The discontinuous boundary problem for transversally-isotopic plates containing linear defects

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ABSTRACT. The main purpose of this paper is to derive relations connecting the discontinuities of the main geometrical and physical characteristics of the stress-state of a plate, containing linear defects with discontinuities of solving functions.

Key words and phrases. The stress-state of a plate, linear defects, discontinuities of solving functions.

1991 Mathematics Subject Classification. Primary 35Q. Secondary 35RO5.

1. Introduction

The extensive use of structures in the form of plates and envelopes in machine-design, shipbuilding and apparatus design demands research in the field of the strength of these structures. Furthermore, many structures represented by plates vibrate due to the harmonic loads applied to them. Experimental investigations show that concentrated loads applied either at the ends of split or refined inclusion cause a sharp concentration of stress. The precise models of vibration theory of plates which take into account the weak displaced stiffness

of the material of the plate have proved to be most suitable when studying the dynamical theory of vibrations of thin-shelled elements of the structure.

In the present paper, abstracting from the well known models of the theory of vibrations of a plate [2, 3, 4], we develop the model depending on the use of the system of differential equations of plate vibrations with weak displaced stiffness (transversally-isotropic plates, given by Ambartsomain [1]).

In [1] the author derived the main system of partial differential equations of the vibration theory of a transversally-isotropic plate in the frame of Ambartsomain modulus:

$$\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} + \rho \frac{12}{h^2} \omega^2 W = -\frac{12}{h^3} q, \tag{1}$$

$$D\frac{\partial}{\partial x}\Delta W - \frac{h^2 D}{10G'}\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 \phi}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 \psi}{\partial x \partial y}\right] + \frac{h^3}{12}\phi$$
$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial q}{\partial x}, \tag{2}$$

$$D\frac{\partial}{\partial y}\Delta W - \frac{h^2 D}{10G'}\left[\frac{\partial^2 \psi}{\partial y^2} + \frac{1-\nu}{2}\frac{\partial^2 \psi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 \phi}{\partial x \partial y}\right] + \frac{h^3}{12}\phi$$
$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial q}{\partial y}. \tag{3}$$

Here $W(x,y)$ is the deflection amplitude, and the functions $\phi(x,y)$, $\psi(x,y)$ up to a fixed multiplier coincide with the transversal forces in the plate. The functions $W(x,y), \phi(x,y)$ and $\psi(x,y)$ must also satisfy the following boundary conditions;

$$x = 0, \ a; \ W = -D\frac{\partial^2 W}{\partial x^2} + \frac{h^5}{60}\frac{G}{G'}\frac{\partial \phi}{(1-\nu)G'\partial x} = \psi = 0, \tag{4}$$

$$y = 0, \ b; \ W = -D\frac{\partial^2 W}{\partial y^2} + \frac{h^5}{60}\frac{G}{(1-\nu)G'}\frac{\partial \psi}{\partial y} = \phi = 0, \tag{5}$$

for the case of the rectangular plate

$$(x,y) \in \overline{\Omega^2} = \left\{(x,y) \in \mathbb{R}^2 : 0 \le x \le a; \ 0 \le y \le b\right\}.$$

Here $D = \frac{Eh^3}{12(1-\nu^2)}$ is the cylindrical stiffness of the plate, h is the thickness of the plate and E, E', ν, ν' are the young modulus and the Poisson coefficient, respectively, for pressure-tension with respect to the xy-isotropic plane and in the direction which is perpendicular to this plane, $q(x,y) \in \mathcal{C}^1$ is the amplitude of the external load, ω is the vibrations frequency. In the case of the band plates

$$(x,y) \in \overline{E^2} = \left\{(x,y) \in \mathbb{R}^2 : 0 \le x \le a, \ -\infty \le y \le \infty\right\},$$

the functions $W(x,y), \phi(x,y)$ and $\psi(x,y)$ must be decreasing as $|y| \longrightarrow \infty$, $W(x,y)$ together with its partial derivatives through third order and $\phi(x,y)$ and $\psi(x,y)$ together with its partial derivatives through second order. The solutions of the system (1)–(3) must satisfy the boundary conditions (4).

The presence of the linear defects in the plate field which are parallel to the y–axis leads to the appearance of jump discontinuities in the main geometrical and physical characteristics of the stress-state of the plate.

In the case of harmonic vibrations, for discontinuities we introduce the following notations for the amplitude discontinuities.

$$\langle W(L,y) \rangle := W(L-0,y) - W(L+0,0) = W_0(y);$$
$$\langle A_X(L,y) \rangle = A_1(y); \quad \langle A_Y(L,y) \rangle = A_2(y);$$
$$\langle M_X(L,y) \rangle = M_1(y); \quad \langle H(L,y) \rangle = H(y); \tag{6}$$
$$\langle N_X(L,y) \rangle = N(y)$$

In the case of a rectangular plate, $0 < C_1 \leq y \leq C_2 < b$. For band plate $0 < y < +\infty$, in this connection, $0 < L < a$. Here $W_0(y)$ is the discontinuity of the deflection amplitude of the mid-plane of the plate; $A_{1,2}(y)$ is the discontinuity of amplitudes of the rotation angles of the plate elements, at first perpendicular to its mid-plane, in the directions of the x and y axes respectively; $M(y)$ is the amplitude discontinuity of the bending moment in the plate in the direction of y axis; $H(y)$ is the amplitude discontinuity of a torque; $N(y)$ is the amplitude discontinuity of the transversal force in the plate in the direction of x axis.

The presence of the discontinuities (6) leads to the appearance of the analogous discontinuities for the solving functions $W(x,y)$, $\phi(x,y)$, $\psi(x,y)$:

$$\left\langle \frac{\partial W(1,y)}{dx^i} \right\rangle = W_i(y); \quad i = 1,2,$$
$$\left\langle \frac{\partial^k \phi(1,y)}{dx^k} \right\rangle = \phi_k(y); \quad \left\langle \frac{\partial^i \psi(1,y)}{dx^j} \right\rangle = \psi_j(y), \quad (k,j) = (0,1). \tag{7}$$

In this paper we obtain the relations connecting the discontinuities (6) of the main physical characteristics of the stress-state of the plate, with the discontinuities (7) of the solving functions.

The system of differential equations (1)–(3), the boundary conditions (4), (5) and the zone of linear defects determined the discontinuous boundary problem which is convenient to rewrite in the following operator form:

$$(\hat{A}_\omega u)(x,y) = (Qq)(xy); \tag{8}$$

in the domains $\widehat{\Omega}^2 = \overline{\Omega^2} \smallsetminus \{x = L; \; 0 < C_1 \leq y \leq C_2 < b\}$ for rectangular plates, or in $\widehat{E}^2 = \overline{E^2} \smallsetminus \{x = L; 0 \leq y < \infty\}$, $0 < L < a$ for band plates, where the vector function $u(x, y)$ is defined as $u(x, y) = (W, \phi, \psi)$. The equations (1)–(3) and the boundary conditions (4)–(5) determine the ordinary (continuous) boundary problem which can also be written in the operator form:

$$(A_\omega u)(x, y) = (Q_q)(x, y) \tag{9}$$

in the domains $\overline{\Omega^2}$ and $\overline{E^2}$. We call the generalized eigenvalues of the operator the values of the parameters $\omega \in \mathbb{R}_+$ for which the equation $(A_\omega u)(x, y) = 0$ has nontrivial solution $(A_\omega u)(x, y)$.

2. The statement of the discontinuous boundary problem and the relation between the test and auxiliary functions

We assume that the transversally-isotropic plate which takes up the domain $\overline{\Omega^2} = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq a; \; 0 \leq y \leq b\}$ and is freely supported at $x = 0, a$; $y = 0, b$ is bending under the action of the dynamical load of the vertically distributed intensity $Z(x, y, t) \in C^1(\overline{\Omega^2} \times [0, \infty))$. In this case the bending angles are expressed in terms of the solving functions $W(x, y, t)$, $\phi(x, y, t)$, $\psi(x, y, t)$ by the formulas

$$\alpha_X(x, y) \approx \tan \alpha_X = \left. \frac{\partial u_X}{\partial z} \right|_{z=0} = -\frac{\partial \omega}{\partial x} + \frac{h^2}{8G'}\phi; \tag{10}$$

$$\alpha_Y(x, y) \approx \tan \alpha_Y = \left. \frac{\partial u_Y}{\partial z} \right|_{z=0} = -\frac{\partial \omega}{\partial y} + \frac{h^2}{8G'}\psi; \tag{11}$$

The bending moments and the torque are

$$
\begin{aligned}
M_X(x, y, t) = {}& -D\left(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}\right) + \frac{h^5}{50}\frac{G}{(1-\nu)G'}\left(\frac{\partial \phi}{\partial x} + \nu \frac{\partial \psi}{\partial y}\right) \\
& + \frac{\nu' E}{(1-\nu)E'}\frac{h^2}{10}z(x, y, t) + \rho\frac{\nu' E}{(1-\nu)E'}\frac{h^3}{12}\frac{\partial^2 \omega}{\partial t^2};
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
M_Y(x, y, t) = {}& -D\left(\frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2}\right) + \frac{h^2}{60}\frac{G}{(1-\nu)G'}\left(\frac{\partial \psi}{\partial y} + \nu \frac{\partial \phi}{\partial x}\right) \\
& + \frac{\nu' E}{(1-\nu)E'}\frac{h^2}{10}z(x, y, t) + \rho\frac{\nu' E}{(1-\nu)E'}\frac{h^3}{12}\frac{\partial^2 \omega}{\partial t^2};
\end{aligned} \tag{13}
$$

$$H_{XY}(x,y) = \int_{h/2}^{h/2} \tau_{XY} z \, dz$$

$$= -D(1-\nu)\frac{\partial^2 \omega}{\partial x \partial y} + \left(\frac{h^5}{120}\frac{G}{G'}\right)\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right)$$

(14)

The transversal forces are

$$N_X(x,y) = \int_{-h/2}^{h/2} \tau_X z \, dz = \frac{h^3}{12}\phi(x,y),$$

(15)

$$N_Y(x,y) = \int_{-h/2}^{h/2} \tau_Y z \, dz = \frac{h^3}{12}\psi(x,y).$$

(16)

The functions $W(x,y)$, $\phi(x,y)$, and $\psi(x,y)$ satisfy the system of partial differential equations

$$\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} + \rho\frac{12}{h^2}\frac{\partial^2 \omega}{\partial t^2} = -\frac{12}{h^3}z(x,y,t)$$

(17)

$$D\frac{\partial}{\partial x}\Delta\omega - \frac{h^2}{10}\frac{D}{G'}\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 \phi}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 \psi}{\partial x \partial y}\right] + \frac{h^3}{12}\phi$$

$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial z(x,y)}{\partial x}$$

(18)

$$D\frac{\partial}{\partial}\Delta\omega - \frac{h^2}{10}\frac{D}{G'}\left[\frac{\partial^2 \psi}{\partial y^2} + \frac{1-\nu}{2}\frac{\partial^2 \psi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 \phi}{\partial x \partial y}\right] + \frac{h^3}{12}\phi$$

$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial z(x,y)}{\partial y}$$

(19)

and the boundary conditions

$$x = 0, a; \quad \omega = -D\frac{\partial^2 \omega}{\partial x^2} + \frac{h^5}{60}\frac{G}{1-\nu}\frac{\partial \phi}{G'\partial x} = \psi = 0; \quad y \in [0,b]$$

(20)

$$y = 0, b; \quad \omega = -D\frac{\partial^2 \omega}{\partial y^2} + \frac{h^5}{60}\frac{G}{1-\nu}\frac{\partial \phi}{G'\partial y} = \phi = 0; \quad x \in [0,a]$$

(21)

As in [4, 5], the linear defects of random nature in the plate we will call a segment of a straight line in the domain which is taken up by the plate, at passage through which discontinuities of the first kind, with the exception of arbitrarily small neighborhoods of the segment, ends simultaneously all the main unknown functions transversal force, bending moment, torque, bending angles and the deflections of the mid-plane.

For definiteness we consider the case, when the linear defect in the plate is the segment

$$L := \{(x,y) \in \mathbb{R}^2 : x = L; \ 0 < L < a; \ 0 < C_1 \le y \le C_2 < b\},$$

on which the solving functions have discontinuities. We introduce the notations:

$$\langle \omega(L, y, t)\rangle = \omega(L - 0, y, t) - \omega(L + 0, y, t) = \omega_0(y, t), \quad C_1 \le y \le C_2;$$
$$\langle \alpha_X(L, y, t)\rangle = \alpha_1(y, t); \quad \langle \alpha_Y(L, y, t)\rangle = \alpha_2(y, t);$$
$$\langle M_X(L, y, t)\rangle = m(y, t); \quad \langle H_{XY}(L, y, t)\rangle h(y, t); \tag{22}$$
$$\langle N_X(L, y, t)\rangle = n(y, t)$$

We shall also call the auxiliary functions $\omega(x, y, t)$, $\phi(x, y, t)$, $\psi(x, y, t)$, which are the solution of the system (17)–(19), and in terms of which the main unknown functions are expressed. In this case, $\omega(x, y, t)$, the deflection of the mid-plane, simultaneously belongs to the main and auxiliary functions.

The presence of the linear defect in the plate leads to the appearance of the discontinuities in the auxiliary functions as well, namely:

$$\left\langle \frac{\partial^i \omega(L, y, t)}{\partial x^i} \right\rangle = \omega_i(y); \quad i = 1, 2; \tag{23}$$

$$\left\langle \frac{\partial^k \phi(L, y, t)}{\partial x^k} \right\rangle = \phi_k(y); \quad k = 0, 1; \tag{24}$$

$$\left\langle \frac{\partial^j \psi(L, y, t)}{\partial x^j} \right\rangle = \psi_j(y); \quad i = 0, 1; \tag{25}$$

From (15), (10), (11), (14) and (13) it is easy to deduce the following relations which connect the discontinuities (22) of the physical and geometrical values at passage through the linear defect with the discontinuities (23)–(25) of the auxiliary functions $\omega(x, y, t)$, $\phi(x, y, t)$ and $\psi(x, y, t)$:

$$n(y, t) = \frac{h^3}{12}\phi_0(y, t); \quad \alpha_1(y, t) = -\omega(y, t) + \frac{h^2}{8G'}\phi_0(y, t);$$

$$\alpha_2(y, t) = -\frac{\partial \omega_0(y, t)}{\alpha_y} + \frac{h^2}{8G'}\psi_0(y, t);$$

$$h(y, t) = -D(1 - \nu)\frac{\partial \omega_1(y, t)}{\partial y} + \frac{h^5}{120}\frac{G}{G'}\left[\frac{\partial \phi_0(y, t)}{\partial y} + \psi_1(y, t)\right]; \tag{26}$$

$$m(y, t) = -D\left[\omega_2(y, t) + \nu \frac{\partial^2 \omega_0(y, t)}{\partial y^2}\right]$$
$$+ \frac{h^5}{60}\frac{G}{(1 - \nu)G'}\left[\phi_1(y, t) + \nu \frac{\partial \psi_0(y, t)}{\partial y}\right] + \rho \frac{\nu'}{(1 - \nu)}\frac{E}{E'}\frac{h^3}{12}\frac{\partial^2 \omega_0(y, t)}{\partial t^2}.$$

Usually it is known beforehand from the statement of the problem which of the values ω_0, α_2, m, h and n are different from zero, and it is necessary to explain how they are connected with the discontinuities ω_1, $\phi_{0,1}$ and $\psi_{0,1}$ of

the functions $\omega(x,y,t)$, $\phi(x,y,t)$, and $\psi(x,y,t)$. Therefore it is advisable to convert the relations (26).

Taking into account the equations (17), after elementary transformations we get

$$
\phi_0(y,t) = \frac{12}{h^3} n(y,t); \quad \omega_1(y,t) = \frac{3}{2G'h} n(y,t) - \alpha_1(y,t);
$$

$$
\psi_0(y,t) = \frac{8G'}{h^2} \left[\frac{\partial \omega_0(y,t)}{\partial y} + \alpha_2(y,t) \right];
$$

$$
\phi_1(y,t) = \rho \frac{12}{h^2} \frac{\partial^2 \omega_0(y,t)}{\partial t^2} - \frac{8G'}{h^2} \frac{\partial}{\partial y} \left[\frac{\partial \omega_0(y,t)}{\partial t} + \alpha_2(y,t) \right];
$$

$$
\psi_1(y,t) = \frac{18}{h^3} \frac{\partial n(y,t)}{\partial y} + \frac{120G'}{h^5 G} h(y,t) - \frac{20}{h^2} \frac{\partial \alpha_1(y,t)}{\partial y};
$$

$$
\omega_2(y,t) = -\frac{m(y,t)}{D} - \nu \frac{\partial^2 \omega_0(y,t)}{\partial y^2} - \frac{12}{15}(1-\nu)\frac{\partial}{\partial y} \left[\frac{\partial \omega_0(y,t)}{\partial y} + \alpha_2(y,t) \right]
$$

$$
- \rho \frac{h^3}{D(1-\nu)} \left[\frac{G}{5G'} + \frac{\nu' E}{12E'} \right] \frac{\partial^2 \omega_0(y,t)}{\partial t^2}.
$$

(27)

Now let the external dynamical load change by the harmonic law with respect to time $Z(x,y,t) = q(x,y)\exp(iwt)$, where ω is the frequency of forced vibrations.

Thanks to the linearity of the equations (17)–(19), the functions $\omega(x,y,t)$, $\phi(x,y,t)$ and $\psi(x,y,t)$ can be represented in the form:

$$
\omega(x,y,t) = W(x,y,)e^{iwt}, \quad \psi(x,y,t) = \Psi(x,y)e^{iwt}, \quad \phi(x,y,t) = \Phi(x,y)e^{iwt}.
$$

Then

$$
\alpha_X(x,y,t) = A_X(x,y)e^{iwt}; \qquad \alpha_Y(x,y,t) = A_Y(x,y)e^{iwt};
$$

$$
M_X(x,y,t) = M_X(x,y)e^{iwt}; \qquad M_Y(x,y,t) = M_Y(x,y)e^{iwt};
$$

$$
H_{XY}(x,y,t) = H(x,y)e^{iwt};
$$

$$
N_X(x,y,t) = N_X(x,y)e^{iwt}; \qquad N_Y(x,y,t) = N_Y(x,y)e^{iwt}.
$$

The amplitudes $A_X(x,y)$, $A_Y(x,y)$, $M_X(x,y)$, $M_Y(x,y)$, $H(x,y)$, $N_X(x,y)$ and $N_Y(x,y)$ are connected with the functions $W(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ as follows:

$$
A_X(x,y) = -\frac{\partial W(x,y)}{\partial x} \frac{h^2}{G'} \phi(x,y);
$$

$$
A_Y(x,y) = -\frac{\partial W(x,y)}{\partial y} \frac{h^2}{G'} \psi(x,y);
$$

(28)

$$M_X(x,y) = -D\left[\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial y^2}\right]W + \frac{h^5}{60}\frac{G}{(1-\nu)G'}\left[\frac{\partial\phi}{\partial x} + \nu\frac{\partial\psi}{\partial y}\right]$$
$$+ \frac{\nu'}{1-\nu}\frac{E}{E'}\frac{h^2}{10}q(x,y) - \rho\omega^2\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{h^2}{12}\omega(x,y);\tag{29}$$

$$M_Y(x,y) = -D\left[\frac{\partial^2}{\partial y^2} + \nu\frac{\partial^2}{\partial x^2}\right]W + \frac{h^5}{60}\frac{G}{(1-\nu)G'}\left[\frac{\partial\psi}{\partial y} + \nu\frac{\partial\phi}{\partial x}\right]$$
$$+ \frac{\nu'}{1-\nu}\frac{E}{E'}\frac{h^2}{10}q(x,y) - \rho\omega^2\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{h^3}{12}W(x,y);\tag{30}$$

$$N_X(x,y) = \frac{h^3}{12}\phi(x,y);\quad N_Y(x,y) = \frac{h^3}{12}\psi(x,y);\tag{31}$$

$$H(x,y) = -D(1-\nu)\frac{\partial^2 W}{\partial x\partial y} + \frac{h^5}{120}\frac{G}{G'}\left[\frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial x}\right].\tag{32}$$

Moreover, the amplitudes $W(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ must satisfy the system of differential equations

$$\frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y} + \rho\frac{12}{h^2}\omega^2 W = -\frac{12}{h^3}q(x,y)\tag{33}$$

$$D\frac{\partial}{\partial x}\Delta W - \frac{h^2 D}{10G'}\left[\frac{\partial^2\phi}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2\phi}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2\psi}{\partial x\partial y}\right] + \frac{h^3}{12}\phi$$
$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial q(x,y)}{\partial x},\tag{34}$$

$$D\frac{\partial}{\partial y}\Delta W - \frac{h^2 D}{10G'}\left[\frac{\partial^2\psi}{\partial y^2} + \frac{1-\nu}{2}\frac{\partial^2\psi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2\phi}{\partial x\partial y}\right] + \frac{h^3}{12}\psi$$
$$= \frac{h^2}{10}\frac{\nu'}{1-\nu}\frac{E}{E'}\frac{\partial q(x,y)}{\partial y}\tag{35}$$

and the boundary conditions

$$x = 0, a;\ W = -D\frac{\partial^2 W}{\partial x^2} + \frac{h^5}{60}\frac{G}{(1-\nu)G'}\frac{\partial\phi}{\partial x} = \psi = 0,\ y\in[0,b]\tag{36}$$

$$y = 0, b;\ W = -D\frac{\partial^2 W}{\partial y^2} + \frac{h^5}{60}\frac{G}{(1-\nu)G'}\frac{\partial\psi}{\partial y} = \phi = 0,\ x\in[0,a].\tag{37}$$

In the zone of a linear defect, the amplitudes $W(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ can have discontinuities $(0 < C_1 \le y \le c_2 < b)$ (7) which are connected with the possible discontinuities $W_0(y)$, $A_1(y)$, $A_2(y)$, $M(y)$, $H(y)$ and $N(y)$

of the amplitudes of the corresponding deflection $\omega(x, y, t)$, the bending angles $\alpha_X(x, y, t)$ and $\alpha_(Y)(x, y, t)$, the bending moment $M_X(x, y, t)$, the torque $H_{XY}(x, y, t)$ and the transversal forces $N_X(x, y, t)$ according to the relations

$$\phi_0(y) = \frac{12}{h^3} N(y); \ \ W_1(y) = \frac{3}{2G'h} N(y) - A_1(y);$$

$$\psi_0(y) = \frac{8G'}{h^2} \left[\frac{d}{dy} W_0(y) + A_2(y) \right];$$

$$\phi_1(y) = -\frac{12}{h^2} \rho \omega^2 W_0(y) - \frac{8G'}{h^2} \frac{d}{dy} \left[\frac{d}{dy} W_0(y) + A_2(y) \right];$$

$$\psi_1(y) = \frac{18}{h^3} \frac{d}{dy} N(y) + \frac{120}{h^5} \frac{G'}{G} H(y) - \frac{20}{h^2} \frac{d}{dy} A_1(y);$$

$$W_2(y) = -\frac{M(y)}{D} - \nu \frac{d^2}{dy^2} W_0(y) - \frac{12}{15} (1 - \nu) \frac{d}{dy} \left[\frac{d}{dy} W_0(y) + A_2(y) \right]$$
$$- \rho \frac{h^3}{D(1 - \nu)} \omega^2 \left[\frac{1}{5} \frac{G}{G'} + \frac{\nu'}{12} \frac{E}{E'} \right] W_0(y).$$

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