Revista Colombiana de Matemáticas
Vol. XIX (1985), pág. 1 - 8

ANTONIO MONTEIRO

1907 - 1980

Roberto Cignoli

In the following pages, I will try to give at least a faint image of the personality of Antonio Monteiro and the important role he played in the development of Mathematics and Mathematical Logic in Latin America.

Monteiro was a person firmly convinced that the scientific development of a country was a necessary condition for its economical and social development. Consequently, the major task of his life was to arouse the consciousness of the people on this issue and to promote scientific research, first in Portugal and later on in Brazil and Argentina.

Some of the leading mathematicians of these three countries, as José Sebastião e Silva and Hugo Ribeiro in Portugal, Leopoldo Nachbin and Mauricio Peixoto in Brazil, and Orlando Villamayor in Argentina, among others, were initially trained by him.

He initiated the publication of several journals and series of monographs on mathematics. Some of this publicacions are *Portugalia Mathematicae*, *Gazeta Matematica* and the series *Cadernos de Análise Geral* in Portugal; the series *Notas de Matemática*, formely published by the Universidade Nacional do Brasil and now published as an international series under the guidance of Leopoldo Nachbin, the journal *Revista Cuyana de Matemática* (Mendoza, Argentina), and the series *Notas de Lógica Matemática* (Bahía Blanca, Argentina).

One of the best mathematical libraries of Latin America, at the Instituto de Matemática in Bahía Blanca, is a result of Monteiro's personal effort.

His strict moral statute and his compromise with the development of scientific research caused him serious personal problems in the three countries where he worked. At the begining of his professional life, Monteiro refused to sign a document supporting the corporate Portuguese constitution of 1933, and the government prevented him from occupying any position at the universities. Between 1939 and 1943 he did a tremendous effort to develop modern mathematics in Portugal, but he had to earn his living by giving private lessons and cataloguing the scientific bibliography existent in Portugal. He took this last job very seriously and learned a lot about the organization of libraries. Of course, those were years of great deprivations for his family. Almost at the end of his life, in 1974, the goverment-appointed rector of the University at Bahía Blanca fired him from his position of Emeritus Professor. Since then, he was explicitly forbidden to enter the university building, not even to consult books and journals in the library that had been built up through his effort.

His complete name was Antonio Aniceto Ribeiro Monteiro and he was born in Mossamedes, Angola, on May 31, 1907. After his father's death, when Monteiro was seven years old, his family returned to Portugal. He obtained his degree of Licenciado em Ciências Matemáticas from the University of Lisbon in 1930, and immediatly earned a fellowship from the Ministry of Education of Portugal that allowed him to study at the Institut Henri Poincaré, Paris. In 1936, he got his "Docteur en Sciences" degree from the Sorbonne, with the thesis "Sur l'additivité des noyaux de Fredholm" written under the guidance of Maurice Fréchet.

Monteiro returned to Portugal, where he organized the "seminário de Análise Geral" in Lisbon, and later the "Junta de Investigação Matemática" in Porto, with the aim of training researchers in mathematics. As explained before, these activities had no support from the university.

In December of 1945 Monteiro was appointed professor at the Universidade Nacional do Brazil (now Universidade Federal de Rio de Janeiro). Later he was also a professor at the Fundação Getulio Vargas and participated in the organization of the Centro Brasileiro de Pesquisas Físicas.

In 1949 he had to leave Brazil, and was appointed professor at the Universidad Nacional de Cuyo, in San Juan, Argentina. At this university he contributed greatly to the creation of an Institute of Mathematics, entirely devoted to research, which played a very important role in the development of mathematics in Argentina. Two important events, of a wide latin-american projection, were organized by this Institute, with UNESCO support: a course for trainning latin-american professors in mathematics (February-March, 1955) and a Symposium (July, 1955) in which almost all the mathematicians that were at the time in Latin-America participated. Unfortunately, at the end of 1955 the Institute was deactivated by the university authorities. In 1956 Monteiro moved to Bahía Blanca, where, at the Universidad Nacional del Sur, he organized the Institute of Mathematics and developed the programs of "Licenciatura" and "Doctorado". In 1969-1970 he received a grant from the Consejo Nacional de Investigaciones to visit several European universities. This was his first visit to Europe after 25 years in Latin-America. In 1970, at the age of 65, he was appointed Emeritus Professor; he was fired in 1974. In 1977 he went to Portugal to return later to Bahía Blanca in 1979, where he died on October 29, 1980.

During his years in Paris, Monteiro was in touch with some of the leaders of the classical French School of Analysis, like E. Borel, H. Lebesgue, H. Hadamard and, simultaneously, was a witness to the moderns trends in the study of algebraic and topological structures. He participated in the Julia's Seminar which was the kernel of the N. Bourbaki group. At that time (late thirties), the papers by M. Stone on the topological representacion of Boolean algebras and distributive lattices, those by G. Birkhoff on the foundations of lattice theory and universal algebra, and those by A. Tarski on Boolean algebras and the relations between deductive systems and closure operators appeared. They strongly influenced Monteiro and, according to his own words, he devoted his work to the study of topological spaces, lattices, and the relations among them.

The first papers Monteiro wrote, after his return to Portugal, were on the foundations of general topology. His research on the characterization of closure operators and continuous functions naturaly led him to the theory of partially ordered sets and lattices.

The integers **Z** form a lattice, under the order relation of divisibility. The filters of this lattice are precisely the ideals of **Z** as a ring. The maximal filters are the sets of multiples of prime numbers and the prime filters are the sets of multiples of prime powers. The basic arithmetic properties of **Z** can be expressed in terms of filters. For instance, the decomposition of an integer into prime factors is equivalent to the fact that each filter in the lattice **Z** is a finite intersection of prime filters. Thus lattices can be considered as generalization of the integers, and the study of the properties of the filters of a lattice can be considered as an "arithmetic" for this lattice.

The filters of a lattice, ordered by inclusion, from a new lattice. Monteiro and his coworker's research in Brazil was mainly devoted to the study of the relationship between a lattice and the lattice of its filters. For instance, they characterized the lattice of filters and prime filters of several classes of lattices. An important property proved by Monteiro (1947) is that a lattice L is distributive if and only if each filter of L is an intersection of prime filters. Consequently, we can say that distributive lattices are exactly those lattices in which the analogue of the factorization of an integer holds.

Real numbers generalize the integers, and topological spaces, in a sense, generalize the real line. Then we may consider topological spaces as generalizationsof the integers, and it then appears natural to look for some type of arithmetic properties in topological spaces. This is the point of view adopted by Monteiro in his paper *Arithmêtique des espaces topologiques* (1950)*. Given a topological space X, the closed sets of X, ordered by inclusion, form a dis-

* Submitted to the French Mathematical Society for a contest in honor of Maurice Fréchet, it was among the four best papers presented(Bull.Soc.Math.France, 1951, XXXIX-XL). Twenty years later, published in "Notas de Lógica Matemática" of Bahía Blanca, Nº 29 (1974), under the title: *L'Arithmétique des filtres et les espaces topologiques - I.*

tributive lattice L(X). Monteiro viewed the filters of L(X) as generalized integers. He showed how some of the separation axioms currently considered in topology can be interpreted in terms of arithmetic properties of L(X). For instance, X is a normal space if and only if each prime filter of L(X) is contained in a unique maximal filter. In the lattice **Z**, this property is obvious because prime filters correspond to prime powers, and maximal filters to prime numbers. Moreover, X is a completely normal space if and only if given a prime filter P of L(X), the set of filters F of L(X) such that $P \subseteq F$ is totally ordered by inclusion. This property is stronger than the previous one, and it is also a property of the lattice **Z**. These two examples illustrate Monteiro's idea: the spaces X such that L(X) has arithmetic properties closer to those of the lattice **Z**, should be considered better generalizations of the intergers.

The lattices of closed sets are more than distributive lattices: they are Brouwerian algebras. Their duals, Heyting algebras, are the algebraic counterparts of the intuitionistic propositional calculus, and they play for intuitionistic logic the same role as that of Boolean algebras for classical logic. In the previously mentioned paper, Monteiro considered Brouwerian and Heyting algebras in detail. For instance, he gave the interesting algebraic result that the class of Boolean algebras coincides with the class of semi-simple Heyting algebras.

During his first years in Argentina, Monteiro continued his research on Brouwerian and Heyting algebras. Around 1955, Halmos introduced polyadic algebras as a tool for the algebraic analysis of quantifiers, and immediately Monteiro showed that it was possible to give a non-trivial generalization of the theory of monadic Boolean algebras to monadic Heyting algebras.

Throughout the study of Boolean and Heyting algebras, Monteiro became interested in the algebraic aspects of logic, this interest having been aroused by direct contact with Roman Sikorski and Helena Rasiowa when they visited Bahía Blanca in 1958. Monteiro applied his mathematical experience to the study of algebraic systems related to non-classical logics. He was convinced that the algebraic methods in logic would have important technological applications in the future, as a consequence of the development of computers. In view of such prospective applications, he tried to use finitistic and combinatorial methods in studying classes of algebras, whenever possible.

Given a class of algebras K, it was a basic problem for him to decide if the finitely generated free algebras in K were finite, and if so, to find explicitly the number of its elements as a function of the number of free generators. In general, to achieve this goal it is necessary to have a deep understanding of the structure of the algebras in K. As an example, let me reproduce the formula giving the number of elements $L_n(r)$ of the free algebra with r free generators in the class K_n corresponding to the n-valued Lukasiewicz propositional calculus (n an integer $\geqslant 2$). Here, D is the set of divisors of n-1, M(d) is the set of maximal divisors of d, $\wedge$ denotes the greatest common divisor and $|X|$ is the number of elements of X:

$$L_n(r) = \prod_{d\in D} (d+1)^{\left\{\sum_{X\subseteq M(d)} (-1)^{|X|} \left(\bigwedge_{m\in X} m+1\right)^r\right\}}$$

This formula was obtained by Monteiro in 1969, solving in this way a problem which was open since 1930.

In the same vein, he determined the structure of the finite De Morgan and Nelson algebras. Monteiro investigated in depth the structure of Nelson algebras (introduced by H. Rasiowa as the algebraic counterparts of the constructive logic with strong negation considered by Nelson and Markhoff), and he proved that the class of semi-simple Nelson algebras coincides with the class of algebras corresponding to three-valued Lukasiewicz logics. Thus he showed that, from the algebraic point of view, the three-valued Lukasiewicz logic stands in the same relation to constructive logic with strong negation as classical logic does to intuitionistic logic.

Monteiro also proved that each three-valued Lukasiewicz algebra can be represented as a suitable monadic Boolean algebra. He liked this result very much, because it meant that Lukasiewicz three-valued propositional calculus had an interpretation in the classical monadic functional calculus. This results is analogous to the construction of Euclidean models of non-Euclidean geometry.

It is worthwhile to mention here that Monteiro also characterized the class of monadic Boolean algebras as the class of semi-simple closure algebras.

Monteiro won the 1978 prize for scientific and technological achievements from the Gulbenkian Foundation in Lisbon, for a manuscript on symmetric Heyting algebras, where he gave a rather detailed account of some of his work done in the previous years*

I have tried to give here a brief indication of what I consider Monteiro's main contributions to mathematics, but there are many more. Usually, he just published short summaries or abstracts of his main results. The details as well as the underlying ideas of his work were given in his courses and seminars, and they are dispersed in the notes taken by his students.

I am sure that Monteiro will live for ever in the memory of all who, like myself, have had the privilege of receiving his teaching on both, moral conduct and mathematics.

---

* To appear in Portugalia Mathematicae.

SCIENTIFIC PAPERS OF ANTONIO MONTEIRO

1. *Sur les noyaux additifs dans la théorie des équations intégrales,* C.R. Acad, Sci. Paris (1934), 1737.
2. *Sur une classe de noyaux développables en séries de noyaux principaux,* C.R. Acad. Sci. Paris 200, 1er. Sem.(1935), 2143.
3. *Sur l'additivité des noyaux de Fredholm* (Tesis de Doctorado), Portugal Math. 1 (1937), 1-174.
4. *Sur l'additivité dans un anneau,* Portugal Math. 1 (1940), 189-192.
5. (with H. Ribeiro) *Sur l'axiomatique des espaces V,* Portugal Math. 1 (1940), 275-288.
6. *Caractérisation des Espaces de Hausdorff au moyen de l'opération de dérivation,* Portugal Math. 1 (1938-1940), 333-349.
7. (with Armando Gibert) *Os conjuntos mutuamente connexos e os fundamentos de la Topologia Integral,* Comunicación del Congreso Luso-Español para el progreso de las Ciencias - Zaragoza - 1940, Las Ciencias, Año VII, Nº 2, 1940.
8. *Les ensembles fermés et les fondements de la topologíe générale,* Portugal Math. 2 (1941), 290-298.
9. *La notion de fermeture et les axiomes de séparation,* Portugal Math. 2 (1941), 290-298.
10. (with H. Ribeiro) *L'operation de fermeture et ses invariants dans les systèmes partiellement ordonnés,* Portugal Math. 3 (1942), 171-184.
11. *Caracterisation de l'opération de fermeture par un seul axiome,* Portugal Math. 4 (1944), 158-160.
12. (with H. Ribeiro) *La Notion de fonction continue,* Summa Mathematicae Brasiliensis 1, fac.1 (1945), 1-8.
13. *Arithmétique des Filtres Premiers,* C.R. Acad. Sci. Paris 225 (1947), 846-848.
14. *Filtros e Ideais, Vol. 1,* Notas de Matemática, Nº 2, Río de Janeiro, 1948.
15. *Filtros e Ideais, Vol. 2,* Notas de Matemática, Nº 5, Río de Janeiro, 1948.
16. *Réticules distributifs de dimensioñ linéaire n,* C.R. Acad. Sci. Paris 226 (1948), 1658-1660.
17. (with Mauricio Peixoto) *Note on uniform continuity,* Proceding of the International Congress of Mathematicians (1950), Nº 1, pág. 385.
18. *Arithmétique des espaces topologiques,* Submitted to the French Mathematical Society for a contest in honor of M. Fréchet. Published finally in Notes de Lógica Matemática, Bahía Blanca, Nº 29 (1974).
19. (with Mauricio Peixoto) *Le nombre de Lebesgue et la continuité uniforme,* Portugal Math. 10 (1951), 105-113.
20 *Les filtres des espaces compacts,* Gazeta Matemática 50 (1951), 95-96.
21 *Propiedades características de los filtros de un Algebra de Boole,* Acta Cuyana de Ingeniería 1, Nº 5 (1951), 1-7.
22 *L'arithmétique des filtres et les espaces topologiques,* Segundo Simposio sobre algunos problemas matemáticos que se están estudiando en Latinoamérica,

(Villavicencio), Unesco, Montevideo (1954), 129-162.

23. *Axiomes indépendents pour les algèbres de Brouwer*, Rev. Un.Mat. Argentina 17 (1955), 149-160.

24. *Les ensembles ordennés Compacts*, Revista Matemática Cuyana 1, fac.3 (1955), 187-194.

25. (with Oscar Varsavsky) *Algebras de Heyting Monádicas*, Actas de las X Jornadas de Un. Mat. Argentina, Bahía Blanca (1957), 52-59.

26. *Normalidad en las álgebras de Heyting monádicas*, Actas de las X Jornadas de la Un. Mat. Argentina (1957), 50-51.

27. *Algebras Monádicas*, Actas do Segundo Coloquio Brasileiro de Matemática, São Paulo, Brazil (1960), 33-52.

28. *Matrices de Morgan caractéristiques pour le calcul propositionnel classique*, An. Acad. Brasil. Ciênc. 32 (1960), 1-7.

29. *Linéarisation de la logique positive de Hilbert-Bernays*, Rev. Un. Mat. Argentina 20 (1962), 308-309.

30. (with Diana Brignole) *Caracterización de las Algebras de Nelson por Igualdades*, Rev. Un. Mat. Argentina 19, N° 5 (1962), 361.

31. (with Luisa Iturrioz) *Representación de las Algebras de Tarski Monádicas*, Rev. Un. Mat. Argentina 19, N° 5 (1962), 361.

32. *Construcción de las Algebras de Nelson Finitas*, Rev. Un. Mat. Argentina 19, N° 5 (1962), 361.

33. *Algebras de Nelson semi-simples*, Rev. Un. Mat. Argentina 21, N° 3 (1963), 145-146.

34. *Sur la définition des algèbres de Lukasiewicz trivalentes*, Bull Math. Soc. Sci. Math. R.S. Roumaine (N.S.) 7 (55), N°s. 1-2, (1963), 1-12.

35. *Construction des algèbres de Nelson finies*, Bull. Acad. Polon. Sci. 11, N° 6 (1963), 359-362.

36. *El Cálculo proposicional trivalente de J. Lukasiewicz y la lógica clásica*, Rev. Un. Mat. Argentina 22, N° 1 (1964), 43-44.

37. *Relations between Lukasiewicz Three Valued Algebras and Monadic Boolean Algebras*, International Congress for Logic, Methodology and Philosophy of Sciences, Program and Abstracts, the Hebrew University of Jerusalem (1964), 16-17.

38. (with Luisa Iturrioz) *Cálculo Proposicional implicativo clásico con n variables proposicionales*, Rev. Un. Mat. Argentina 22 (1965), 146.

39. (with R. Cignloi) *Construcción geometrica de las álgebras de Lukasiewicz libres*, Rev. Un. Mat. Argentina 22 (1965), 151.

40. *Generalización de un teorema de Sikorski sobre Algebras de Boole*, Rev. Un. Mat. Argentina 22 (1965), 151.

41. (with R. Cignoli) *Boolean elements in Lukasiewicz Algebras II*, Japan Aca. 41, N° 8 (1965), 676-680.

42. *Généralisation d'un théorème de Sikorski sur les algèbres de Boole*, Bull. Sci. Math. (2) 89 (1965), 65-74.
43. *Algebras de Boole involutivas*, Rev. Un. Mat. Argentina 32, Nº 1 (1966), 39.
44. (with Diana Brignole) *Caractérisation des Algèbres de Nelson par des égalités*, Proc. Japan Acad. 43. Nº 4 (1967), 279-283.
45. *Construction des algèbres de Lukasiewicz trivalentes dans les algèbres de Boole monadiques I*, Math. Japon 12, Nº 1 (1967), 1-23.
46. (with Luis Monteiro) *Algebras de Stone Libres*, Rev. Un. Mat. Argentina XXIII, Nº 4 (1968), 201.
47. *Generadores de reticulados distributivos finitos*, Actas del Simposio Panamericano de Matemática Aplicada, Buenos Aires (1968), 465.
48. (with Oswaldo Chateaubriand) *Les Algèbres de Morgan libres*, Notas de Lógica Matemática, Bahía Blanca, Nº 26 (1969).
49. *La semi-simplicité des algèbres de Boole topologiques et les systèmes deductifs*, Rev. Un. Mat. Argentina Vol. 23 (1971), 427-448.
50. *Algèbres de Boole Cycliques*, Rev. Roum. Math. Pures Appl. 23 (1978), 71-76.
51. *Les éléments réguliers d'un N-lattice*, An. Acad. Brasil. Ciênc. 52 (1980), 653-656.

***

IMECC
Universidade Estadual de Campinas
Campinas, S.P., Brazil.