact space, then the projection of $X \times \beta X$ onto X is an example of such a mapping.

In this paper, we shall define a new type of mapping, called paranormal, which preserves normality in the above sense, and we shall discuss some ramifications of this definition. We also prove some product theorems for normal space. Finally, we discuss some counterexamples relevant to the definitions and theorems in this paper. We assume that every space throughout this paper is at least T_1. For terminology not define here, see [2] and [8].

§1. Paranormal Mappings.

DEFINITION 1.1. If $\mathcal{A} = \{A_\alpha | \alpha \in \Lambda\}$ and $\mathcal{B} = \{B_\alpha | \alpha \in \Lambda\}$ are two collections of open subsets of a space X, then $\mathcal{A}$ is called a *shrinking* of $\mathcal{B}$ provided that for each $\alpha \in \Lambda$, $\mathrm{Cl} A_\alpha \subset B_\alpha$.

Note that this concept has been considered in the literature (see [8], Def. 15.9).

DEFINITION 1.2. A continuous mapping $f: X \to Y$ is called *paranormal* if for every locally finite open cover $\mathcal{U}$ of X, there is a locally finite open cover $\mathcal{V}$ of Y such that for each v in $\mathcal{V}$, the counter image $f^{-1}(v)$ is covered by a shrinking of a subfamily of $\mathcal{U}$.

THEOREM 1.3. *Let* f *be a continuous mapping from a space* X *onto a normal space* Y. *Then* X *is normal if and only if* f *is paranormal.*

Proof. Suppose X is normal and $f:X \to Y$ is a continuous mapping. Let $\mathcal{U}$ be a locally finite open cover of X. Then $\mathcal{U}$ has a shrinking $\mathcal{W}$ which covers X.

Now $\{Y\}$ is a locally finite open cover of Y such that $f^{-1}(Y)$ is contained in the union of members of $\mathcal{W}$ and $\mathcal{W}$ is a shrinking of $\mathcal{U}$. Thus f is a paranormal mapping.

Conversely, suppose that $f:X \to Y$ is paranormal and Y is normal. To show that X is normal, it suffices to show that every locally finite open cover has a closed locally finite refinement (see [2]). Let $\mathcal{U}$ be such an open cover, then there is a locally finite open cover $\mathcal{V}$ of Y such that for each v in $\mathcal{V}$, $f^{-1}(v)$ is covered by a shrinking of a subfamily of $\mathcal{U}$, say $\mathcal{W}_v$. Since Y is normal, $\mathcal{V}$ has a shrinking $\mathcal{H}$ which covers Y. For each $H \in \mathcal{H}$, choose $V(H) \in \mathcal{V}$ such that $\mathrm{Cl}H \subset V(H)$. Let

$$\mathcal{S} = \{f^{-1}(\mathrm{Cl}H) \cap \mathrm{Cl}w \mid w \in \mathcal{W}_{V(H)},\ H \in \mathcal{H}\}$$

Now, $\mathcal{H}$ and $\mathcal{W}_{V(H)}$ are locally finite. By lemma 20.4, page 145, of [8], it follows that $\{\mathrm{Cl}H \mid H \in \mathcal{H}\}$ and $\{\mathrm{Cl}W \mid w \in \mathcal{W}_{V(H)}\}$ are locally finite. Therefore one can easily see that $\mathcal{S}$ is a closed locally finite refinement of $\mathcal{U}$ which covers X. Hence X is normal. ∎

REMARKS 1.4. (i) The example given in the introduction of this paper shows that a perfect mapping need not to be paranormal. (ii) The projection π of $\mathbb{R}\times\mathbb{R}$ onto $\mathbb{R}$ is paranormal (indeed, $\mathbb{R}\times\mathbb{R}$ is normal); on the other hand, π is not perfect. In fact

π is neither closed nor the $\pi^{-1}(y)$'s are compact.

The definition of paranormal mapping suggests the following property of topological subspaces.

DEFINITION 1.5. A subset F of a space (X,τ) *satisfies the condition* (α) *relative to* X, if every locally finite open cover of F by members of τ has a shrinking by members of τ which covers F.

The above definition resembles the following property known to be equivalent to normality: *every locally finite open cover has a shrinking which is also a cover*. In this connection we shall also introduce the following:

DEFINITION 1.6. A subset F of a space X is called *normal relative to* X if for every two disjoint closed sets C_1 and C_2 of F there exists disjoint open sets U_1 and U_2 of X such that $C_1 \subset U_1$ and $C_2 \subset U_2$. (Note that this concept has been considered in the literature, see [3]).

Observe that if $F \subset X$ is normal relative to X, then it is normal with the relative topology. However, the converse is not true (see examples 3.1, 3.2 and 3.3 in section 3).

THEOREM 1.7. *For any closed subset* A *of a space* X, *the following are equivalent:*

(i) A *satisfies the condition* (α) *relative to* X.

(ii) *For any closed subset* C *of* A *which is con-*

tained in an open subset U *of* X, *there exists an open subset* V *of* X *such that* $C \subset V \subset \mathrm{Cl}V \subset U$.

<u>*Proof*</u>. (i) ⇒ (ii). Suppose that C is a closed subset of A and $C \subset U$ where U is open in X. $\{X-C\} \cup \{U\}$ is a locally finite open cover of A in X, so it has a shrinking $\{G,H\}$ in X which covers A. Let $G \subset \mathrm{Cl}G \subset X-C$ and $H \subset \mathrm{Cl}H \subset U$; therefore $C \subset H \subset \mathrm{Cl}H \subset U$.

(ii) ⇒ (i). Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \Lambda\}$ be a locally finite open cover of A in X. Well order the set Λ; for convenience, then, suppose $\Lambda = \{1,2,3,..,\alpha,..\}$. Now construct $\{V_\alpha \mid \alpha \in \Lambda\}$ by transfinite induction as follows: Let $F_1 = A \bigcup_{\alpha>1}(A \cap U_\alpha)$. Then $F_1 \subset U_1$; so there is an open set V_1 such that $F_1 \subset V_1$ and $\mathrm{Cl}_X V_1 \subset U_1$. Suppose that V_β has been defined for $\beta < \alpha$. Now let $F_\alpha = A \setminus [\bigcup_{\beta<\alpha}(V_\beta \cap A)) \cup (\bigcup_{\gamma>\alpha}(U_\gamma \cap A))]$. Then F_α is closed in A and $F_\alpha \subset U_\alpha$ so we have an open set V_α in X such that $\mathrm{Cl}_X V_\alpha \subset U_\alpha$ and $F_\alpha \subset V_\alpha$. Now it is easy to see that $\mathcal{V} = \{V_\alpha \mid \alpha \in \Lambda\}$ is a shrinking of $\mathcal{U}$ which covers A. ■

Note that the second part of the proof of theorem 1.7 is modeled on the proof of theorem 15.10, page 104, of [8].

COROLLARY 1.8. *Let* A *be a closed subset of a given space* X. *If* A *satisfies the condition* (α) *relative to* X, *then* A *is normal relative to* X.

The above corollary follows from theorem 1.7 (ii). The converse is no true (see example 3.1

and the observation following example 3.3).

DEFINITION 1.9. [5] A cover $\mathcal{U}$ of a space X is called *bounded locally finite*, if there is a positive integer n such that every point x of X has a neighborhood which intersects at most n members of $\mathcal{U}$.

THEOREM 1.10. *Let f be a closed continuous mapping from a space X onto a normal space Y. If $A = f^{-1}(B)$ where B is a closed subset of Y, then the following are equivalent:*

(i) *A is normal relative to X.*

(ii) *A satisfies condition (α).*

(iii) *Any cover $\mathcal{U}$ of A which is bounded locally finite in X and whose members are open in X has a shrinking $\mathcal{V}$ in X which covers A.*

(iv) *Any finite cover $\mathcal{U}$ of A whose members are open in X has a shrinking $\mathcal{V}$ which covers A.*

<u>*Proof*</u>. We will show that (i) ⇒ (ii), the other implications are easy. Let C be a closed subset of A and $C \subset U$, where U is open in X. Now C and $(X-U) \cap A$ are two disjoint closed subsets of A, therefore there exists disjoint open set H_1, H_2 of X such that $C \subset H_1$ and $(X-U) \cap A \subset H_2$. Let $K = (X-U)-H_2$, hence $K \cap A = \emptyset$. By the normality of Y, there is an open set G of Y such that $B \subset G \subset ClG \subset Y-f(K)$. Let $V = H_1 \cap F^{-1}(G)$. Then $C \subset V \subset ClV \subset U$. This ends the proof. ∎

It is easy to see that a normal closed subset M of a space X need not to be normal relative to X (see section 3 for examples). But we have the following theorem:

THEOREM 1.11. *Let* M *be a closed normal subset of a space* X. *If* F *is closed in the interior* G *of* M, *then* F *is normal relative to* X, *in fact,* F *even satisfies the condition* (α).

<u>*Proof*</u>. Suppose that A is a closed subset of F and $A \subset U$, where U is open in X. Since $A \subset U \cap G$ and M is normal, there is an open set V in M such that $A \subset V \subset Cl_M V \subset U \subset G$. Therefore, $A \subset V \cap G \subset Cl_M V \cap G \subset U \cap G \subset U$. Let $N = V \cap G$ then $Cl_X N \subset Cl_X V \cap Cl_X G$ so $Cl_X N \subset Cl_X V \cap N = Cl_M V$. Hence $A \subset N \subset Cl_X N \subset U$. This ends the proof. ■

We shall now apply the above results to prove and sometimes improve some of the results of Hanai [3]. We shall start with:

THEOREM 1.12. (Hanani [3]). *Let* f *be a closed continuous mapping from a space* X *onto a paracompact* T_2*-space* Y. *Then* X *is normal if and only if for each* y *in* Y, $f^{-1}(y)$ *is normal relative to* X.

<u>*Proof*</u>. It suffies to show that f is paranormal. Let $\mathcal{U}$ be a locally finite open cover of X. Since $f^{-1}(y)$ satisfies the condition (α), $\mathcal{U}$ has a shrinking $A_y = \{A_{y,\alpha} \mid \alpha \in \Lambda\}$ in X which covers $f^{-1}(y)$. For each y in Y, let $O_y = Y - f(X - \bigcup_{\alpha \in \Lambda_y} A_{y,\alpha})$. Then Then for each y in Y, $f^{-1}(O_y) \subset \bigcup_{\alpha \in \Lambda_y} A_{y,\alpha}$ and O_y is open. Now $\{O_y \mid y \in Y\}$ has an open locally finite refinement which covers y, say $\{W_y \mid y \in Y'\}$, $Y' \subset Y$.

and for each W_y, $f^{-1}(W_y)$ is covered by a shrinking of a subfamily of $\mathcal{U}$. Hence f is paranormal mapping. ∎

DEFINITION 1.13. [1] A subset F of a space X is called *σ-paracompact (paracompact) relative to* X if every open cover of F in X has an open σ-locally finite (locally finite) refinement in X.

We shall now prove:

THEOREM 1.14. (i) *Let* f *be a closed continuous mapping of a* T_2*-space* X *onto a paracompact* T_2 *space* Y, *such that* $f^{-1}(y)$ *is normal and the boundary* $\partial f^{-1}(y)$ *is paracompact relative to* X, *for each* y *in* Y. *Then* X *is normal.*

(ii) *Let* f *be a closed continuos mapping of a regular space* X *onto a paracompact* T_2 *space* Y *such that* $f^{-1}(y)$ *is normal and the boundary* $\partial f^{-1}(y)$ *is σ-paracompact relative to* X, *for each* y *in* Y. *Then* X *is normal.*

Proof. (i) Let A,B be two disjoint closed subsets of $f^{-1}(y)$; then by the normality of $f^{-1}(y)$, there exists open sets G and H of X that $A \subset f^{-1}(y) \cap G$, $B \subset f^{-1}(y) \cap H$ and $f^{-1}(y) \cap G \cap H = \emptyset$. Since $\partial f^{-1}(y)$ is paracompact relative to X and X is T_2, then by theorem 5 of Aull [1] there exist open sets G_0 and H_0 of X such that $\partial f^{-1}(y) \cap A \subset G_0$, $\partial f^{-1}(y) \cap B \subset H_0$ and $H_0 \cap G_0 = \emptyset$. Now let $G' = [\text{Int } f^{-1}(y) \cap G] \cup [G_0 \cap G]$ and $H' = [\text{Int } f^{-1}(y) \cap H] \cup [H_0 \cap H]$. Then $A \subset G'$, $B \subset H'$ and $G' \cap H' = \emptyset$.

(ii) The proof follows by the same method used in (i), and theorem 13 of Aull [1]. ■

Observe that theorem 1.14 improves a similar result of Hanai [3], which he proves with the stronger hypothesis in (i) that $\partial f^{-1}(Y)$ is compact, and the stronger hypothesis in (ii) that $\partial f^{-1}(y)$ is Lindelöf.

DEFINITION 1.15. Let m be an infinite cardinal number. A subset F of a space X is called *m-paracompact relative to* X if every open cover of F in X with cardinality at most m has an open locally finite refinement in X. ■

We shall prove:

THEOREM 1.16. *For an any closed set* A *of a topological space* X*, the following are equivalent:*
(i) A *is* m*-paracompact relative to* X;
(ii) A *is* m*-paracompact and* ∂A *is* m*-paracompact relative to* X.

<u>*Proof*</u>. (i) ⇒ (ii). It is clear that A is m-paracompact. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \Lambda\}$, $|\Lambda| \leq m$, be an open cover of ∂A in X. Let $V = A - \partial A$; then there exists and open set U in X such that $V = U \cap A$. Now $\mathcal{B} = \{U\} \cup \mathcal{U}$ is an open cover of A in X. Therefore, $\mathcal{B}$ has a locally finite open refinement in X. Now it is easy to see that ∂A is m-paracompact relative to X.

(ii) ⇒ (i). Let $\mathcal{U} = \{U_\alpha \mid \alpha \in \Lambda\}$, $|\Lambda| \leq m$, be an open cover of A in X. Since A is m-paracompact, there is a locally finite open refinement $\{W_\beta \mid \beta \in \Gamma\}$ of $\{A \quad U_\alpha \mid \alpha \in \Lambda\}$. Let for each β, $W'_\beta = W_\beta$ Int A. Since ∂A is m-paracompact relative to X, $\mathcal{U}$ has an open refinement $\mathcal{H}$ wich is locally finite in X and covers ∂A. Now $\{W'_\beta \mid \beta \in \Gamma\} \cup \mathcal{H}$ is an open locally finite refinement of $\mathcal{U}$ in X which covers A. ■

THEOREM 1.17. (i) *Let* f *be a closed continuous mapping of a* T_2 *space* X *onto a paracompact* T_2 *space* Y *such that* $f^{-1}(y)$ *is paracompact and the boundary* $\partial f^{-1}(y)$ *is paracompact relative to* X, *for each* y *in* Y. *Then* X *is paracompact and normal.*
(ii) *Let* f *be a closed continuos mapping of a regular space* X *onto a paracompact space* Y *such that* $f^{-1}(y)$ *is paracompact and* $\partial f^{-1}(y)$ *is* σ*-paracompact relative to* X, *for each* y *in* Y. *Then* X *is paracompact and normal.*

Proof. It is easy to see that X is paracompact. The normality of X follows from theorem 1.14. ■

Observe that theorem 1.17 improves a similar result of Hanai [3], which he proves with the stronger hypothesis in (i) that the boundary $\partial f^{-1}(y)$ is compact, and the stronger hypothesis in (ii) that the boundary $\partial f^{-1}(y)$ is Lindelöf.

DEFINITION 1.18. [4] A collection A of subsets of a space X is called *closure preserving* if, for every subcollection $\mathcal{B}$ A, the union of closures is the closure of the union: $\bigcup\{\mathrm{Cl}B \mid B \in \mathcal{B}\} = \mathrm{Cl}[\bigcup\{B \mid B \in \mathcal{B}\}]$.

Finally we prove:

THEOREM 1.19. *Let* f *from* X *onto* Y *be continuous mapping and let* Y *have two coverings* $\{V_i \mid i \in I\}$ *and* $\{H_i \mid i \in I\}$ *with the following properties:*
(i) *for each* $i \in I$, H_i *is open*, V_i *is closed and* $f^{-1}(V_i)$ *satisfies the condition* (α) *relative to* X;
(ii) *for each* $i \in I$, $V_i \subset H_i$ *and* $\{H_i \mid i \in I\}$ *is closure preserving. Then* X *is normal.*

Proof. Let $\mathcal{U}$ be a locally finite open cover of X. Since for each $i \in I$, $f^{-1}(V_i)$ satisfies the condition (α) relative to X, $\mathcal{U}$ has a shrinking $\mathcal{W}_{V_i}$ in X which covers $f^{-1}(V_i)$. Let $S = \{f^{-1}(H_i) \cap W \mid W \in \mathcal{W}_{V_i},\ i \in I\}$; then S is a shrinking of $\mathcal{U}$. Hence X is normal. ∎

§2. Normality of the Products. We shall apply the results in the previuos section to obtain some product theorems. The first one follows from Theorem 1.19.

THEOREM 2.1. *Let* X *and* Y *be two spaces such that* Y *is normal and* X *has two covering* $\{V_i \mid i \in I\}$ *and* $\{H_i \mid i \in I\}$ *with the following properties:*
(i) *for each* $i \in I$, H_i *is open*, V_i *is closed and* $V_i \times Y$ *satisfies the condition* (α) *relative to* X×Y;
(ii) *for each* $i \in I$, $V_i \subset H_i$ *and* $\{H_i \mid i \in I\}$ *is closure preserving. Then* X×Y *is normal.*

THEOREM 2.2. *Let* X *and* Y *be normal spaces. Then* X×Y *is normal if and only if* X *has an open cover* $\{V_\alpha \mid \alpha \in \Lambda\}$ *which is point finite closure* *preserving and for each* α Λ, $\mathrm{ClV}_\alpha \times Y$ *is normal*.

Proof. Since X is normal $\{V_\alpha \mid \alpha \in \Lambda\}$ has a shrinking $\{M_\alpha \mid \alpha \in \Lambda\}$ For each α Λ, $M_\alpha \subset \mathrm{ClM} \subset V_\alpha \subset \mathrm{ClV}_\alpha$. Therefore $M_\alpha \times Y \subset \mathrm{ClM}_\alpha \times Y \subset V_\alpha \times Y \subset \mathrm{ClV}_\alpha \times Y$. Since $\mathrm{ClV}_\alpha \times Y$ is normal, $\mathrm{ClM}_\alpha \times Y$ satisfies the condition (α) relative to X×Y. Therefore $\{\mathrm{ClM}_\alpha \mid \alpha \in \Lambda\}$ is a closed cover of X and $\{V_\alpha \mid \alpha \in \Lambda\}$ is an open cover of X which is closure preserving and for each $\alpha \in \Lambda$, $\mathrm{ClM}_\alpha \subset V$ and $\mathrm{ClM}_\alpha \times Y$ satisfies the condition (α) relative to X×Y. Hence, by theorem 2.1, X×Y is normal. ∎

THEOREM 2.3. *Let* X *be locally compact, metric space. Let* Y *be normal countably paracompact space. Then* X×Y *is normal.*

Proof. X has an open cover $\{V_\alpha \mid \alpha \in \Lambda\}$ such that ClV_α is compact. Since X is paracompact, $\{V_\alpha \mid \alpha \in \Lambda\}$ has an open locally finite refinement $\{F_\gamma \mid \gamma \in \Gamma\}$ Now for each $\gamma \in \Gamma$, there is $\alpha \in \Lambda$ such that $\mathrm{ClF}_\gamma \subset \mathrm{ClV}_\alpha$. Therefore, for each γ, ClF_γ is compact. By theorem 21.4 of [8], $\mathrm{Clf}_\gamma \times Y$ is normal. Thus X has an open locally finite refinement $\{F_\gamma \mid \gamma \in \Gamma\}$ such that for each $\gamma \in \Gamma$, $\mathrm{ClF}_\gamma \times Y$ is normal. Hence by Theorem 2.2, X×Y is normal. ∎

Observe that theorem 2.3 is a generalization of the well known theorem of Dowker ([8], Theorem 21.4).

§3. <u>Counterexamples</u>. In this section we discuss some counterexamples relevant to the definitions and theorems in the previuos sections. Concerning Definition 1.6, it is easy to see that a normal closed subset need not to be normal relative to X. In fact, take any non-normal space X having two disjoint closed normal subspaces A and B that can not be separated by open sets. Then $M = A \cup B$ is normal and closed but it is not normal relative to X.

A particular case of this situation can be realized in each of the following three examples taken from [6].

3.1. TYCHONOFF PLANK. The Tychonoff Plank is defined to be $T = [0,\Omega]\times[0,\omega]-\{(\Omega,\omega)\}$ where both ordinal spaces $[0,\Omega]$ and $[0,\omega]$ are given the interval topology. Let $A = \{(\Omega,n) \mid 0 \leqslant n < \omega\}$ and $B = \{(\alpha,\omega) \mid 0 \leqslant \alpha < \Omega\}$. Both A and B are normal and closed in T, but on the other hand, A and B can not be separated by open sets (thus $M = A \cup B$ is normal and closed in T but M is not normal relative to T).

3.2. MICHAEL'S PLANE. Let X = R, define a topology τ on X in the following way: G τ if and only if $G = U \cup V$ where U is an open subset of R in usual topology and V is any subset of the irrationals. Let Y be the space of irrationals. Now take $A = \{(x,y) \mid x \text{ is rational}\}$ and $B = \{(x,x) \mid x \text{ is irrational}\}$, then A and B are normal and closed in $X \times Y$, but cannot be separated by open sets (thus again $M = A \cup B$ is a normal and closed subset of $X \times Y$ but is not normal relative to $X \times Y$).

3.3. NIEMYTZKI'S SPACE. Let $T = P \cup L$ where P is the open upper half plane with the euclidean topology τ, and L is the real axis, we generate a topology τ^* on T by adding to τ all sets of the form $\{x\} \cup D$, where $x \in L$ and D is an open disc in P which is tangent to L at x. Now the real axis L is a normal and closed subset of T which is not normal relative to T. This can be seen by writing $L = Q \cup I$, where Q is the rationals and I is the irrationals. Q and I are normal closed subset which can not be separated by open sets.

Observe that Michael's plane (example 3.2) shows that theorem 2.3 is not valid for non-locally compact metric spaces.

In theorem 1.8 it has been shown that if A satisfies the condition (α), then A is normal relative to X. However, the converse is not true. An example of this can be found in any non-normal space X having two disjoint closed sets A and B which can not be separated by open sets and such that B is normal and A is normal relative to X. In this case A does not satisfy the condition (α) (indeed, A satisfying the condition (α) implies that A and B can be separated by open sets). This situation can be realized, for example in the Tychonoff Plank with the same sets A and B of example 3.1. Both A

and B are normal relative to T. The proof for B is obvious; for A use the fact that given two disjoint closed subsets of A one of them will be compact. But both A and B do not satisfy the condition (α). This shows also that the union of disjoint closed subspaces which are normal relative to a space need not to be normal relative to the space.

Note also that each of the example 3.2 and 3.3 can be used here for the same purpose. One has to rely on the statement: *In a regular space* X *any closed countable subspace* X_0 *is normal relative to* X. A proof of the above statement is obtained by a straightforward adaptation of the well-known argument (due Tychonoff [8]) that a regular Lindelöf space is normal. Indeed, let X_0 be a countable closed subspace of a regular space X. Let $A = \{a_1, a_2, \ldots\}$ and $B = \{b_1, b_2, \ldots\}$ be two disjoint closed subsets of X_0. For each n, choose U_n, V_n open subsets of X such that $a_n \in U_n$, $\mathrm{Cl}U_n \cap B = \emptyset$ and $b_n \in V_n$, $\mathrm{Cl}V_n \cap A = \emptyset$. Now put

$$U = (U_1 - \mathrm{Cl}V_1) \cup (U_2 - (\mathrm{Cl}V_1 \quad \mathrm{Cl}V_2)) \quad \ldots$$

$$V = (V_1 - \mathrm{Cl}U_1) \cup (V_2 - (\mathrm{Cl}U_1 \quad \mathrm{Cl}U_2)) \quad \ldots$$

Then $A \subset U$, $B \subset U$, and $U \cap V = \emptyset$.

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