USE OF GENERALIZED HYPERGEOMETRIC FUNCTIONS
IN ANALYTIC STELLAR MODELS

by

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Abstract: The present paper deals with the technique of integration theory of special functions applied to two simple analytic stellar models. We consider two cases, one with a non-linear dependence of the radial density and the other with a generalized energy generation rate. The integration theory of the generalized hypergeometric functions is applied to evaluate analitically the rate of nuclear energy generation. Some known results follow as particular cases of our formulae established here.

§1. Introduction. The study of some problems in the theory of internal structure of stars [1,2], motivates the interest in obtaining simple analytic stellar models.

In this work we shall apply the technique of integration theory of special functions [3,4] for treating special solutions of the equations of stellar structure.

The model we are concerned is a spherically symmetric purely gaseous star, which is generating nuclear energy and it is in quasi-static equilibrium [5].

In general the stellar structure models leads to a system of non-linear differential equations which can not

be solved in a closed form. However under some restricted conditions, such as the separation of the conditions of hydrostatic equilibrium mass conservation, and energy conservation from the consideration of the mode of energy transports one can obtain analytic models. The advantage of such analytic solution with energy conservation is to determine the central conditions on the star which concern the equation of the state and the rate of energy generations.

In section 2, we shall have a first stellar model with a radial density inside the star given by

$$\rho(\hbar) = \rho_c (1 - \frac{\hbar}{R})^{\nu}, \qquad \nu > -1 \tag{1}$$

where $\rho_c = \rho(0)$ is the central density of the star and R is the radius of the star. In-equilibrium the average energy production per unit mass at the radius $\hbar$, $\varepsilon(\hbar)$, is assumed, for the sake of simplicity, proportional to certain powers of the density and temperature, that is

$$\varepsilon(\hbar) = \varepsilon_0(\rho_0, T_0) \left(\frac{\rho(\hbar)}{\rho_0}\right)^a \left(\frac{T(\hbar)}{T_0}\right)^b, \tag{2}$$

where $T(\hbar)$ is the temperature distribution function inside the star defined later, ρ_0 and T_0 are the reference density and temperature respectively. And a and b are two real numbers.

The total net rate of nuclear energy generation which is equal to the luminosity of the star,

$$L = 4\pi \int_0^R \hbar^2 \rho(\hbar) \varepsilon(\hbar) d\hbar \tag{3}$$

will be evaluated for four different cases of the analytic representations of the nuclear energy generation rate $\varepsilon(\hbar)$.

In Section 3, we shall consider a second stellar model with a linear dependence of the radial density that is,

$$\rho(\hbar) = \rho_c (1 - \frac{\hbar}{R}), \tag{4}$$

and the rate of nuclear energy generation is given by

$$\varepsilon(\mathit{n}) = \varepsilon_0(\rho_0, T_0) \left(\frac{\rho(\mathit{n})}{\rho_0}\right)^a \left(\frac{T(\mathit{n})}{T_0}\right)^b \exp\left(-\xi\frac{\rho(\mathit{n})}{\rho_0}\right), \tag{5}$$

where $\xi \geqslant 0$.

The luminosity function $L(\mathit{n})$ defined by (3) will be calculated in four different analytic forms of $\varepsilon(\mathit{n})$.

The resuitd given recently by Haubold and Mathai [5] follows as particular cases of our generalized stellar models.

§2. Stellar model with a non-linear dependence of the radial density.

First we shall evaluate the distribution of mass $M(\mathit{n})$ of the star which is given by

$$M(\mathit{n}) = 4\pi\int_0^{\mathit{n}} \mathit{n}^2 \rho(\mathit{n})\, d\mathit{n}.$$

By using (1), we have

$$M(\mathit{n}) = 4\pi\rho_c \int_0^{\mathit{n}} \mathit{n}^2 \left(1 - \frac{\mathit{n}}{R}\right)^\nu d\mathit{n}.$$

By invoking the integral representation for Gauss' hypergeometric function $_2F_1$ [6] or simply the result [7, pp.30, (3)]

$$\int_0^x \frac{x^{\lambda-1}}{(x+\gamma)^\alpha}\, dx = \frac{\gamma^{-\alpha}}{\lambda} x^\lambda {}_2F_1\left(\alpha, \lambda; \lambda+1; \frac{-x}{\gamma}\right), \quad \text{Re } \lambda > 0, \tag{6}$$

we have

$$M(\mathit{n}) = \frac{4\pi}{3} \rho_c \mathit{n}^3 {}_2F_1\left(-\nu, 3; 4; \frac{\mathit{n}}{R}\right), \quad \nu > -1. \tag{7}$$

If M is the total mass of the star, that is $M(R) = M$, then from (7) we get

$$\rho_c = \frac{M(\nu+3)(\nu+2)(\nu+1)}{8\pi R^3} \tag{8}$$

From the basic equation of the hydrostatic equilibrium between the gravitational force exerted on the mass and the gas pressure force directed outward, we have

$$P(\pi) = P(0) - \int_0^\pi \frac{GM(\pi)}{\pi^2} \rho(\pi) d\pi \tag{9}$$

where G is the gravitational constant. Let

$$A(\pi) = \int_0^\pi \frac{GM(\pi)}{\pi^2} \rho(\pi) d\pi.$$

From (1) and (7), $A(\pi)$ can be rewritten as

$$A(\pi) = \frac{4}{3}\pi\rho_c^2 G \int_0^\pi \pi(1 - \frac{\pi}{R})^\nu {}_2F_1(-\nu,3;4;\frac{\pi}{R}) d\pi$$

By using (6) and the series representation of Gauss' hypergeometric function we can write $A(\pi)$ as follows

$$A(\pi) = \frac{4}{3}\pi\rho_c^2 G \sum_{k=0}^{\infty} \frac{(-\nu)_k(3)_k}{(4)_k k!(k+2)} (\frac{\pi}{R})^k \pi^2 {}_2F_1(-\nu,2+k;3+k;\frac{\pi}{R}) . \tag{10}$$

From the boundary condition $P(R) = 0$, we have $P(0) = A(R)$ and therefore (9) can be written as

$$P(\pi) = \frac{4\pi}{3}\rho_c^2 GR^2 \left[\frac{\Gamma(\nu+1)}{\Gamma(\nu+3)} {}_3F_2(-\nu,3,2;4,\nu+3;1) \right.$$
$$\left. - \sum_{k=0}^{\infty} (\frac{\pi}{R})^{k+2} {}_2F_1(-\nu,2+k;3+k;\frac{\pi}{R})\right]. \tag{11}$$

For linear dependence of the radial density inside the star, $P(\pi)$ can be written as,

$$P(\pi) = \frac{\pi}{36}\rho_c^2 GR^2 \left[5 - 24(\frac{\pi}{R})^2 + 28(\frac{\pi}{R})^3 - 9(\frac{\pi}{R})^4\right], \tag{12}$$

which is in agreement with the equation (2.3) of [5]. By virtue of the equation of state of the perfect gas

$$P = \frac{\kappa}{\mu M_u} \rho T , \tag{13}$$

where κ is the Boltzman constant, μ is the mean molecular weight, M_u is the atomic weight unit and T is the temperature, the distribution function of the temperature inside the star $T(\pi)$ can be written in the following form ((1) is

expressed as a series),

$$T(\hbar) = \frac{4\pi\mu M_u \rho_c G R^2}{3\kappa}\left[\frac{\Gamma(\nu+1)}{\Gamma(\nu+3)}{}_3F_2(-\nu,3,2;4;\nu+3;1)\right.$$

$$\left. - \sum_{k=0}^{\infty}(\tfrac{\hbar}{R})^{k+2}{}_2F_1(-\nu,2+k;3+k;\tfrac{\hbar}{R})\right]\sum_{n=0}^{\infty}\frac{(\nu)_n}{n!}(\tfrac{\hbar}{R})^n \qquad (14)$$

The expression for $T(\hbar)$ can be expressed as a rapidly converging series of the form

$$T(\hbar) = T_c\sum_{i=0}^{\infty}c_i(\tfrac{\hbar}{R})^i, \qquad (\tfrac{\hbar}{R} < 1) \qquad (15)$$

where

$$T_c = \frac{4\pi\mu M_u L \rho_c G R^2}{3\kappa(\nu+2)(\nu+1)}\,{}_3F_2(-\nu,3,2;4,\nu+3;1) \qquad (16)$$

In the linear case ($\nu = 1$) equation (15) becomes

$$T(\hbar) = \frac{5\pi}{36}\frac{G\mu M_u}{\kappa}\rho_c R^2\sum_{i=0}^{\infty}c_i(\tfrac{\hbar}{R})^i,$$

where the coefficients, can be easily determined:

$$c_0 = c_1 = 1, \quad c_2 = -\frac{19}{5}, \quad c_3 = \frac{9}{5} \text{ and } c_i = 0 \text{ for } i \geqslant 4.$$

As usual we define the luminosity of the star $L(R)$ as defined by (3),

$$L(R) = 4\pi\int_0^R \hbar^2\rho(\hbar)\varepsilon(\hbar)\,d\hbar,$$

where the rate of the nuclear generation $\varepsilon(\hbar)$ is given by (2).

By using (1) and (2) and setting $x = \frac{\hbar}{R}$, we can rewrite $L(R)$ in the following form:

$$L(R) = 4\pi\varepsilon_0(\rho_0,T_0)\left(\frac{\rho_c}{\rho_0}\right)^a\left(\frac{T_c}{T_0}\right)^b\rho_c R^3 I, \qquad (18)$$

where

$$I = \int_0^1 x^2(1-x)^{\nu(a+1)}\left(\sum_{i=0}^{\infty}c_i x^i\right)^b dx \qquad (19)$$

In order to evaluate (19), we shall consider the more general integral of the form

$$I(\nu,a,b,d) = \int_0^1 x^d (1-x)^{\nu(a+1)} \left(\sum_{i=0}^{\infty} c_i x^i \right)^b dx; \tag{20}$$

In the general case, we can not evaluate (20) in a closed form. However under some special cases the infinite series of the integrand can be summed and that leads to evaluation of $I(\nu,a,b,d)$ in terms of generalized hypergeometric functions.

Case 1. We shall consider here thst $x \ll 1$; then the series $\sum_{i=0}^{\infty} c_i x^i$ can be approximated by $\sum_{i=1}^{\infty} c_i x^i$ for $k < \infty$. Therefore we have

$$\left(\sum_{i=1}^{k} c_i x^i \right)^b = (1 + c_1 x + c_2 x^2 + \ldots + c_k x^k)^b \tag{21}$$

$$= (1 + a_1 x)^b (1 + a_2 x)^b \ldots (1 + a_k x)^b$$

where $-\dfrac{1}{a_1}, -\dfrac{1}{a_2}, \ldots, -\dfrac{1}{a_k}$ are the roots of the polynomial equation $1 + c_1 x + c_2 x^2 + \ldots + c_b x^k = 0$.

There seems to be an error in [5, pp.376]. The numbers $a_1, a_2, \ldots, a_k$ should be replaced by $-\dfrac{1}{a_1}, -\dfrac{1}{a_2}, \ldots -\dfrac{1}{a_k}$ as the roots of the polynomial equation

$$1 + c_1 x + c_2 x^2 + \ldots + c_k x^k = 0$$

The ordinary binomial expansion with factorial notation can be used to write (21) as follows

$$\left(\sum_{i=0}^{\infty} c_i x^i \right)^b = \sum_{r_1=0}^{\infty} \frac{(-b)_{r_1}}{r_1!} (-a_1 x)^{r_1} \sum_{r_2=0}^{\infty} \frac{(-b)_{r_2}}{r_2!} (-a_2 x)^{r_2}$$

$$\cdots \sum_{r_k=0}^{\infty} \frac{(-b)_{r_k}}{r_k!} (-a_k x)^{r_k}, \tag{22}$$

where $|a_j| < 1$, $j = 1, 2, \ldots, k$ or b is a positive integer. Substituting in (20), we obtain

$$I_k(v,a,b,d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_k=0}^{\infty} \frac{(-b)_{n_1}(-b)_{n_2}\cdots(-b)_{n_k}}{n_1! n_2! \cdots n_k!}(-a_1)^{n_1}(-a_2)^{n_2}\cdots$$
$$\cdots(-a_k)^{n_k}\int_0^1 x^{d+n_1+n_2+\cdots+n_k}(1-x)^{v(a+1)}dx . \tag{23}$$

Further we have the beta integral,

$$\int_0^1 x^{d+n_1+n_2+\cdots+n_k}(1-x)^{v(a+1)} = \frac{\Gamma(va+v+1)\Gamma(d+1)(d+1)_{n_1+n_2+\cdots+n_k}}{\Gamma(d+va+v+2)(d+va+v+2)_{n_1+n_2+\cdots+n_k}} \tag{24}$$

From $[8, \text{pp.}449,(15)]$ and $[4]$ and by using (24), we can write $I_k(v,a,b,d)$ in terms of the Lauricella function F_D as follows

$$I_k(v,a,b,d) = \frac{\Gamma(va+v+1)\ (d+1)}{\Gamma(d+va+v+2)}\ F_D^{(k)}(d+1,-b,-b,\ldots,-b;d+va+v+2 ;$$
$$-a_1,-a_2,\ldots,-a_k) \tag{25}$$

where $|a_j| < 1$ for $j = 1,2,\ldots,k$ or b is a positive integer, $Re(d+1) > 0$ and $v(a+1) > -1$. $F_D^{(n)}$ is defined in the series representation in the following form

$$F_D^{(n)}(a,b_1,\ldots,b_n;c,z_1,\ldots,z_n) = \sum_{k_1,\ldots,k_n=0}^{\infty} \frac{(a)_{k_1+\cdots+k_n}(b_1)_{k_1}(b_2)_{k_2}\cdots(b_n)_{k_n}}{(c)_{k_1+k_2+\cdots+k_n}}$$
$$\times \frac{z_1^k\cdots z_n^{k_n}}{k_1!\cdots k_n!} \tag{26}$$

where $|z_j| < 1$ for $j = 1,2,\ldots,n$.

We mention some particular cases of (25).

(i) Let $a_1 = a_2 =\ldots= a_k = a'$, in this case the Lauricella function F_D reduces to the Gauss hypergeometric function $_2F_1$ and that (25) becomes

$$I_k(v,a,b,d) = \frac{\Gamma(va+v+1)(d+1)}{\Gamma(d+va+v+2)}\ _2F_1(d+1,-kb;d+va+v+2;-a') \tag{27}$$

(ii) For $a_1 = a_2 =\ldots= a_k = -1$, (25) reduces to

$$I_k(\nu,a,b,d) = \frac{\Gamma(\nu a+\nu+kb+1)\,\Gamma(d+1)}{\Gamma(d+\nu a+\nu+kb+2)} \tag{28}$$

(iii) If $k = 2$, then the Lauricella function F_D reduces to the first Appell's function F_1 [9], and (25) takes the following form,

$$I_k(\nu,a,b,d) = \frac{\Gamma(\nu a+\nu+1)\Gamma(d+1)}{\Gamma(d+\nu a+\nu+2)}\, F_1(d+1,-b,-b;d+\nu a+\nu+2;-a_1,-a_2) \tag{29}$$

$$|a_1| < 1, \quad |a_2| < 1, \quad Re(d+1) > 0, \quad \nu(a+1) > -1.$$

Case 2. Suppose the series expansion $\sum\limits_{i=0}^{\infty} c_i x^i$ has this form

$$\sum_{i=0}^{\infty} c_i x^i = (1-px)^{-q},$$

where $|p| < 1$, p and q are known. Substituting in (20), we obtain, by using [6,9], the following result

$$I_q(\nu,a,b,d) = \frac{\Gamma(\nu a+\nu+1)\Gamma(d+1)}{\Gamma(d+\nu a+\nu+2)}\, {}_2F_1(bq,d+1;d+\nu a+\nu+2;p) \tag{30}$$

where $|p| < 1$, $Re(d+1) > 0$ and $\nu(a+1) > -1$.

Case 3. Let

$$\left(\sum_{i=0}^{\infty} c_i x^i\right) = (1-z_1 x)^{-\delta_1}(1-z_2 x)^{-\delta_2}\ldots(1-z_m x)^{-\delta_m} \tag{31}$$

for m finite, $|z_j| < 1$; $j = 1,2,\ldots,m$ or δ_j a negative integer; substituting (31) in (20), we get, by using the same method as in Case 1, the following

$$I_\delta(\nu,a,b,d) = \frac{\Gamma(\nu a+\nu+1)\,(d+1)}{\Gamma(d+\nu a+\nu+2)}\, F_D^{(m)}(d+1,b\delta_1,b\delta_2,\ldots,b\delta_m; \tag{32}$$
$$d+\nu a+\nu+2;z_1,z_2,\ldots,z_m)$$

where

$$Re(d+1) > 0 \text{ and } \nu(a+1) > -1.$$

The integral representation of Lauricella's function $F_D^{(n)}$ is as follows [8, pp.452,(57)],

$$F_D^{(n)}(a, b_1, b_2, \ldots; c; z_1, z_2, \ldots, z_n)$$

$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 u^{a-1}(1-u)^{c-a-1}(1-uz_1)^{-b_1}\ldots(1-uz_n)^{-b_n} du$$

where $Re\, c > Re\, a > 0$; $\arg(1-z_n) < \pi$; $n = 1, 2, \ldots, n$.

Case 4. Let

$$(\sum_{i=0}^{\infty} c_i x^i) = \frac{(1-ux)^{\delta}}{(1-vx)^g} \tag{33}$$

where $|u| < 1$ and $|v| < 1$.

Substituting (32) in (20) and by using [8,pp.450,(42)], we have

$$I(v, a, b, d) = \frac{\Gamma(va+v+1)\Gamma(d+1)}{\Gamma(d+va+v+2)} F_1(d+1, -\delta b, gb; d+va+v+2; u, v), \tag{34}$$

for $Re(d+1) > 0$ and $v(a+1) > -1$.

§3. Stellar model with generalized energy rate.

In this section we assume the radial density ρ is a linear function in r, that is

$$\rho(r) = \rho_c(1 - \frac{r}{R}), \tag{35}$$

and the rate of the nuclear generation function $\varepsilon(r)$ is given by

$$\varepsilon(r) = \varepsilon_0(\rho_0, T_0)(\frac{\rho(r)}{\rho_0})^a \exp(-\xi\frac{\rho(r)}{\rho_0})(\frac{T(r)}{T_0})^b \tag{36}$$

where $\xi > 0$.

Hence by virtue of (35) and (36), the luminosity of the star defined by (3), becomes

$$L(R) = 4\pi\varepsilon_0(\rho_0, T_0)(\frac{\rho_c}{\rho_0})^a(\frac{T_c}{T_0})^b \rho_c R^3 \exp(-\xi\frac{\rho_c}{\rho}) I', \tag{37}$$

where

$$I' = \int_0^1 x^2(1-x)^{a+1} \exp(\xi\frac{\rho_c}{\rho_0}x)(\sum_{i=0}^{\infty} c_i x^i)^b. \tag{38}$$

In order to evaluate I', we shall consider the more general integral

$$I'(a,b,d,\xi) = \int_0^1 x^d (1-x)^{a+1} \exp\left(\xi \frac{\rho_c}{\rho_o} x\right) \left(\sum_{i=0}^{\infty} c_i x^i\right)^b dx \tag{39}$$

We shall consider four different representation of the series involved in the integrand. $I'(a,b,d,\xi)$ is evaluated in terms of the confluent hypergeometric functions of several variables.

Case 1. Recall (21) and apply the same technique as in case 1 of section 2 to obtain $I'(a,b,d,\xi)$ in the following form,

$$I'_k(a,b,d,\xi) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_k=0}^{\infty} \frac{(-b)_{n_1} \cdots (-b)_{n_k}}{n_1! \cdots n_k!}$$

$$\times (-a_1)^{n_1} \cdots (-a_k)^{n_k} \int_0^1 x^{d+n_1+\cdots+n_k} (1-x)^{a+1} c^{\xi(\rho_c/\rho_o)x} dx \tag{40}$$

The above equation can be written by using [4], in the following form,

$$I'_k(a,b,d,\xi) \tag{41}$$

$$= \frac{\Gamma(a+2)\Gamma(d+1)}{\Gamma(a+d+3)} \phi_D^{(k+1)}\left(d+1,-b,\ldots,-b;a+d+3;-a_1,\ldots,-a_k,\xi\frac{\rho_c}{\rho_o}\right),$$

where $|a_j| < 1$, $j = 1,\ldots,k$ or b is a positive integer, $Re(a+2) > 0$ and $Re(d+1) > 0$, $\phi_D^{(n)}(\alpha,\beta_1,\ldots,\beta_{n-1};\gamma;x_1,\ldots,x_n)$ is the confluent hypergeometric function of several variables define as

$$\phi_D^{(n)}(\alpha,\beta_1,\ldots,\beta_{n-1};\gamma;x_1,\ldots,x_n)$$

$$= \sum_{m_1,\ldots,m_n=0}^{\infty} \frac{(\alpha)_{m_1+\cdots+m_n}(\beta_1)_{m_1}\cdots(\beta_{n-1})_{m_{n-1}}}{(\gamma)_{m_1+\cdots+m_n} m!\cdots m_n!} x_1^{m_1}\cdots x_n^{m_n}$$

$|x_1| < 1,\ldots,|x_{n-1}| < 1$; x_n any finite value.

Case 2. Consider case

$$\sum_{i=0}^{\infty} c_i x^i = (1 - px)^{-q}.$$

Using $|8, \text{ pp.}451,(48)|$ in (39), we obtain

$$I_q'(a,b,d,\xi) = \int_0^1 x^d (1-x)^{a+1} \exp\left(\xi \frac{\rho_c}{\rho_0} x\right)(1-px)^{-bq} dx \tag{42}$$

$$= \frac{\Gamma(d+1)\Gamma(a+2)}{\Gamma(a+d+3)} \phi_1\left(d+1, bq, a+d+3; p, \xi \frac{\rho_c}{\rho_0}\right)$$

for $Re(d+1) > 0$ and $a > -2$. ϕ_1 is the Humbert function de-fined by

$$\phi_1(\alpha,\beta;\gamma;x,y) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}(1-ux)^{-\beta} e^{uy} du, \tag{43}$$

$Re(\alpha) > 0$ and $Re(\gamma-\alpha) > 0$.

Case 3. Recall (31) and substitute in (39), we get

$$I_\delta'(a,b,d,\xi) = \int_0^1 x^d(1-x)^{a+1}(1-z_1 x)^{-\delta_1 b} \ldots (1-z_m x)^{-\delta_m b} \exp\left(\xi \frac{\rho_c}{\rho} x\right) dx \tag{44}$$

Equation (44) can be expressed in terms of ϕ_D [4] and is given by

$$I_\delta'(a,b,d,\xi)$$
$$= \frac{\Gamma(d+1)\Gamma(a+2)}{\Gamma(a+d+3)} \phi_D^{(m+1)}\left(d+1, \delta_1 b, \ldots, \delta_m b; a+d+3; z_1, \ldots, z_m, \xi \frac{\rho_c}{\rho_0}\right) \tag{45}$$

where $|z_j| < 1$, $j = 1,\ldots,m$ or δ_j is a negative integer, $Re(d+1) > 0$ and $a > -2$.

Case 4. We consider the case when the series $\sum_{i=0}^{\infty} c_i x^i$ can be expressed as

$$\sum_{i=0}^{\infty} c_i x^i = e^{-\lambda x}(1-px)^{-q}.$$

$\lambda > 0$, $|p| < 1$.

Hence, using (43), we obtain

$$I_q'(a,b,d,\xi) = \frac{\Gamma(d+1)\Gamma(a+2)}{\Gamma(a+d+3)} \phi_1(d+1,bq,a+d+3;p,\xi\frac{\rho_c}{\rho_0}-\lambda) \tag{46}$$

for $Re(d+1) > 0$, $a > -2$, $|p| < 1$.

As the results established here involve generalized hypergeometric functions of several variables, special values of the parameters will lead to a number of particular cases.

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