

LOCALIZATION IN BUNDLES OF UNIFORM SPACES

by

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§1. Introduction. In sheaf theory we have at our disposal the so called "germination process" by means of which a sheaf can be obtained, in a natural manner, from data provided by a presheaf. Essentially, "germination" is a stalk producing mechanism that requires direct systems, whose colimits are the desired stalks of the sheaf.

The purpose of this paper is to establish a version of that process in the vastly more general situation of bundles of uniform spaces. Uniformities are described in this work by families of pseudometrics and key constructions depend on category-theoretical considerations. The "localization" procedure to be studied provides a universal arrow from the given presheaf of uniform spaces to the functor assigning to each bundle of uniform spaces the presheaf of its bounded local sections.

In the applications the following data are given:

- (a) a surjection $p: G \rightarrow T$
- (b) a uniformity on G
- (c) a topology on T
- (d) a family \mathcal{L} of selections for p .

A topology on G is sought, with the property the each selection $\alpha \in \Sigma$ becomes continuous. But such a topology generally does not exist, unless G is modified appropriately, through "localization". A family of modified stalks is then obtained in terms of the given uniformity, the neighborhood filters of the topological space T and the family Σ of selections.

§2. Directed colimits.

2.1. Consider the pairs $(X, (d_i^X)_{i \in I})$ where X is an arbitrary set and $(d_i^X)_{i \in I}$ is a family of pseudometrics on X . As usual, each d_i^X takes values in the extended reals $\bar{\mathbb{R}}$ but $d_i^X(x, y) = 0$ does not imply $x = y$ as in the case of a metric.

The family is called the *gauge* of X . Such a family defines on X a uniform structure and conversely, as it is well known, to any uniform space we can assign a family of pseudometrics such that the sets $\{(x, y) : d_i^X(x, y) < \epsilon\}$ with $\epsilon > 0$ and $i \in I$ form a fundamental systems of entourages. So the pair $(X, (d_i^X)_{i \in I})$ will be referred to as a *pseudometric presentation* of X .

We define the homomorphisms sets between two pair, say $(X, (d_i^X)_{i \in I})$ and $(Y, (d_j^Y)_{j \in J})$ as follows: if $I \neq J$, then the corresponding hom-set is empty; but if $I = J$, then f belongs to the hom-set if and only if $f: X \rightarrow Y$ is a map such that $d_j^Y(f(x), f(y)) \leq d_i^X(x, y)$ for every $i \in I$ and every $x, y \in X$.

We call such maps *non-expansive morphisms* or *contractive maps* or *contractions*. The above hom-sets are clearly closed under ordinary composition.

We denote by $Unif$ the category of all pairs $(X, (d_i^X)_{i \in I})$ and non-expanding morphisms. By $Unif_S$ we denote the full subcategory of $Unif$ consisting of all pairs such that X is a Hausdorff space when it is furnished with the uniformity defined by its gauge. In particular, a pair is an object in $Unif_S$ iff $d_i^X(x, y) = 0$ for every $i \in I$ implies $x = y$.

2.2. Consider a direct system in Unif_S ,

$$[(X_\alpha, (d_\alpha^\gamma)_{\gamma \in I})_{\alpha \in A}, (\rho_{\beta\alpha})_{\beta \geq \alpha}],$$

where A is a directed set. Then each $\rho_{\beta\alpha}: X_\alpha \rightarrow X_\beta$ is a contractive map such that

(i) $\rho_{\alpha\alpha} = id_{X_\alpha}$

(ii) if $\gamma \geq \beta \geq \alpha$, then $\rho_{\gamma\alpha} = \rho_{\gamma\beta} \rho_{\beta\alpha}$

Let X be the disjoint union of the family $(X_\alpha)_{\alpha \in A}$. For each define $\delta_\alpha: X \times X \rightarrow \bar{\mathbb{R}}$ by

$$\delta_\alpha(u_\alpha, u_\beta) := \inf_{\substack{\alpha \leq \gamma \\ \beta \leq \gamma}} d_\alpha^\gamma(\rho_{\gamma\alpha}(u_\alpha), \rho_{\gamma\beta}(u_\beta)),$$

then δ_α is a pseudometric (cf. [3]).

Define on X an equivalence relation R by uRv iff $\delta_\alpha(u, v) = 0$ for every $\alpha \in I$. Let \bar{u} be the R -equivalence class of u . Consider the quotient $Z = X/R$ and define for each α , $\bar{\delta}_\alpha(\bar{u}, \bar{v}) = \delta_\alpha(u, v)$. It can be shown that $\bar{\delta}_\alpha$ is a well defined pseudometric on Z (cf. [3]). Moreover, the canonical map $\tau_\alpha: X_\alpha \rightarrow Z$ is contractive.

2.3. $(Z, (\bar{\delta}_\alpha)_{\alpha \in I})$ and the maps $(\tau_\alpha)_{\alpha \in A}$ define a cone for the direct system considered in 2.2. Furthermore, this cone turns out to be the direct colimit of the system. In fact, given another inductive cone, say $(Y, (\delta_\alpha)_{\alpha \in I})$, $\sigma_\alpha: X_\alpha \rightarrow Y$, where α runs through A ; define $\phi: Z \rightarrow Y$ by $\phi(\bar{u}_\alpha) = \sigma_\alpha(u_\alpha)$. One can easily see that ϕ is a well defined map (invoking the Hausdorffness of Y), it is also a contraction satisfying the universal property for Z (cf. [3]). Hence we have the following result.

2.4. **PROPOSITION.** Every direct system in Unif_S has a direct colimit.

§3. Bundles of Uniform Spaces.

3.1. DEFINITION. Let $p:G \rightarrow T$ be a surjective function. A pseudometric for p is a map $d:G \times G \rightarrow \bar{\mathbb{R}}$ such that its restriction to each fiber $G_t = \{u \in G: p(u) = t\}$ is a pseudometric in G_t and $d(u,v) = +\infty$ if $p(u) \neq p(v)$.

We refer to [4], [10] for the definitions of selection, section, local section, etc. We adopt the definition of uniform bundle given in [9]. In particular, the tubes $\mathcal{T}_\varepsilon(\sigma) = \{u \in G: d_\chi(u, \sigma(p(u))) < \varepsilon\}$ around local sections σ , are a basis for the topology of G and the map $t \rightarrow d_\chi(\sigma(t), \tau(t)): U \rightarrow \bar{\mathbb{R}}$ is upper semicontinuous for every pair σ, τ of local sections over U and every $\chi \in I$.

A uniform bundles is said to be Hausdorff if each fiber is Hausdorff with respect to the induced uniformity.

Our aim now is to construct a bundle of uniform spaces from data provided by a "saturated" presheaf in the sense of definition 3.2. Given a topological spaces T , in this paper we will denote by \mathbf{A} the topology or collection of open sets of T and by $\mathbf{V}(t)$ the set of all open neighborhoods of t , $t \in T$.

3.2. DEFINITION. A presheaf $[\sum(u)_{u \in \mathbf{A}}, (\rho_{vu})_{u,v \in \mathbf{A}}]$, or \sum for short, in $Unif$ or $Unif_S$ is said to be saturated if for every $u \in \mathbf{A}$ the gauge $(d_\chi^u)_{\chi \in I}$ of $\sum(u)$ is saturated, i.e. $\max(d_\chi^u, d_j^u)$ belongs to the gauge of $\sum(u)$ for every $\chi, j \in I$.

3.3. REMARK. If a presheaf \sum is saturated, then for every $\chi, j \in I$ there exists $k \in I$ such that $d_k^u = \max(d_\chi^u, d_j^u)$. In fact, the restriction map $\rho_{\phi T}$ can be factored as $\rho_{\phi U} \rho_{UT}$, for every u , and this secures the independence of k from u .

3.4. A construction. Let \sum be a saturated presheaf in $Unif_S$, a uniform (\hat{G}, \hat{p}, T) is to be constructed. Consider for each $t \in T$ the presheaf

$$[\sum(u)_{u \in \mathbf{V}(t)}, (\rho_{vu})_{u,v \in \mathbf{V}(t)}].$$

Let $(\hat{G}_t, \tau_{tU}: \hat{\Sigma}(U) \rightarrow \hat{G}_t, U \in V(t))$ be the direct colimit of this presheaf, as obtained in 2.4. Let \hat{G} be the disjoint union of the family $\{\hat{G}_t: t \in T\}$, i.e., $\hat{G} = \dot{\bigcup}_{t \in T} \hat{G}_t = \{\tau_{tU}(x): t \in T, x \in \hat{\Sigma}(U)\}$ and let $\hat{p}: \hat{G} \rightarrow T$ be given by $\hat{p}(\tau_{tU}(x)) = t$. The family of pseudometrics for \hat{p} are defined as follows. For each $i \in I$, $\hat{d}_i: \hat{G} \times \hat{G} \rightarrow \bar{\mathbb{R}}$ is given by $\hat{d}_i(\tau_{tU}(x), \tau_{sV}(y)) = \infty$ if $s \neq t$, and $\hat{d}_i(\tau_{tU}(x), \tau_{tV}(y)) = \delta_i(\bar{x}, \bar{y}) = \delta_i(x, y)$, where

$$\delta_i(x, y) = \inf_{\substack{W \in V(t) \\ W \subseteq U \cap V}} \delta_i^W(\rho_{WU}(x), \rho_{WV}(y)).$$

Therefore the pseudometric \hat{d}_i coincides with the colimit pseudometric in each fiber.

Using the saturation assumption on the presheaf $\hat{\Sigma}$ it follows by straightforward verification that the family of pseudometrics $(\hat{d}_i)_{i \in I}$ is also saturated (cf. [3]).

We define a family of local selection for \hat{p} as follows. For $U \in \mathbf{A}$ and $x \in \hat{\Sigma}(U)$, let $\hat{x}: U \rightarrow \hat{G}$ be defined as $\hat{x}(t) = \tau_{tU}(x)$. Now, given any element of \hat{G} , say $\tau_{tU}(x) \in \hat{G}_t$, it clearly belongs to the tube with center \hat{x} and radius ε (with respect to the distance \hat{d}_i), i.e., $\hat{d}_i(\tau_{tU}(x), \hat{x}(t)) = 0 < \varepsilon$. This is a condition (a) of Theorem 5 in [9].

On the other hand, $s \mapsto \hat{d}_i(\hat{x}(s), \hat{y}(s)): U \rightarrow \bar{\mathbb{R}}$ is upper semicontinuous, for $x, y \in \hat{\Sigma}(U)$. In fact,

$$\begin{aligned} A &= \{s \in U: \hat{d}_i(\hat{x}(s), \hat{y}(s)) < \varepsilon\} = \{s \in U: \hat{d}_i(\tau_{sU}(x), \tau_{sU}(y)) < \varepsilon\} \\ &= \{s \in U: \inf_{\substack{W \in V(s) \\ W \subseteq U}} \delta_i^W(\rho_{WU}(x), \rho_{WU}(y)) < \varepsilon\} \end{aligned}$$

Let $t \in A$, then there exists $W' \in V(t)$ such that $\delta_i^{W'}(\rho_{W'U}(x), \rho_{W'U}(y)) < \varepsilon$. Then $W' \subseteq A$ because for every $s \in W'$ we have,

$$\hat{d}_i(\hat{x}(s), \hat{y}(s)) = \inf_{\substack{W \in V(s) \\ W \subseteq U}} \hat{d}_i(\rho_{WU}(x), \rho_{WU}(y)) \leq \delta_i^{W'}(\rho_{W'U}(x), \rho_{W'U}(y)) < \varepsilon$$

Then $s \in A$ and A is open. Thus condition (b) of Theorem 5 in [9] is also satisfied.

In conclusion, Theorem 5 in [9] secures the existence of a topology on G such that

- (i) each local selection $\hat{x}:U \rightarrow \hat{G}$ is continuous,
- (ii) (\hat{G}, \hat{p}, T) is a uniform bundle.

3.5. Category Bun of uniform bundles. Let T be a fixed topological space. Denote by Bun the category whose objects are uniform bundles with base space T . Define the morphisms in Bun between a pair of objects $((G, p, T), (d_i)_{i \in I})$ and $((H, q, T), (d_j)_{j \in J})$ as follows. In the case $I = J$, f is in the corresponding hom-set iff f is fiber-preserving and contrative. But in the case $I \neq J$, the corresponding hom-set is defined to be empty.

Bun_S denotes the full subcategory of Bun whose object are Hausdorff uniform bundles.

3.6. To a bundle of uniform spaces (G, p, T) we can associate a sheaf of uniform spaces \sum_p such that for each open set U of T , $\sum_p(U)$ is the space of all local sections whose domain is U , and to each morphism $h:G \rightarrow H$ of bundles of uniform spaces, we can associate a sheaf morphism $\sum_p(h) = 0$ such that if $\alpha \in \sum_p(U)$ then $0_\alpha = h\alpha \in \sum_q(U)$.

Let A denote the topology of T . A presheaf $\sum:A \rightarrow Bun$ is called a *presheaf of local sections* in the uniform bundle (G, p, T) if for every $U \in A$, $\sum(U) \subset \sum_p(U)$. Hence the domain of each local section in $\sum(U)$ is U .

3.7. PROPOSITION. Let $((G, p, T), (d_i)_{i \in I})$ be a uniform bundle, $t \in T$ and $U \in \mathcal{V}(t)$. If $\alpha, \beta \in \sum_p(U)$ then

$$\inf_{W \in \mathcal{V}(t)} d_i(\alpha_W, \beta_W) = d_i(\alpha(t), \beta(t)).$$

Proof. Without loss of generality we can assume that $d_i(\alpha(t), \beta(t)) < \infty$. By the upper semicontinuity of $s \rightarrow d_i(\alpha(s), \beta(s))$, given $\varepsilon > 0$, we can find $W \subseteq U$, open neighborhood of t such that $d_i(\alpha(t), \beta(t)) \leq d(\alpha_W, \beta_W) < d_i(\alpha(t), \beta(t)) + \varepsilon$ where $d(\alpha_W, \beta_W) = \sup\{d_i(\alpha(s), \beta(s)) : s \in W\}$.

It is also clear that (cf. [3]) $\inf_{\substack{w \in V(t) \\ w \subseteq U}} d_i(\alpha_w, \beta_w) = d_i(\alpha(t), \beta(t))$.

3.8. PROPOSITION. Let (G, p, T) and (H, q, T) uniform bundles, \sum a presheaf of local sections in (G, p, T) and \sum' a presheaf of local sections in (H, q, T) . Assume that (H, q, T) is Hausdorff and that \sum is full, in the sense that for each $u \in G$ there exists $\alpha \in \sum(V)$ such that $\alpha(p(u)) = u$. Then a morphism of presheaves $\phi: \sum \rightarrow \sum'$ determines a morphism $f: G \rightarrow H$ of uniform bundles.

Proof. Denote by I the set of indexes of the gauges of $\sum(U)$ and $\sum'(U)$. The giving of $\phi: \sum \rightarrow \sum'$ guarantees that I is also the set of indexes for the family of pseudometrics of the bundles. Define $f: G \rightarrow H$ by $f(x) = \phi_U(\alpha)(t)$ where $p(x) = t$, $\alpha(t) = x$ and $t \in U$. This is a well defined map: suppose $\beta \in \sum(V)$ is such that $\beta(t) = x$. Since for every $i \in I$ $d_i(\alpha(t), \beta(t)) = 0$, $\inf d_i(\alpha_w, \beta_w) = 0$ where w ranges through the set of all open neighborhoods of t such that $w \subset U \cap V$. Thus given $\varepsilon > 0$, there exists $w \in V(t)$ such that $d_i(\alpha_w, \beta_w) < \varepsilon$. Then

$$\begin{aligned} d_i(\phi_U(\alpha)(t), \phi_V(\beta)(t)) &= d_i(\phi_w(\alpha_w)(t), \phi_w(\beta_w)(t)) \\ &\leq d_i(\phi_w(\alpha_w), \phi_w(\beta_w)) \leq d_i(\alpha_w, \beta_w) < \varepsilon. \end{aligned}$$

Hence $d_i(\phi_U(\alpha)(t), \phi_V(\beta)(t)) = 0$ for every $i \in I$. Since (H, q, T) is Hausdorff it follows that $\phi_U(\alpha)(t) = \phi_V(\beta)(t)$. This shows that f is well defined. The map f is contractive fiberwise: in fact, take $x, y \in G$ with $p(x) = p(y) = t$ and let $\alpha \in \sum(U)$, $\beta \in \sum(V)$ be such that $\alpha(t) = x, \beta(t) = y$. For each $w \in V(t)$ with $w \subset U \cap V$, and $i \in I$,

$$\begin{aligned} d_i(f(x), f(y)) &= d_i(\phi_U(\alpha)(t), \phi_V(\beta)(t)) = d_i(\phi_w(\alpha_w)(t), \phi_w(\beta_w)(t)) \\ &\leq d_i(\phi_w(\alpha_w), \phi_w(\beta_w)) \leq d_i(\alpha_w, \beta_w). \end{aligned}$$

By taking inf. over all $w \in V(t)$ with $w \subset U \cap V$ it follows that $d_i(f(x), f(y)) \leq d_i(x, y)$. To establish the continuity

of δ , let $x \in G$, $t = p(x)$ and σ be a local section in the bundle (H, q, T) such that $\delta(x) \in \mathcal{J}_\varepsilon^{(i)}(\sigma) = \{v \in H: d_i(v, \sigma(p(v))) < \varepsilon\}$. Take $\alpha \in \sum(U)$ such that $\alpha(t) = x$, then $W = \{s \in U \cap \text{dom}\sigma: d_i(\phi_U(\alpha)(s), \sigma(s)) < \delta\}$, with $d_i(\delta(\alpha(t)), \alpha(t)) < \delta < \varepsilon$, is an open neighborhood of $t = p(x) = q(\delta(x))$. Now

$$\mathcal{J}_{\varepsilon-\delta}^{(i)}(\phi_W^{(i)}(\alpha_W)) \subset \mathcal{J}_\varepsilon^{(i)}(\sigma);$$

indeed, if $y \in \mathcal{J}_{\varepsilon-\delta}^{(i)}(\phi_W(\alpha_W))$, then $s = q(y) \in W$ and

$$d_i(y, \sigma(s)) \leq d_i(y, \phi_W(\alpha_W)(s)) + d_i(\phi_W(\alpha_W)(s), \sigma(s)) < \varepsilon - \delta + \delta = \varepsilon.$$

Using the contractivity of δ we also have that

$$\delta(\mathcal{J}_{\varepsilon-\delta}^{(i)}(\alpha_W)) \subseteq \mathcal{J}_{\varepsilon-\delta}^{(i)}(\phi_W(\alpha_W)) = \mathcal{J}_{\varepsilon-\delta}^{(i)}(\delta\alpha_W).$$

Thus δ is continuous at x .

3.9. REMARK. Let \sum be a saturated presheaf in Unif_S , $x \in \sum(U)$, $u, v \in \mathbf{A}$ then $\hat{x}|_v = (\rho_{vu}(x))^\wedge$.

Proof. $\hat{x}|_v: v \rightarrow \hat{G}$ and $(\rho_{vu}(x))^\wedge: v \rightarrow \hat{G}$. We have $\hat{x}|_v(t) = \tau_{tU}(x)$ and $(\rho_{vu}(x))^\wedge(t) = \tau_{tV}(\rho_{vu}(x))$. The equality $\tau_{tU}(x) = \tau_{tV}(\rho_{vu}(x))$ follows from properties of the colimit map. So the remark follows. \blacktriangle

3.10. THEOREM. Given a saturated presheaf \sum in Unif_S there exists a Hausdorff bundle of uniform spaces (\hat{G}, \hat{p}, T) and a presheaf $\hat{\sum}$ of local sections such that

- (a) the presheaf $\hat{\sum}$ is full
- (b) there exists a presheaf map $\phi: \sum \rightarrow \hat{\sum}$.

Proof. The construction was made in 3.4. Define $\hat{\sum}(U) := \{\hat{x}: x \in \sum(U)\} \subseteq \hat{\sum}_p(U)$. The restriction maps are the ordinary restrictions, $\rho_{vu}: \hat{\sum}(U) \rightarrow \hat{\sum}(V)$, $\rho_{vu}(\hat{x}) = \hat{x}|_v$. Note that $\hat{x}|_v$ is in fact an element of $\hat{\sum}(V)$ due to the previous remark. For each $u \in \mathbf{A}$, take $\phi_u: \sum(U) \rightarrow \hat{\sum}(U)$ with $\phi_u(x) := \hat{x}$ and $\hat{x}(t) = \tau_{tU}(x)$. It is clear that ϕ_u is contractive and hence a morphism. Moreover, these maps are compatible with

the restrictions maps and consequently ϕ is a presheaf morphism.

3.11. THEOREM. Let Σ be a saturated presheaf in Unif_S and $F: \text{Bun}_S \rightarrow \text{Presh}$ be the functor defined in 3.6 between the category Bun_S of Hausdorff uniform bundles and the category Presh of presheaves of uniform spaces defined in 3.6. The pair $(\hat{G}, \hat{p}, T), \phi: \Sigma \rightarrow \hat{\Sigma}_p$ is a universal arrow from Σ to F .

Proof. See Theorem 12.1 in [3].

3.12. PROPOSITION. Let G be a set and $p: G \rightarrow T$ be a surjection. If $(d_i)_{i \in I}$ is a saturated family of pseudometrics for p and G_x is Hausdorff for each $t \in T$, then given any presheaf Σ of local sections for p , the application of the localization process provides the following additional information

- (i) for every $\alpha, \beta \in \Sigma(U), d_i(\hat{\alpha}, \hat{\beta}) = d_i(\alpha, \beta)$,
- (ii) $\phi: \Sigma \rightarrow \hat{\Sigma}$ is a presheaf isomorphism.

Proof. (i) The maps ρ_{VU} with $V \subseteq U$ are ordinary restrictions maps. The inequality $d_i(\hat{\alpha}, \hat{\beta}) \leq d_i(\alpha, \beta)$ is obtained directly from the localization process. Conversely, given U and $t \in U$ we have

$$d_i(\alpha(t), \beta(t)) \leq \inf_{\substack{W \in V(t) \\ W \subseteq U}} d_i(\alpha_W, \beta_W)$$

where as usual α_W is the restriction of α to W . Then for every t , $d_i(\alpha(t), \beta(t)) \leq \hat{d}_i(\hat{\alpha}(t), \hat{\beta}(t))$ and hence $d(\alpha, \beta) \leq \hat{d}(\hat{\alpha}, \hat{\beta})$.

(ii) is just a restatement of (i).

3.13. EXAMPLE. Let T be a topological space, Y a Hausdorff uniform space with a saturated family of pseudometrics $(d_i)_{i \in I}$, $p: T \times Y \rightarrow T$ the canonical projection onto T and \mathcal{S} a nonempty set of subsets of T . Define a family of pseudometrics for p as follows. For each $S \in \mathcal{S}$, and $i \in I$, let $d_S^i: (T \times Y) \times (T \times Y) \rightarrow \bar{\mathbb{R}}$ be such that for $u = (t, x), v = (s, y)$,

$$d_S^i(u, v) = \begin{cases} d_i(x, y) & \text{if } p(u) = t = s = p(v) \in S \\ 0 & \text{if } p(u) = t = s = p(v) \notin S \\ \infty & \text{if } p(u) = t \neq s = p(v) \end{cases}$$

We can now define pseudometrics between selections (denoted also by d_S^i)

$$d_S^i(\alpha, \beta) = \sup_{t \in S} d_t^i(\alpha(t), \beta(t)).$$

Note that the selections are, in this example, in a bijective correspondance with T^Y . This family of pseudometrics as i ranges through I and S ranges through \mathcal{S} determines the uniformity of uniform convergence in the sets of \mathcal{S} . Since Y is Hausdorff each fiber of $T \times Y$ is Hausdorff too.

Let $\hat{\mathcal{L}}$ be the presheaf defined by $\hat{\mathcal{L}}(U) = \{\alpha_U : \alpha \in \mathcal{H}\}$, \mathcal{H} being the set of all global selections for p and α_U the restriction of α to U .

Since for $\alpha, \beta \in \mathcal{H}$ we cannot in general secure the upper semicontinuity of $t \mapsto d_S^i(\alpha(t), \beta(t))$, $(T \times Y, p, T)$ is not in general a uniform bundle, but through localization we can construct a uniform bundle $((T \times Y)^\wedge, \hat{p}, \hat{T})$ and a presheaf $\hat{\mathcal{L}}$ of local sections for \hat{p} , such that $\hat{d}(\hat{\alpha}, \hat{\beta}) = d(\alpha, \beta)$.

Nevertheless, there is an instance, the uniformity of pointwise convergence, in which we do have upper semicontinuity of the pseudometrics, provided that T is a T_0 space. If that is the case, we have automatically a uniform bundle.

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(Recibido en febrero de 1989, la versión revisada en enero de 1990).